



Engineering Mechanics *Statics*

Robert W. Soutas-Little
Daniel J. Inman

Engineering Mechanics

STATICS

Robert W. Soutas-Little

Michigan State University

Daniel J. Inman

Virginia Polytechnic Institute and State University

Prentice Hall
Upper Saddle River, New Jersey 07458

Library of Congress Cataloging-in-Publication Data

Soutas-Little, Robert W.
Engineering mechanics. Statics/Robert W. Soutas-Little, Daniel
J. Inman
p. cm.
Includes bibliographical references and index.
ISBN 0-13-769001-0
I. Statics. I. Inman, D. J. II. Title.
TA351.S58 1998 1998
620.1'03—dc21

98-18729
CIP

Acquisitions editor: William Stenquist
Editorial/production supervision: Rose Kernan
Editor-in-chief: Marcia Horton
Managing editor: Bayani Mendoza de Leon
Copy editing: Abigail Baker
Art director: Jayne Conte
Cover designer: Bruce Kenselaar
Cover photo: Sydney Harbour Bridge, New South Wales, Australia by Dave Jacobs,
Tony Stone Images
Director of production and manufacturing: David W. Riccardi
Manufacturing buyer: Julia Meehan and Pat Brown
Editorial assistant: Meg Weist
Composition: WestWords, Inc.



© 1999 by Prentice-Hall, Inc.
Simon & Schuster/A Viacom Company
Upper Saddle River, New Jersey 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and test of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Printed in the United States of America

10 9 8 7 6 5 4 3 2



ISBN 0-13-769001-0

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Prentice-Hall, *Upper Saddle River, New Jersey*

Preface

A new undergraduate text in mechanics should reflect the changing aspects of engineering. New developments have occurred in the use of mechanics in practice, fundamental changes are occurring in engineering and mathematics education, and computational software is being taught to students and is used in an increasing number of engineering courses. We have not tried to revolutionize statics, but to reform the presentation and bring the material up to date with modern practice. Most texts currently available contain little new material, as it is felt that no new developments have occurred. However, the practice of mechanics has evolved, and we have incorporated this evolution into this text. The use of computational software is now as significant a change to introductory mechanics as were the slide rule and the calculator in earlier times.

Our presentation focuses on the conceptual understanding of mechanics. This is accomplished, in part, by graphing results so that better insights can be gained into the effects of changing parameters. Such parametric studies are the basic foundation of engineering design, and we introduce them early in the text. Much greater insight can be gained by having students solve a problem for an angle or dimension and then graphing the solution to see the effects of parametric changes. Traditional texts rely too heavily on homework problems that specify geometry and loading and miss the transition to design concepts. The process of design requires the use of parametric studies to understand the importance of various parameters. A good mechanics book should encourage students to try “what-if” calculations. The student’s focus needs to be on modeling, using free-body diagrams, and then writing the equations of equilibrium. Computational software can then be used to solve particular numerical problems and investigate design parameters.

A computer icon () next to a problem number indicates those instances where the use of a computer could benefit the student, although a computer is not absolutely necessary to solve the problem. Two computer icons () indicates a problem where the use of a computer is necessary in order to solve the problem.

Our treatment of statics uses vector mathematics. Traditionally, for most two-dimensional problems, the equilibrium equations are formu-

lated in scalar notation rather than vector notation. When the equilibrium equations are written in vector notation, they may be solved directly or expanded and solved with matrix methods taking full advantage of a vector formulation. This approach eliminates the common confusion about linear independence.

An essential computation in statics is the solution of systems of linear equations that result from the application of equilibrium conditions. These systems of linear equations can now be handled by introducing a matrix formulation. Historically, many methods in statics have been developed to reduce the complexity of the simultaneous equations that have to be solved. For example, students are taught to take moments about support points, so the reactions do not appear explicitly in the equations. These simplifications often lead to confusion. If the students are shown that systems of linear equations can be solved by using matrix methods, the number of equations becomes of minimal importance. Matrices are introduced in Section 2.7 and matrix methods are available in all computational software packages, as well as on some calculators. Matrix notation and matrix methods are compatible with advanced courses in structures and mechanical systems. The fundamental concepts of linear algebra are presented on an “as needed” basis, so as not to digress from the main principles of mechanics.

Computational software enables easy solution of systems of nonlinear algebraic and transcendental equations. This allows consideration of equilibrium problems where, for example, the wires supporting an object deform. A smooth transition is provided to mechanics of materials courses by introducing the basic concepts of springs and spring constants. Students, therefore, gain a better understanding of the concepts of deformable materials and static indeterminacy. The ability to solve nonlinear equations allows an increase in the number of applications that students will encounter in engineering practice.

One significant change in mechanics has occurred because of advances in computer technology. Computational software presents visualization and solution possibilities not available previously. Students can now learn, think, and see in both two- and three-dimensional space. Because of this, the number of three-dimensional problems in the chapter on equilibrium of the rigid body and the chapter on structures has been increased. Computational software allows us to improve the type of problems and include solutions in terms of general geometries and loadings. This is much closer to the actual methods used in engineering practice. Students can now focus on the free-body diagram and the equilibrium conditions. The analytical solution of the equilibrium equations afford students good insights and leads to discussions of design alternatives. Computational software encourages students to examine their solutions and take corrective measures. The result is students build better intuition about engineering mechanics.

Photo Credits

Cover Photo: Dave Jacobs, Tony Stone Images

Chapter 1 Opening Page: Doug Armand, Tony Stone Images

Chapter 2 Opening Page: William Saliaz, Duomo Photography

Chapter 3 Opening Page: William Saliaz, Duomo Photography

Chapter 4 Opening Page: Wayne Eastep, Tony Stone Images

Chapter 5 Opening Page: Dr. E.R. Degginger

Chapter 6 Opening Page: Index Stock Photography, Inc.

Chapter 6 Text Photos: Dan Templeton, Still Life Stock, Inc. And Dave Jacobs, Tony Stone Images

Chapter 7 Opening Page: AP/World Wide Photos

Chapter 8 Opening Page: James Balog, Tony Stone Images

Chapter 9 Opening Page: Gerard Planchenault, Allsport Photography (USA), Inc.

Chapter 10 Opening Page: Dr. E.R. Degginger

Contents

Preface	x
Chapter 1. Introduction	1
1.1 Mechanics	1
1.2 Basic Concepts	2
Problems 1.1–1.4	5
1.3 Units	6
SI Units	6
U. S. Customary Units	7
Conversion between Systems of Units	9
Problems 1.5–1.9	12
1.4 Numerical Calculations	13
1.5 Problem-solving Strategy	14
1.6 Computational Software	15
Chapter 2. Vectors and Force Systems Acting on a Particle	16
2.1 Introduction	16
2.2 Vectors	17
Definition of a Scalar and Vector	17
Vector Addition	18
Multiplication of a Vector by a Scalar	22
Vector Components	22
Resolution of a Vector into Components	29
2.3 Forces and Their Characteristics	34
Concurrent Coplanar Forces	36
Problems 2.1–2.21	38
2.4 Three-dimensional Cartesian Coordinates and Unit Base Vectors	42
Unit Base Vectors	43
Vector Equality in Component Notation	45
Vector Addition by Components	45

Multiplication of a Vector by a Scalar	46
Vector Subtraction	46
General Unit Vectors	46
Vector Directions in Space	47
Matrix Notation for Vectors	49
2.5 Computation of Vector Operations	51
Problems 2.2–2.59	54
2.6 Components of a Vector in Nonorthogonal Directions	59
2.7 Systems of Linear Equations	62
Matrices	64
Problems 2.60–2.77	69
2.8 Scalar Product of Two Vectors	71
Applications of the Scalar Product	72
Problems 2.78–2.94	76
2.9 Free-body Diagrams of a Particle	79
Problems 2.95–2.104	83
2.10 Equilibrium of a Particle	85
Problems 2.105–2.113	90
2.11 Springs	93
2.12 Statically Indeterminate Problems	97
Problems 2.114–2.118	100
Chapter Summary	101
Vectors	101
Free-body Diagrams and Equilibrium of a Particle	102
Review Problems 2.119–2.131	103

Chapter 3. Rigid Bodies: Equivalent Force Systems **106**

3.1 Rigid Bodies	106
3.2 Modeling of Rigid Bodies and Moment of a Force	108
Problems 3.1–3.10	112
3.3 Vector Product or Cross Product	114
Multiple Products of Vectors	117
Problems 3.11–3.20	121
3.4 Moment of a Force about a Point in Space	122
Direct Vector Solutions	124
3.5 Varignon's Theorem	127
Problems 3.21–3.47	130

3.6	Moment of a Force about an Axis	134
	Problems 3.48–3.60	137
3.7	Moment of a Couple	139
	Problems 3.61–3.70	142
3.8	Equivalent Force Systems	144
	Problems 3.71–3.74	147
3.9	Special Equivalent Force Systems	148
	Concurrent Force Systems	150
	Coplanar Force Systems	150
	Parallel Force Systems	152
3.10	General Equivalent Force Systems	157
	The Wrench	159
	Problems 3.75–3.94	166
	Chapter Summary	170

Chapter 4. Distributed Forces: Centroids and Center of Gravity **172**

4.1	Introduction	172
4.2	Center of Mass and Center of Gravity	175
	Center of Mass	175
	Center of Gravity	177
4.3	Centroids of Areas, Volumes, and Lines; The First Moment	179
	Centroid of an Area	179
	Centroid of a Volume	183
	Centroid of a Line	185
	Centroid of a Curve in Space	186
	Problems 4.1–4.25	189
4.4	Theorems of Pappus and Guldinus	192
	Problems 4.26–4.32	
4.5	Centroids of Composite Bodies	196
	Problems 4.33–4.32	202
4.6	Distributed Loads on Beams	206
	Problems 4.52–4.62	208
4.7	Forces Due to Fluid Pressure Acting on a Submerged Surface	210
	Buoyancy	211
	Problems 4.63–4.70	214
	Chapter Summary	216
	Centroid	216
	Center of Mass	216

Centroids of Composite Bodies	217
Distributed Loads on a Beam	217
Focus on a Submerged Surface	218
Buoyancy	218
Chapter 5. Equilibrium of Rigid Bodies	219
5.1 Introduction	219
5.2 Supports for a Two-dimensional Model	221
5.3 Supports for a Three-dimensional Model	225
5.4 Free-Body Diagram	229
Problems 5.1–5.10	233
5.5 Equilibrium of a Rigid Body in Two Dimensions	236
Solution Strategy	237
A Two-Force Member	238
A Three-Force Member	238
Problems 5.11–5.51	246
5.6 Equilibrium of a Rigid Body in Three Dimensions	257
Constraints	258
5.7 Statically Indeterminate Reactions and Improper Constraints	264
Problems 5.52–5.80	272
Chapter Summary	282
Equilibrium	282
Free-body Diagram	282
Two-force Member	282
Three-force Member	282
Equilibrium of a Rigid Body in Three Dimensions	283
Chapter 6. Analysis of Structures	284
6.1 Introduction	284
6.2 Planar Trusses	286
6.3 Simple Trusses	289
6.4 Method of Joints	290
6.5 Method of Joints Using Matrix Techniques	296
Problems 6.1–6.30	300
6.6 Method of Sections	305
Problems 6.31–6.55	308
6.7 Space Trusses	312

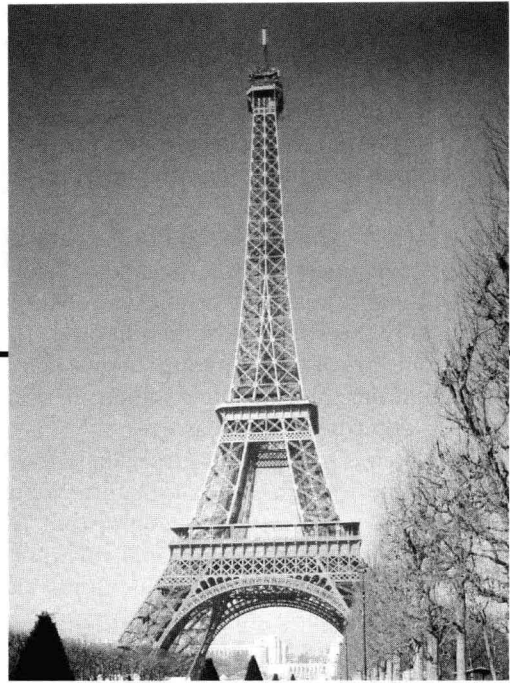
Problems 6.56–6.66	316
6.8 Compound Trusses	319
6.9 Frames and Machines	319
Problems 6.67–6.97	324
Chapter Summary	335
Trusses	335
Frames and Machines	335
Chapter 7. Internal Forces in Structural Members	336
7.1 Introduction	336
7.2 Internal Forces in a Member	337
Problems 7.1–7.17	343
7.3 Types of Loading and Supports in Beams	346
7.4 Shear and Bending Moments in Beams	347
Relationship between the Load Distribution, the Shear Force, and the Bending Moment	349
Problems 7.18–7.41	355
7.5 Discontinuity Functions for Beam Equations	359
Problems 7.42–7.53	366
7.6 Cables	368
Cable Subjected to Concentrated Loads	368
Cables Supporting Loads Distributed Uniformly along a Horizontal Line	371
Cables Supporting Loads Distributed Uniformly along Its Own Length	373
Problems 7.54–7.70	376
Chapter Summary	380
Beams	380
Discontinuity Functions	380
Cables	381
Chapter 8. Friction	383
8.1 Introduction	383
8.2 Coulomb Friction	387
Problems 8.1–8.70	396
8.3 Wedges	406
Problems 8.71–8.89	411
8.4 Square-Threaded Screws	413
Problems 8.90–8.104	418

8.5	Belt Friction	421
	V-belts	423
8.6	Bearings	425
8.7	Thrust Bearings, Collars, and Clutches	428
8.8	Rolling Resistance	429
	Problems 8.105–8.125	432
	Chapter Summary	435
	Belt Friction	436
	Roller Bearings	436
	Thrust Bearings, Collars, and Clutches	436
Chapter 9.	Moments of Inertia	437
9.1	Introduction	437
9.2	Second Moment of an Area	438
	Determination of the Second Moment of an Area by Integration	439
9.3	Polar Moment of Inertia	439
9.4	Second Moment of an Area about Centroidal Axes for Specific Areas	441
	Problems 9.1–9.21	442
9.5	Parallel-Axes Theorem for the Second Moment of Area	444
9.6	Radius of Gyration of an Area	446
	Problems 9.22–9.32	448
9.7	Second Moments of Composite Areas	449
	Problems 9.33–9.45	452
9.8	Principal Second Moments of Area	454
	Product Moment of an Area, or Product Moment of Inertia	454
	Rotation of Axes	455
	Problems 9.46–9.59	460
9.9	Mohr's Circle to Determine Principal Second Moments of Area	462
	Problems 9.60–9.73	465
9.10	Eigenvalue Problem	467
	Problems 9.74–9.87	470
9.11	Mass Moments of Inertia	472
	Parallel-Axis Theorem	473
	Problems 9.88–9.102	476

Chapter Summary	478
Principal Second Moment of Area	479
Mass Moments of Inertia	480
Chapter 10. Virtual Work	481
10.1 Introduction	481
Work-By a Force or Moment	481
10.2 Virtual Work	483
10.3 Principle of Virtual Work for a System of Connected Rigid Bodies	484
Problems 10.1–10.28	489
10.4 Finite Work of a Force and Moment	494
Problems 10.29–10.34	498
10.5 Conservative Forces and Potential Energy	499
10.6 Potential Energy and Equilibrium	503
10.7 Stability of Equilibrium	504
Problems 10.35–10.60	508
Chapter Summary	511
Conservative Forces	512
Appendix: Solution of Systems of Linear Equations	514
Glossary	523
Problem Statements	534
Index	560

1

Introduction



The 984-foot-high Eiffel Tower was designed by the French engineer Alexandre Gustave Eiffel (1832–1923) and erected for the Paris Exposition of 1889. Eiffel also designed the framework of the Statue of Liberty, which is found in New York Harbor. A miniature model of the statue stands in the middle of the River Seine on a small island near the Eiffel Tower.

1.1 Mechanics

Mechanics is the oldest area of study in physics, originating with the Greek culture in 400 to 300 BC. The earliest record of the study of mechanical problems was the work of Aristotle (384–322 BC). Although historical questions remain as to the authorship of some of these early works, considerations of the equilibrium of vertical forces and the action of levers were evident in these writings. Archimedes of Syracuse (287–212 BC) introduced the formal study of levers and the concept of the center of gravity. He also examined concepts of geometry and wrote about the theory of buoyancy and the equilibrium of floating bodies. Many later scientists and mathematicians, including Kepler and Galileo, contributed to the development of mechanics, but the formal presentation of the principles of mechanics was made by Sir Isaac Newton (1642–1727).

Mechanics is a basic engineering science that lays the foundation for machine design, structural analysis, stress analysis, vibrations, electromagnetic field theory, and fluid and solid mechanics. In particular, these

courses use the methods of modeling, the techniques of vector algebra and calculus, and the computational methods presented in introductory mechanics. Mechanics is the branch of physical science that deals with motion and the effects of forces on gases, liquids, and solid bodies. For ease of study, mechanics has been divided into the study of deformable solid bodies, the study of fluids and gases, and the study of rigid bodies. To further simplify the approach, the mechanics of rigid bodies is subdivided into statics, which deals with rigid bodies at rest or moving at a constant velocity, and dynamics, which is the study of rigid bodies undergoing acceleration. Although statics may be considered a special case of dynamics, it is helpful to study this special case first in order to obtain a firm understanding of the concepts of forces and moments, learn the methods of modeling physical situations, and become skilled at using the mathematical tools necessary to describe these ideas.

Statics, which is based upon surprisingly few basic principles, employs a field of mathematics called vector algebra. We will introduce another field of mathematics, called matrix algebra, that proves useful when one is working with common computational software. It is not necessary to use matrix notation to solve problems in statics, but most presently available commercial software uses this notation.

1.2 Basic Concepts

The starting point of the study of mechanics is an examination of the basic concepts upon which Newton based his three laws of motion. These concepts are as follows.

Space is a boundless expanse in which objects and events occur and have relative position and direction. Space allows the measurement of length, area, and volume. The measurement of length is made by comparison with another object of known or standard length, such as a ruler. Area is defined as the product of two lengths and allows the measurement of a two-dimensional space. Volume is three-dimensional space and is the product of three lengths. Newton considered space to be infinite, homogeneous, isotropic, and absolute. The last property, absoluteness, allowed Newton to assume the existence of a primary inertial frame of reference, which is not moving relative to the “fixed” stars and has an origin located at the center of mass of the universe. If space is isotropic, the properties of a closed system at any point in space are unaffected by the orientation of the system. “Homogeneous” means that the space is the same at every point and does not change from point to point. Our present-day understanding of space is somewhat different from Newton’s, owing to Einstein’s theory of relativity; however, in this book, we will assume the same idea of space that Newton did and neglect any small deviations owing to relativistic effects.

Time is the concept used to order the flow of events. Time is assumed to be absolute—that is, time is the same for all observers and is independent of all objects in the world. Physicists now consider time to be an abstraction arrived at by changes in the physical world; in other words, the flow of a series of events *defines* time, and time depends upon that flow of events. We use Newton's definition of absolute time, and time is measured by comparison with some repeatable event, such as the rotation of the Earth, the oscillation of a pendulum, or atomic vibrations (i.e., a transition frequency in the element cesium).

Newton defined **mass** as a “quantity of matter” that related an object's volume to its density. He stated that *gravitational mass*, as defined by the law of gravitational attraction, is equivalent to *inertial mass*, which measures an object's resistance to being accelerated. In current times, this statement is called the “principle of equivalence” and is a postulate of Einstein's theory of relativity. In this book, we assume that the mass of a body is independent of its motion, neglecting any relativistic effects.

Force is defined as the action of one body on another. This action may be the result of direct contact between the two bodies, or it may arise from gravitational, magnetic, or electrical effects between two bodies separated by a distance. Newton postulated that forces always occur in pairs, equal and opposite, each acting on one of the two bodies. Force is not measured directly; only the effect a force produces can be measured. For example, the force required to stretch a spring is measured by determining the distance the spring stretches.

Newton formulated four axioms, or laws, that are the basis of the study of rigid-body statics and dynamics. The first three laws, known as **Newton's laws** of motion, may be stated as follows:

1. *Every body or particle continues in a state of rest or of uniform motion (constant velocity) in a straight line, unless it is compelled to change that state by forces acting upon it.* That is, the body will remain at rest or continue to move with the same speed and direction if no net external forces act on it.
2. *The change of motion of a body is proportional to the net force imposed on the body and is in the direction of the net force.* That is, the net force is equal to the change in the product of the mass and the velocity. The mass is the resistance of the body to acceleration.
3. *If one body exerts a force on a second body, then the second body exerts a force on the first that is equal in magnitude, opposite in direction, and collinear.*

Newton's final law is the law of universal gravitational attraction.

4. *Any two particles are attracted to each other with a force whose magnitude is proportional to the product of their gravitational masses and inversely proportional to the square of the distance between them.* An example of a gravitational attraction force is shown in Figure 1.1.

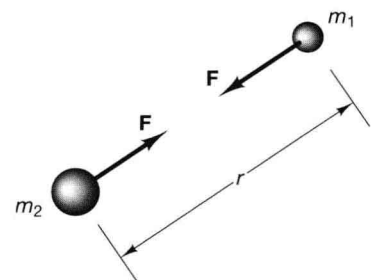


Figure 1.1 The gravitational force F between two bodies of masses m_1 and m_2 separated by a distance r between their centers.

The magnitude of the gravitational force F is stated mathematically as

$$F = \frac{Gm_1m_2}{r^2} \quad (1.1)$$

where

$$G = 66.73 \times 10^{-12} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

is the **universal gravitational constant**.

The **weight** of a body on the Earth's surface is the force due to the gravitational attraction of an object to the mass of the Earth. Weight is expressed as

$$W = \frac{GmM}{R^2} \quad (1.2)$$

where G is the universal gravitational constant, m is the mass of the object (in kilograms), M is the mass of the Earth (in kilograms), and R is the radius of the Earth (in meters). For the gravitational attraction of an object on the surface of the Earth, the ratio

$$g = (GM)/R^2$$

is taken to be a constant, and the value of g is 9.807 m/s^2 or 32.17 ft/s^2 . This value differs at different points on the Earth, as the Earth is not a perfect sphere of radius R , and elevations vary. However, for most engineering problems, g is considered to be a constant. The weight of an object near the surface of the Earth is caused by the gravitational attraction and is equal to

$$W = mg \quad (1.3)$$

The constant g has the units of acceleration and is sometimes called the **gravitational acceleration** or *the acceleration due to gravity*. This concept will be discussed in detail when the subject of particle dynamics is presented. For the present, consider g to be a constant that relates the mass of an object to its weight on the surface of the Earth.