CONCEPTS AND APPLICATIONS OF FINITE ELEMENT ANALYSIS

Second Edition

Robert D. Cook

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E8054129

John Wiley & Sons

New York / Chichester / Brisbane / Toronto / Singapore

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Library of Congress Cataloging in Publication Data:

Cook, Robert Davis.
Concepts and applications of finite element analysis.

Includes bibliographical references and index.
1. Structures, Theory of. 2. Finite element method. I. Title.
TA646.C66 1981 624.1'71 80-26255
ISBN 0-471-03050-3

Printed in the United States of America

10 9 8 7 6 5

Preface

The finite element method is a generally applicable method for getting numerical solutions. Problems of stress analysis, heat transfer, fluid flow, electric fields, and others have been solved by finite elements. This book emphasizes stress analysis and structural mechanics. Other areas are treated in a way that is easy for stress analysts to understand. The formulation and computation procedures of finite elements are much the same in all areas of application.

This text is introductory and is inclined toward practical application. Theory is presented as needed. The book contains enough material for a two-semester course sequence. By studying this book, an engineer can learn to use finite elements with confidence and effectiveness, and those who want to do advanced work will have a sound physical understanding from which to proceed.

The background for the book is as follows. Undergraduate courses in statics, dynamics, and mechanics of materials must be mastered. Mathematically, little is needed but the ability to differentiate and integrate sines, cosines, and polynomials. I presume knowledge of matrix operations such as multiplication, transposition, differentiation, and the meaning of an inverse (the calculation procedure for inversion is not essential). Students should be competent in Fortran programming, to the extent of being able to use subroutines, COMMON blocks, and storage such as disc files. Other areas—theory of elasticity, plates and shells, energy methods, numerical analysis—are desirable but not essential. Fortunately, we seldom call on these areas and, when we do, it is usually for their elementary concepts.

The following list more fully describes the content and orientation of the book.

- It is more concerned with analysis of continua than with special methods for framed structures. However, truss and beam elements are often used as simple but useful vehicles for explanations.
- Linear static analysis is emphasized.
- Elements discussed in detail are based on assumed displacement fields, are not restricted to a special shape, and have no ''excessive'' nodal continuities. Of the multitude of elements available, only a few are discussed with emphasis on the isoparametric type.
- To show unambiguously the steps of certain processes, Fortran coding is given when it is simple, helpful, and not very long. This coding may not be the most recent or most efficient. Complete programs are available from other sources.
- Topics are emphasized that seem useful and well enough established to be enduring. Less durable topics, such as explanations of currently popular computer programs, are omitted.

I have tried to avoid second-edition prolixity. First-edition topics not appropriate to the level and orientation of the book have been omitted except for citations of the literature. New topics have been added only if clearly suitable (see Chapters 14, 17, and 18). There are more numerical examples. Less obvious are rearrangement of sections and improvement in wording. Also incorporated are useful thoughts culled over the years from the literature and from the comments of students and colleagues. Many homework problems have been added; answers are given in the back of the book. The problems illustrate principles and procedures and foster insight. Most require neither a computer nor extensive numerical calculations.

The practical success of the finite element method depends on a reliable computer program. Students in a first course say that a programming project is an excellent learning device. They prefer to write and test a simple but complete program instead of coding subroutines. Sample elements and situations on which to base a programming project are listed here. Each can invoke Gauss quadrature for element stiffness formation if so desired. For students with interests outside structural mechanics, nonstructural problems of similar extent are possible.

1. Standard beam element, two d.o.f. (degrees of freedom) per node.
2. Same as item 1, but add internal d.o.f.
3. Shear beam on an elastic foundation, one d.o.f. per node. (See Fig. 9.5.2. For the beam, consider only w and energy stored by γ_{xy}.)
4. Beam as a degenerate isoparametric plate element (Figs. 9.5.1 or 9.5.2).
5. Tapered bar element, two nodal and one nodeless d.o.f.
6. Plane disc of constant or variable thickness, with annular elements and torsional loads only.
7. Same as item 6, but add nodeless d.o.f.
8. Same as item 6, but use radial loads only.
9. Same as item 6, but add an elastic foundation and allow only transverse shear stiffness and lateral load.
10. Same as item 5, but consider torsional action only.
11. Plane frame, three d.o.f. per node.
12. Same as item 11, but use the element of part 4.
13. Rectangular elements, one d.o.f. per corner, for the harmonic equation (soap film, seepage flow, and so on).
14. Application of Eq. 4.10.4 to a tapered bar with a distributed axial load.

A programming project can also be assigned in a second course. A possible project involves Fourier series components of loading and superposition of the separate solutions. Examples include plane stress problems that involve circular regions with annular elements and asymmetric loads, and finite strip analysis of simply supported plates. A project that involves natural frequencies of vibration or dynamic response is

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also good. Alternatively, if a general purpose program with interactive graphics is available, students will profit by using it to solve a practical problem.

I am grateful to the students whose questions led to better explanations and improved homework problems. The substance of the book comes mostly from published papers. Their authors have my appreciation. In language matters, I have become inordinately sensitive to the words "that" and "which", but thank the Wiley editors for teaching me the distinction. Rules given by Strunk and White in *The Elements of Style,* especially Rule 13, helped tighten the prose. Of the six typists who worked on the manuscript, Pat Klitzke did quality work quickly.

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Notation

This is a list of principal symbols. Locally used notation and modifications (as by addition of a subscript) are defined where used. Similarly, a symbol that has different meanings in different contexts is defined where it is used. Matrices are denoted by boldface type.

MATHEMATICAL SYMBOLS

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[ ]	A rectangular or square matrix.
⌈ ⌋	A diagonal matrix.
⌊ ⌋	A row vector.
{ }	A column vector. *Note.* $\begin{Bmatrix} u \\ v \end{Bmatrix} \equiv \{u \ v\}$.
$[\ ]^{-1}$	Matrix inverse.
$[\ ]^T$	Matrix transpose (also applies to row and column vectors).
$[\ ]^{-T}$	Inverse transpose; $[\ ]^{-T} \equiv ([\ ]^{-1})^T \equiv ([\ ]^T)^{-1}$.
	Note. The foregoing brackets and braces may be omitted from submatrices and from the separate matrices of a matrix product that is bracketed.
$\dot{}$	Time differentiation; for example, $\dot{u} = du/dt$, $\ddot{u} = d^2u/dt^2$.
,	Partial differentiation if the following subscript(s) is literal; for example, $w_{,x} = \partial w/\partial x$, $w_{,xy} = \partial^2 w/\partial x \partial y$.
$\bar{}$	Amplitude; for example, $u = \bar{u} \sin \omega t$. (Other meanings are numerous.)
$\left\{\dfrac{\partial \Pi_p}{\partial \mathbf{a}}\right\}$	Represents $\left\{\dfrac{\partial \Pi_p}{\partial a_1} \dfrac{\partial \Pi_p}{\partial a_2} \cdots \dfrac{\partial \Pi_p}{\partial a_n}\right\}$, where Π_p is a scalar function of parameters $a_1, a_2, \ldots, a_n$.

LATIN SYMBOLS

A	Area.
$[\mathbf{A}]$	Relates $\{\mathbf{d}\}$ to $\{\mathbf{a}\}$; $\{\mathbf{d}\} = [\mathbf{A}]\{\mathbf{a}\}$.
$\{\mathbf{a}\}$	Generalized coordinates.
B	Semibandwidth of a matrix.
$[\mathbf{B}]$, $[\mathbf{B}_a]$	The "strain-displacement" matrix (Section 4.3).
C_n	Continuity of degree n (Section 4.2).
$C(\mathbf{K})$	Condition number of $[\mathbf{K}]$ (Section 15.6).
$[\mathbf{C}]$	Damping matrix or constraint matrix.
D	Displacement.
d.o.f.	Degree (or degrees) of freedom.
$\{\mathbf{D}\}$	Nodal d.o.f. of a structure (global d.o.f.).

{**d**}	Nodal d.o.f. of an element.	
[$\mathcal{D}$]	Flexural rigidity matrix of a plate.	
e	Elongation.	
E, E_s	Elastic modulus (E), secant modulus (E_s) (Fig. 13.10.1).	
[**E**]	Matrix of elastic stiffnesses (Section 1.5).	
{**F**}	Body forces per unit volume.	
{**f**}	A displacement field; {**f**} = {u v w} in 3-space.	
G	Shear modulus.	
[**H**]	Used in the way that [**Q**] is used.	
I	Moment of inertia of a beam.	
[**I**]	The unit matrix (also called the identity matrix).	
J	Determinant of [**J**], known as the Jacobian.	
[**J**]	The Jacobian matrix.	
k	Spring stiffness. Thermal conductivity (Chapter 17).	
[**K**]	Structure (global) stiffness matrix.	
[**k**]	Element stiffness matrix (conductivity matrix in Chapter 17).	
[$\mathbf{K}_\sigma$]	Structure (global) stress stiffness matrix.	
[$\mathbf{k}_\sigma$]	Element stress stiffness matrix.	
L, ℓ	Length.	
ℓ, m, n	Direction cosines.	
[**M**]	Structure (global) mass matrix.	
[**m**]	Element mass matrix.	
M, N	Bending moment (M), membrane force (N).	
MBAND	Same as B.	First used in Chapter 2.
N, NEQ	Number of equations.	
NDOF	Number of d.o.f. per node.	
NUMEL	Number of elements in a structure.	
NUMNP	Number of nodes in a structure.	
[**N**], ⌊**N**⌋	Matrix of shape functions; {**f**} = [**N**]{**d**}.	
O	Order; for example, $O(h^2)$ = a term of order h^2.	
[**O**], {**O**}	Null matrix, null vector.	
p_i, q_i	Concentrated forces on node i (Chapters 1 and 2).	
P	Force.	
{**P**}	Vector of externally applied loads on structure nodes.	
[**Q**], {**Q**}	Matrix of various uses. Defined where used.	
q	Lateral load (surface or line).	
R	Residual (Section 15.9; Chapter 18).	
{**R**}	Total load on structure nodes; {**R**} = {**P**} + Σ {**r**}.	
{**r**}	Forces applied by element to nodes (Eq. 4.3.5).	
{$\bar{\mathbf{r}}$}	{$\bar{\mathbf{r}}$} = −{**r**} (Section 2.4).	
S	Surface.	
s, t	Coordinate directions, usually Cartesian.	
T	Temperature	
t	Thickness. Time.	

[**T**]	Transformation matrix.
U, U_0	Strain energy, strain energy per unit volume.
u, v, w	Displacement components.
V	Volume.
x, y, z	Cartesian coordinates.
x', y', z'	Local Cartesian coordinates.

GREEK SYMBOLS

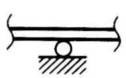
α	Coefficient of thermal expansion.
α, β, γ	Area coordinates (Section 7.9).
β	Angle, relaxation factor, foundation modulus, and the like.
$[\boldsymbol{\Gamma}]$	The Jacobian inverse; $[\boldsymbol{\Gamma}] = [\mathbf{J}]^{-1}$.
Δ	Small change operator; for example, Δt is a time increment.
δ	Virtual operator; for example, δu is a virtual displacement.
$\{\boldsymbol{\epsilon}\}$, $\{\boldsymbol{\epsilon}_0\}$	Strains, initial strains (Section 1.6).
θ	Angle. Circumferential coordinate.
$\{\boldsymbol{\kappa}\}$	Vector of curvatures (as in plate bending).
λ	An eigenvalue. A Lagrange multiplier.
ν	Poisson's ratio.
ξ, η, ζ	Isoparametric coordinates (Chapter 5).
Π	A functional. (Π_p = total potential energy).
π	3.1415926536 . . .
ρ	Mass density.
$\{\boldsymbol{\sigma}\}$, $\{\boldsymbol{\sigma}_0\}$	Stresses, initial stresses (Section 1.6).
ϕ	A dependent variable. Meridian angle of a shell (Chapter 10).
$\{\boldsymbol{\Phi}\}$	Vector of surface tractions (Section 1.3).
ω	Circular frequency in radians per second.

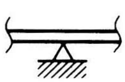
GRAPHIC SYMBOLS

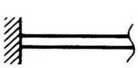

Force or displacement vector.


Moment or rotation vector (by the right-hand rule).


Spring or elastic support.


Roller support (resists positive or negative normal force).


Pinned support (resists all forces but does not resist moment).


Fixed support (resists all forces and moments).

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