

Effective Groundwater Model Calibration

With Analysis of Data, Sensitivities,
Predictions, and Uncertainty



Mary C. Hill
Claire R. Tiedeman

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**MARY C. HILL
CLAIRE R. TIEDEMAN**



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We dedicate this book to the groundwater modelers and software developers of the U.S. Geological Survey. These men and women devote their careers to providing sound scientific analyses for policy makers and to enabling others in the government and the private sector to do the same. We are honored to be their colleagues.

We also dedicate this book to the United States taxpayers, to whom we are ultimately accountable. They have supported our educations, salaries, field work and students. We hope our efforts have improved the understanding and management of their groundwater resources.

With love, we also dedicate this book to our husbands and families.

PREFACE

This book is intended for use in undergraduate and graduate classes, and is also appropriate for use as a reference book and for self-study. Minimal expertise in statistics and mathematics is required for all except a few advanced, optional topics. Knowledge of groundwater principles is needed to understand some parts of the exercises and some of the examples, but students from other fields of science have found classes based on drafts of the book to be very useful.

This book has been more than 12 years in the making. Progressively more mature versions have been used to teach short courses most years since 1991. The short courses have been held at the U.S. Geological Survey National Training Center in Denver, Colorado; the International Ground Water Modeling Center at the Colorado School of Mines in Golden, Colorado; the South Florida Water Management District in West Palm Beach, Florida; the University of Minnesota, in Minneapolis, Minnesota; the Delft University of Technology, The Netherlands; Charles University in Prague, the Czech Republic; University of the Western Cape in Belleville, South Africa; and Utrecht University, The Netherlands. A version also was used to teach a semester course at the University of Colorado in Boulder, Colorado in the fall of 2000. Much of what the book has become results from our many wonderful students. We thank them for their interest, enthusiasm, good humor, and encouragement as we struggled to develop many of the ideas presented in this book.

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All errors and omissions are the sole responsibility of the authors.

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