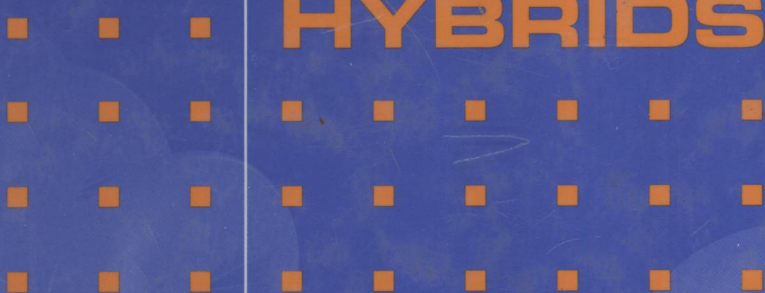
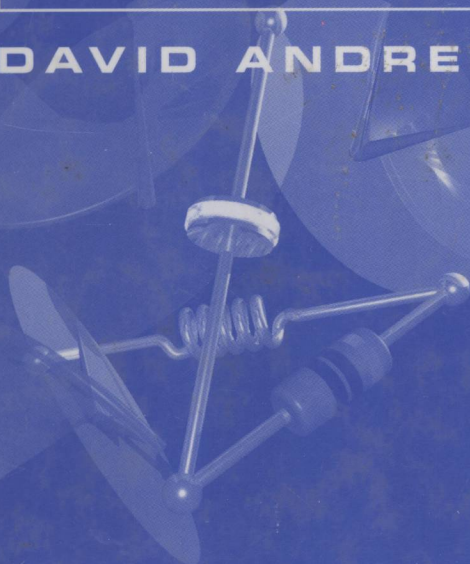


# LUMPED ELEMENT QUADRATURE HYBRIDS



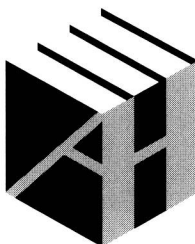
DAVID ANDREWS



TN454  
A565

# Lumped Element Quadrature Hybrids

David Andrews



**ARTECH  
HOUSE**

BOSTON | LONDON  
artechhouse.com



E2009003721

**Library of Congress Cataloging-in-Publication Data**

A catalog record for this book is available from the U.S. Library of Congress.

**British Library Cataloguing in Publication Data**

A catalogue record for this book is available from the British Library.

**Cover design by Yekaterina Ratner**

**© 2006 ARTECH HOUSE, INC.**

**685 Canton Street**

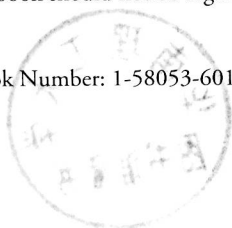
**Norwood, MA 02062**

All rights reserved. Printed and bound in the United States of America. No part of this book may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information storage and retrieval system, without permission in writing from the publisher.

All terms mentioned in this book that are known to be trademarks or service marks have been appropriately capitalized. Artech House cannot attest to the accuracy of this information. Use of a term in this book should not be regarded as affecting the validity of any trademark or service mark.

International Standard Book Number: 1-58053-601-8

10 9 8 7 6 5 4 3 2 1



## **Lumped Element Quadrature Hybrids**



For a complete listing of recent titles in the *Artech House Microwave Library*,  
turn to the back of this book.

# Preface

Quadrature hybrids have wide application in radio frequency (RF) and microwave circuits and systems. In answer to this need, considerable attention has been paid to distributed circuits with the quadrature properties, particularly for microwave applications. RF engineers too find quadrature hybrids useful, although they prefer lumped element circuits for reasons of size. However, they have been disappointed with the availability of material on the subject, which has received far less attention than the distributed counterparts. This is a pity as there is much to say on the subject. In addition, with the trend towards miniaturization of microwave circuits, lumped element circuits are of increasing interest. Microwave engineers will be surprised by the breadth of applications for lumped element quadrature hybrids, which offer the prospects of reduction in circuit size, ease of fabrication, and remarkable performance. RF engineers will also find the material presented in this book useful as, at the time of writing, there has been no other substantial text written on the subject.

The inspiration for this work grew originally out of my experience with RF and microwave amplifier design. Quadrature hybrids are a useful component for this purpose, where two amplifier stages placed between two quadrature hybrids form what is known as a *balanced stage*, after an article by K. Kurokawa, "Design Theory of Balanced Transistor Amplifiers," in the *Bell System Technical Journal*, Vol. 44, No. 8, October 1965, pages 1675–1698. The structure exhibits enhanced input and output match characteristics and double the power output of a single stage. In a system, the performance of a

cascade of balanced stages is greatly enhanced, as the reflection between stages is much reduced. However, the desire for wider operating frequency ranges places severe limitations on the use of distributed circuits, so other solutions were sought. The quest for improved bandwidth led to my personal discovery that lumped element circuits of modest complexity could perform the quadrature hybrid function over much greater relative frequency ranges. It was discovered too that there was a lack of information on the subject, and this became the grounds for a research project at the University of Surrey, England. During the course of the research, the lack of information was confirmed, although it became evident that there was a little information on the subject that had not been publicized well. This book is based largely on my Ph.D. thesis of a similar title, although more material has been added since, particularly on the subject of phase delay circuits. Some material found in the thesis has been omitted from this book, as the focus has been on equipping the designer with the most useful aspects of the subject. There was no value in adding “man of straw” arguments to the book.

The book is structured in a similar manner to the treatment of filter theory, because the subjects have much in common. Chapter 1 gives an overview of the various forms of quadrature hybrids and their applications, then shows a method for assessing the relative performance of a particular design. Chapter 2 examines the constraints that theory places on quadrature hybrid circuits and, more particularly, lumped element forms. Chapter 3 is a treatment of the subject of approximation, a concept familiar to the filter designer. Quadrature hybrids are also filter circuits, and their performance is one of optimization rather than perfection. Chapter 4 deals with the subject of circuit synthesis and shows how the various approximation functions can be given their expression in electrical networks. Chapter 5, titled “Practical Design,” might also be titled “Realizations” and shows how the theoretical circuits can be made in practice. A number of concept circuits are described, illustrating most of the aspects described in the theoretical chapters. The final chapter, “Special Topics,” shows how the theory and application of quadrature hybrids can be extended to related matters, which are of themselves also useful.

Like any other text on filter theory, this book makes considerable use of mathematics. The reader is encouraged to grasp the theory as it will improve his or her understanding of the subject. However, in recognition that many engineers prefer a quick route to design, graphs and tables have been provided to help with the simpler applications. To illustrate the mathematics, a number of worked examples are provided. The reader might notice a lack of reference to computer-aided design (CAD) tools in the book. This is because

these vary widely in scope and capability. CAD may be used at any stage to check element values, optimize numerical solutions, and compensate for practical limitations. Such tools are no substitute for an understanding of the underlying principles. This is not to say that CAD tools are not useful in the design of quadrature hybrids. Indeed, mathematical and circuit design software were both used in the solution of numerical problems and optimization of element values for circuits described in this book.

The reader is assumed to have a graduate-level knowledge of RF and microwave circuits and theory. I never intended to take the material back to first principles. For further treatment of the underlying principles, the references and bibliography should be consulted. Where references have been used, they usually refer to the original work. However, this is not always the case, as the original work is sometimes contained in an obscure publication, or it contains numerous errors and, as such, is likely to lead to confusion. In such cases, the original reference is replaced with a more readily available or accurate one.

In the preparation of circuit diagrams, use has been made of an international style of symbols such as might be found in electronic magazines. The main reason for this is cosmetic; in addition, because engineers have used them for some time now, the eye more readily appreciates them. Thus, the rectangular box has been discarded in favor of the zigzag pattern for the resistor symbol, and the former has been reserved to denote generic impedances. The curly form of the inductor symbol is used rather than a series of semicircles, which was only introduced for the convenience of the drawing offices of a bygone era. The book makes frequent use of mutual inductance, indicated by a double-headed arrow between the coupled inductors. The coupling is indicated either as an absolute value or a coupling factor. To avoid confusion, coupled inductors are always drawn in such a way as to make the dot notation unnecessary.

I would like to acknowledge the help given during the preparation of this book. Mr. Allen Podell, who himself has authored several patents on hybrid circuits, has helped me particularly with the practical aspects of phase delay circuits. I would like to thank Mr. Mike Davis, a fellow director at Vectawave Technology Limited, who fabricated several of the circuit boards used in the original research, the results of which are featured in the practical design section of this book. I would like to acknowledge the contribution of my former supervisor, Professor Colin Aitchison, who guided the development of the original research upon which this book is based. Finally, I would like to acknowledge the help of those many engineers and teachers who have inspired me along the way.



*Generalized Filter Design by Computer Optimization*, Djuradj Budimir

*High-Linearity RF Amplifier Design*, Peter B. Kenington

*High-Speed Circuit Board Signal Integrity*, Stephen C. Thierauf

*Intermodulation Distortion in Microwave and Wireless Circuits*,  
José Carlos Pedro and Nuno Borges Carvalho

*Introduction to Modeling HBTs*, Matthias Rudolph

*Lumped Elements for RF and Microwave Circuits*, Inder Bahl

*Lumped Element Quadrature Hybrids*, David Andrews

*Microwave Circuit Modeling Using Electromagnetic Field Simulation*,  
Daniel G. Swanson, Jr. and Wolfgang J. R. Hoefer

*Microwave Component Mechanics*, Harri Eskelinen and  
Pekka Eskelinen

*Microwave Differential Circuit Design Using Mixed-Mode  
S-Parameters*, William R. Eisenstadt, Robert Stengel,  
and Bruce M. Thompson

*Microwave Engineers' Handbook, Two Volumes*,  
Theodore Saad, editor

*Microwave Filters, Impedance-Matching Networks, and Coupling  
Structures*, George L. Matthaei, Leo Young, and E.M.T. Jones

*Microwave Materials and Fabrication Techniques, Second Edition*,  
Thomas S. Laverghetta

*Microwave Mixers, Second Edition*, Stephen A. Maas

*Microwave Radio Transmission Design Guide*, Trevor Manning

*Microwaves and Wireless Simplified, Third Edition*,  
Thomas S. Laverghetta

*Modern Microwave Circuits*, Noyan Kinayman and M. I. Aksun

*Modern Microwave Measurements and Techniques, Second Edition*,  
Thomas S. Laverghetta

*Neural Networks for RF and Microwave Design*, Q. J. Zhang and K. C. Gupta

*Noise in Linear and Nonlinear Circuits*, Stephen A. Maas

*Nonlinear Microwave and RF Circuits, Second Edition*, Stephen A. Maas

*QMATCH: Lumped-Element Impedance Matching, Software and User's Guide*, Pieter L. D. Abrie

*Practical Analog and Digital Filter Design*, Les Thede

*Practical Microstrip Design and Applications*, Günter Kompa

*Practical RF Circuit Design for Modern Wireless Systems, Volume I: Passive Circuits and Systems*, Les Besser and Rowan Gilmore

*Practical RF Circuit Design for Modern Wireless Systems, Volume II: Active Circuits and Systems*, Rowan Gilmore and Les Besser

*Production Testing of RF and System-on-a-Chip Devices for Wireless Communications*, Keith B. Schaub and Joe Kelly

*Radio Frequency Integrated Circuit Design*, John Rogers and Calvin Plett

*RF Design Guide: Systems, Circuits, and Equations*, Peter Vizmuller

*RF Measurements of Die and Packages*, Scott A. Wartenberg

*The RF and Microwave Circuit Design Handbook*, Stephen A. Maas

*RF and Microwave Coupled-Line Circuits*, Rajesh Mongia, Inder Bahl, and Prakash Bhartia

*RF and Microwave Oscillator Design*, Michal Odyniec, editor

*RF Power Amplifiers for Wireless Communications*, Steve C. Cripps

*RF Systems, Components, and Circuits Handbook*, Ferril A. Losee

*Stability Analysis of Nonlinear Microwave Circuits*, Almudena Suárez and Raymond Quéré

*TRAVIS 2.0: Transmission Line Visualization Software and User's Guide, Version 2.0*, Robert G. Kaires and Barton T. Hickman

*Understanding Microwave Heating Cavities*, Tse V. Chow Ting Chan  
and Howard C. Reader

For further information on these and other Artech House titles,  
including previously considered out-of-print books now available  
through our In-Print-Forever® (IPF®) program, contact:

Artech House Publishers

685 Canton Street

Norwood, MA 02062

Phone: 781-769-9750

Fax: 781-769-6334

e-mail: [artech@artechhouse.com](mailto:artech@artechhouse.com)

Artech House Books

46 Gillingham Street

London SW1V 1AH UK

Phone: +44 (0)20 7596 8750

Fax: +44 (0)20 7630 0166

e-mail: [artech-uk@artechhouse.com](mailto:artech-uk@artechhouse.com)

Find us on the World Wide Web at: [www.artechhouse.com](http://www.artechhouse.com)

---

# Contents

	<b>Preface</b>	<b><i>ix</i></b>
<b>1</b>	<b>Overview of Quadrature Hybrids</b>	<b>1</b>
1.1	From Distributed to Lumped Element Design	1
1.2	Applications	8
1.3	Analysis of Quadrature Error	13
1.4	Conclusion	15
	References	16
<b>2</b>	<b>Basic Theory</b>	<b>19</b>
2.1	Lossless Circuits	19
2.2	Hybrid Circuits	20
2.3	Lumped Element Quadrature Hybrids	24
2.4	Approximate-Phase Quadrature Hybrids	30
2.5	Conclusion	33

	References	34
<b>3</b>	<b>Approximations</b>	<b>35</b>
3.1	Optimization Goal	36
3.2	Polynomial Form of $F(\omega)$	36
3.3	Rational Form of $F(\omega)$	38
3.4	Second-Order Optimum Rational Function	45
3.5	Higher-Order Optimum Rational Functions	48
3.6	Transfer Functions in Terms of the Complex Variable	64
3.7	Numerator Part of the Optimum Transfer Functions	67
3.8	Transfer Functions of Optimum Approximate-Phase Hybrids	71
3.9	Conclusion	72
	References	73
<b>4</b>	<b>Passive Synthesis</b>	<b>75</b>
4.1	Even- and Odd-Mode Analysis	76
4.2	The First-Order Quadrature Hybrid	79
4.3	Higher-Order Lowpass Prototypes	81
4.4	Use of Transmission Lines	93
4.5	Optimum Rational Function Second-Order Prototypes	96
4.6	Higher-Order Optimum Rational Function Hybrids	104
4.7	Hybrid Synthesis Using Cascaded Sections	107
4.8	Cascades of First-Order Sections	109
4.9	Cascades of Second-Order Sections	112
4.10	Further Cascade Arrangements	113
4.11	Approximate-Phase Hybrids	117

4.12	Approximate-Amplitude Hybrids Based on Phase Delay Networks	124
4.13	Conclusion	126
	References	127
<b>5</b>	<b>Practical Design</b>	<b>129</b>
5.1	The First-Order Circuit Coupled Inductor Design	130
5.2	A 435-MHz Ground Inductor Hybrid Design	135
5.3	A 1.27-GHz First-Order Microstrip Hybrid	137
5.4	A 100–200-MHz Third-Order Polynomial-Based Hybrid	138
5.5	A 1–3-GHz Mixed Element Hybrid	140
5.6	A 2.5–6-GHz Hybrid with Coupled Transmission Lines	144
5.7	Optimum Second-Order Hybrids	146
5.8	A 50–150-MHz Second-Order Hybrid	150
5.9	Higher-Order Symmetrical Optimum Hybrids	151
5.10	A 2–32-MHz First-Order Cascade Hybrid	152
5.11	A 10–100-MHz Second-Order Cascade Hybrid	154
5.12	A 10–100-MHz Approximate-Phase Hybrid	157
5.13	A 1–300-MHz Approximate-Amplitude Hybrid Based on Phase Delay Networks	164
5.14	Conclusion	167
	Reference	168
<b>6</b>	<b>Special Topics</b>	<b>169</b>
6.1	Active Circuits	169
6.2	Unequal Division and Nonquadrature Hybrids	181
6.3	Power Handling	190

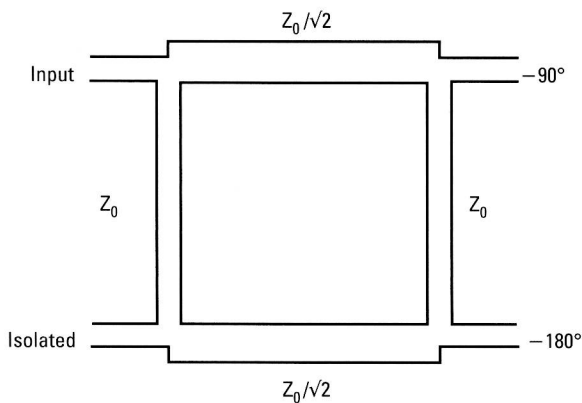
6.4	Graphical Techniques for Cascaded Couplers	192
6.5	A 50–550-MHz Hybrid with Sections of Different Topology	196
6.6	Conclusion	201
	References	202
	<b>Glossary</b>	<b>203</b>
	<b>Bibliography</b>	<b>207</b>
	<b>About the Author</b>	<b>211</b>
	<b>Index</b>	<b>213</b>

# 1

## Overview of Quadrature Hybrids

### 1.1 From Distributed to Lumped Element Design

The reader is likely familiar with distributed versions of quadrature hybrids, as these have been covered extensively in the literature, both in books and technical papers. A simple example of these is the branch line hybrid, as shown in Figure 1.1 [1]. At center frequency, power applied to the input port is transferred equally to the two output ports, with the fourth port in isolation. Input match is perfect at this frequency. The upper output as shown in the diagram leads the lower output by  $90^\circ$ . Acceptable performance



**Figure 1.1** Quarter-wave branch line hybrid circuit.



is achieved at frequencies on either side of the center frequency, but performance degrades rapidly thereafter. Figure 1.2 shows the insertion loss to the two outputs, Figure 1.3 shows the phase balance between outputs, and Figure 1.4 shows the input match and isolation.

Another form of distributed quadrature hybrid makes use of an in-phase divider with a quarter-wave line extension to one output. The in-phase divider may be of the Wilkinson type, as shown in Figure 1.5. In this circuit, a quarter-wave line has been added to the lower output to give a  $90^\circ$  offset. The circuit gives good input match over a reasonable bandwidth, but phase balance deviates rapidly from quadrature on either side of the center

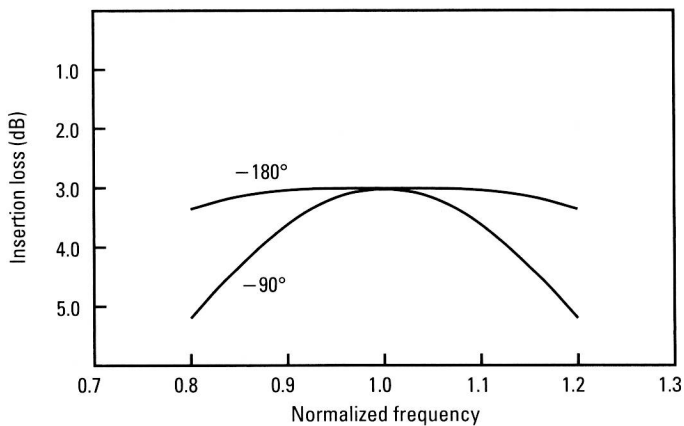


Figure 1.2 Branch line hybrid insertion loss.

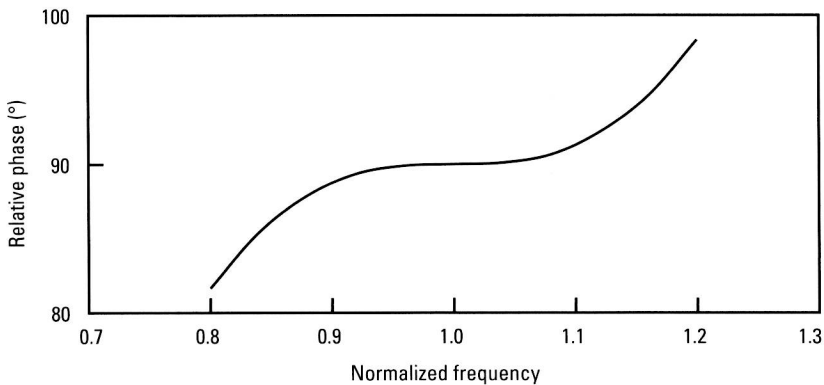


Figure 1.3 Branch line hybrid phase balance.