RELATIVITY THERMODYNAMICS AND COSMOLOGY

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I

INTRODUCTION

1. The subject-matter

It is the threefold purpose of this essay, first to give a coherent and fairly inclusive account of the well-known and generally accepted portions of Einstein's theory of relativity, second to treat the extension of thermodynamics to special and then to general relativity, and third to consider the applications both of relativistic mechanics and relativistic thermodynamics in the construction and interpretation of cosmological models.

The special theory of relativity will first be developed in the next three chapters, which are devoted respectively to the kinematical, mechanical, and electromagnetic consequences of the two postulates of special relativity. In Chapter II, under the general heading 'The Special Theory of Relativity', the two postulates of the theory will be presented, together with a brief statement of the confirmatory empirical evidence in their favour; their kinematical consequences will then be developed, firstly using the ordinary language which refers kinematical occurrences to some selected set of three Cartesian axes and the set of clocks that can be pictured as moving therewith, and secondly using the more powerful quasi-geometrical language provided by the concept of a four-dimensional space-time continuum. In Chapter III, Special Relativity and Mechanics, we shall develop first the mechanics of a particle and then those of a mechanical continuum from a postulatory basis which is obtained by adding the ideas of the conservation of mass and of the equality of action and reaction to the kinematics of special relativity. No appeal to analogies with electromagnetic results will be needed to obtain the complete treatment, and the considerations will be maintained on a macroscopic level throughout. Finally, in Chapter IV, Special Relativity and Electrodynamics, we shall complete our treatment of the more familiar subject-matter of the special theory, by developing the close relationships between special relativity and electromagnetic theory. The first part of this chapter will be devoted to the incorporation of the Lorentz electron theory in the framework of special relativity, a procedure which tacitly assumes a respectable amount of validity still inherent in classical microscopic considerations in spite of the evident necessity for a successful quantum electrodynamics; and the second part of the chapter will be given to the development of Minkowski's macroscopic theory of moving electromagnetic media based on the extension to special relativity of Maxwell's original treatment of stationary matter.

In Chapter V, Special Relativity and Thermodynamics, we then turn to less familiar consequences of the special theory. In the first part of the chapter we consider the effect of relativity, even on the classical thermodynamics of stationary systems, in providing—through the relativistic relation between mass and energy—a natural starting-point for the energy content of thermodynamic systems, and a method for computing the energy changes accompanying physical-chemical processes from a knowledge of changes in mass. This makes it feasible to consider such problems as the thermodynamic equilibrium between hydrogen and helium, and that between matter and radiation—assuming the possibility of their interconversion—and treatments of these questions are given. In the second part of the chapter we undertake the actual extension of thermodynamics to special relativity in order to obtain a thermodynamic theory for the treatment of moving systems. Although the results which are to be derived by such an application of relativity to thermodynamics were considered by Planck and by Einstein only two years after the original presentation of the special theory, but little further attention has been paid to them. Indeed, the very essential difference between the equation

$$E = \frac{E_0}{\sqrt{(1-u^2/c^2)}} \tag{1.1}$$

giving the energy of a moving particle E in terms of its proper energy E_0 and velocity u, and the quite different equation

$$Q = Q_0\sqrt{(1-u^2/c^2)} \tag{1.2}$$

connecting a quantity of heat Q with proper heat Q_0 and velocity, has apparently not always been appreciated. The common lack of familiarity with this branch of relativity has doubtless been due to the absence of physical situations where its applications were necessary. For the later extension of thermodynamics to general relativity, nevertheless, a knowledge of the Planck-Einstein thermodynamics is essential, and at the end of this chapter we introduce a four-dimensional expression for the second law of thermodynamics in special relativity on which the extension to general relativity can later be based.

In Chapter VI, The General Theory of Relativity, we consider the fundamental principles of the general theory of relativity together with some of its more elementary applications. Part I of the chapter will treat the three corner-stones—the principle of covariance, the principle of equivalence, and the hypothesis of Mach—on which the theory rests. In agreement with the point of view first stated by Kretchsmann, the principle of covariance will be regarded as having a logically formal character which can imply no necessary physical consequences, but at the same time in agreement with Einstein we shall emphasize the importance of using covariant language in searching for the axioms of physics, in order to eliminate the insinuation of unrecognized assumptions which might otherwise result from using the language of particular coordinates. The discussion of the principle of equivalence will emphasize not only its empirical justification as an immediate and natural generalization of Galileo's discovery that all bodies fall at the same rate, but will also lay stress on the philosophical desirability of the principle in making it possible to maintain the general idea of the relativity of all kinds of motion including accelerations and rotations as well as uniform velocities. The designation 'Mach hypothesis' will be used to denote the general idea that the geometry of space-time is determined by the distribution of matter and energy, so that some kind of field equations connecting the components of the metrical tensor $g_{\mu\nu}$ with those of the energy-momentum tensor $T_{\mu\nu}$ are in any case implied. In presenting the field equations actually chosen by Einstein, the cosmological or Λ-term will be introduced and retained in many parts of the later treatment, not because of direct empirical or theoretical evidence for the existence of this term, but rather on account of the logical possibility of its existence and the necessity for its presence in the case of certain cosmological models which at least deserve discussion. Part II of Chapter VI will be given to elementary applications of general relativity. These will include a discussion of the clock paradox which proved so puzzling during the interval between the developments of the special and general theories of relativity. Treatment will also be given to Newton's theory of gravitation as a first and very close approximation to Einstein's theory, and the three crucial tests of general relativity will be considered.

Chapter VII, Relativistic Mechanics, will be divided into two parts on general mechanical principles and on solutions of the field equations. In Part I, after illustrating the nature of the energy-momentum tensor and of the fundamental equations of mechanics by application to the behaviour of a perfect fluid, the equations of mechanics will be re-expressed in the form containing the pseudo-tensor density of potential gravitational energy and momentum $\mathfrak{t}_\mu^\nu$ permitting us then to obtain conservation laws for Einstein's generalized expressions for energy and momentum, to exhibit the relation between energy and gravitational mass, and to show the reduction of the energy of a system in the case of weak fields to the usual Newtonian form including potential gravitational energy. In Part II of Chapter VII, Einstein's general solution for the field equations in the case of weak fields will first be presented. This will then be followed by a discussion of the properties of the solutions that can be obtained in special cases of spherical symmetry and the like, including useful explicit expressions for the Christoffel symbols and components of the energy-momentum tensor which then apply.

Chapter VIII, Relativistic Electrodynamics, will present the further extensions to general relativity both for the Lorentz electron theory and for the Minkowski macroscopic theory. This will be followed by a number of applications including the derivation of an expression for the relativistic energy-momentum tensor for black-body radiation, together with discussions of the gravitational interaction of light rays and particles, and of the generalized Doppler effect, these latter being matters of special importance for the interpretation of astronomical findings.

Chapter IX, Relativistic Thermodynamics, considers the extension of thermodynamics from special to general relativity together with its applications. The principles of relativistic mechanics themselves are taken as furnishing the analogue of the ordinary first law of classical thermodynamics; and the analogue of the second law is provided by the covariant generalization of the four-dimensional form in which the second law can be expressed in the case of special relativity. Since the above choice for the analogue of the first law introduces only generally accepted results of relativity, the whole character of relativistic thermodynamics is determined by the relativistic second law. The axiom chosen for this law is hence carefully examined as to meaning; its present status is discussed as being the direct covariant re-expression and therefore the most probable generalization of the ordinary second law; and its future status as a postulate to be verified or rejected on empirical grounds is emphasized. Following this discussion, applications are made to illustrate the characteristic differences between the results of relativistic thermodynamics, and those which might at first sight seem probable on the basis of a superficial extrapolation of conclusions familiar in the classical thermodynamics. Thus in the case of static systems, although we shall find the physical-chemical equilibrium between reacting substances—as measured by a local observer—unaltered from that which would be predicted classically, we shall find on the other hand as a new phenomenon the necessity for a temperature gradient at thermal equilibrium to prevent the flow of heat from regions of higher to those of lower gravitational potential, in agreement with the qualitative idea that all forms of energy have weight as well as mass. Turning to non-static systems we shall then show the possibility for a limited class of thermodynamic processes which can occur both reversibly and at a finite rate—in contrast to the classical requirement of an infinitely slow rate to secure that maximum efficiency which would permit a return both of the system and its surroundings to their initial state. We shall later find that the principles of relativistic mechanics themselves provide a justification for this new thermodynamic conclusion, since they permit the construction of cosmological models which would expand to an upper limit and then return with precisely reversed velocities to earlier states. Finally, in the case of irreversible processes taking place at a finite rate, we shall discover possibilities for a continuous increase in entropy without ever reaching an unsurpassable value of that quantity—in contrast to the classical conclusion of a final quiescent state of maximum entropy. This new kind of thermodynamic behaviour, which may be regarded as mainly resulting from the known modification of the principle of energy conservation by general relativity, will also find later illustration among the cosmological models predicted as possible by the principles of relativistic mechanics.

In Chapter X, Application to Cosmological Models, we complete the text except for some appendices containing useful formulae and constants. In the first part of this chapter we shall show that the only possible static homogeneous models for the universe are the original ones of Einstein and de Sitter, and shall discuss some of their properties which are important without reference to the adequacy of the models as pictures of the actual universe. We shall then turn to the consideration of non-static homogeneous models which can be constructed so as to exhibit a number of the properties of the actual universe, including, of course, the red shift in the light from the extra-galactic nebulae. Special attention will be given to the method of correlating the properties of such models with the results of astronomical observations although the details for obtaining the latter will not be considered. Attention will also be paid to the theoretically possible properties of such models, without primary reference to their immediate applicability in the correlation of already observed phenomena, since no models at the present stage of empirical observation can supply more than very provisional pictures of the actual universe.

The most important omission in this text, from the subjects usually included in applications of the special theory of relativity, is the relativistic treatment of the statistical mechanics of a gas, as developed by Jüttner and to some extent by the present writer.† The omission is perhaps justified by our desire in the present work to avoid microscopic considerations as far as possible, and by the existing absence of many physical situations where the use of this logically inevitable extension of relativity theory has as yet become needed.

In the case of the general theory of relativity, the most important omission lies in neglecting the attempts which have been made to construct a unified field theory, in which the phenomena of electricity as well as gravitation would both be treated from a combined 'geometrical' point of view. Up to the present, nevertheless, these attempts appear either to be equivalent to the usual relativistic extension of electromagnetic theory as given in the present text, or to be—although mathematically interesting—of undemonstrated physical importance. Furthermore, it is hard to escape the feeling that a successful unified field theory would involve microscopic considerations which are not the primary concern of this book.

The most important inclusions, as compared with older texts on relativity, consist in the extension of thermodynamics to general relativity, and the material on non-static models of the universe. Other additions are provided by the calculations of thermodynamic

† Jüttner, *Ann. d. Physik*, **34**, 856 (1911); Tolman, *Phil. Mag.*, **28**, 583 (1914).