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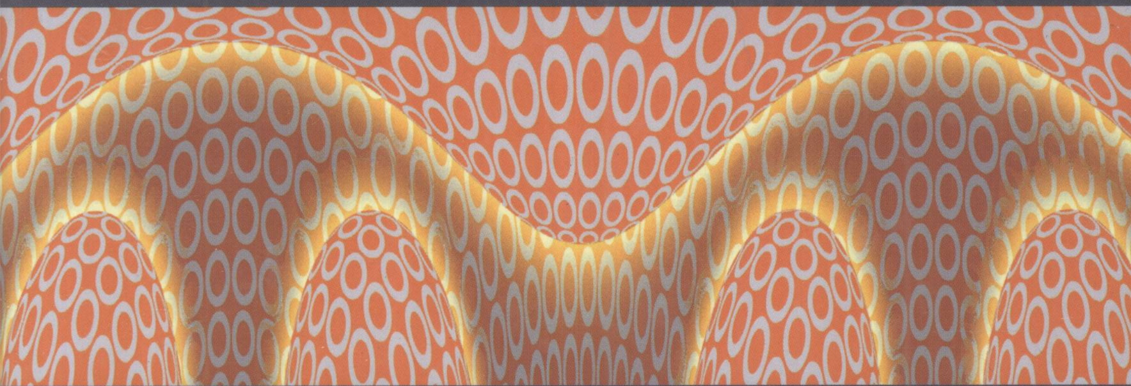

Time–Frequency Analysis

Concepts and Methods

Edited by Franz Hlawatsch and François Auger

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Preface

Is time-frequency a mathematical utopia or, on the contrary, a concept imposed by the observation of physical phenomena? Various "archetypal" situations demonstrate the validity of this concept: musical notes, a linear chirp, a frequency shift keying signal, or the signal analysis performed by our auditory system. These examples show that "frequencies" can have a temporal localization, even though this is not immediately suggested by the Fourier transform. In fact, very often the analyzed phenomena manifest themselves by oscillating signals *evolving with time*: to the examples mentioned above, we may add physiological signals, radar or sonar signals, acoustic signals, astrophysical signals, etc. In such cases, the time-domain representation of the signal does not provide a good view of multiple oscillating components, whereas the frequency-domain representation (Fourier transform) does not clearly show the temporal localization of these components. We may conjecture that these limitations can be overcome by a *time-frequency analysis* where the signal is represented as a joint function of time *and* frequency – i.e., over a "time-frequency plane" – rather than as a function of time *or* frequency. Such an analysis should constitute an important tool for the understanding of many processes and phenomena within problems of estimation, detection or classification.

We thus have to find the mathematical transformation that allows us to map the analyzed signal into its time-frequency representation. Which "generalized Fourier transform" establishes this mapping? At this point, we find ourselves confronted with a fundamental limitation, known as the *uncertainty principle*, that excludes any *precise* temporal localization of a frequency. This negative result introduces some degree of uncertainty, or even of arbitrariness, into time-frequency analysis. One of its consequences is that we can never consider a transformation as *the only* correct time-frequency transformation, since time-frequency localization cannot be verified in an exact manner.

Is time-frequency an ill-posed problem then? Maybe, since it does not have a unique solution. However, this ambiguity and mathematical freedom have led to the definition of a great diversity of time-frequency transformations. Today, the chimeric concept of time-frequency analysis is materialized by a multitude of different transformations (or representations) that are based on principles even more diverse than the domains from which they originated (signal processing, mathematics, quantum mechanics, etc.). These principles and signal analysis or processing methods are just as useful in real-life applications as they are interesting theoretically.

Thus, is time-frequency a reality today? This is what we attempt to demonstrate in this book, in which we describe the principles and methods that make this field an everyday fact in industry and research. Written at the end of a period of approximately 25 years in which the discipline of time-frequency analysis witnessed an intensive development, this tutorial-style presentation is addressed mainly to researchers and engineers interested in the analysis and processing of non-stationary signals. The book is organized into two parts and consists of 13 chapters written by recognized experts in the field of time-frequency analysis. The first part describes the fundamental notions and methods, whereas the second part deals with more recent extensions and applications.

The diversity of viewpoints from which time-frequency analysis can be approached is demonstrated in Chapter 1, *"Time-Frequency Energy Distributions: An Introduction"*. Several of these approaches – originating from quantum mechanics, pseudo-differential operator theory or statistics – lead to the same set of fundamental solutions, for which they provide complementary interpretations. Most of the concepts and methods discussed in this introductory chapter will be developed in the following chapters.

Chapter 2, entitled *"Instantaneous Frequency of a Signal"*, studies the concept of a "time-dependent frequency", which corresponds to a simplified and restricted form of time-frequency analysis. Several definitions of an instantaneous frequency are compared, and the one appearing most rigorous and coherent is discussed in detail. Finally, an in-depth study is dedicated to the special case of phase signals.

The two following chapters deal with *linear* time-frequency methods. Chapter 3, *"Linear Time-Frequency Analysis I: Fourier-Type Representations"*, presents methods that are centered about the short-time Fourier transform. This chapter also describes signal decompositions into time-frequency "atoms" constructed through time and frequency translations of an elementary atom, such as the Gabor and Wilson decompositions. Subsequently, adaptive decompositions using redundant dictionaries of multi-scale time-frequency atoms are discussed.

Chapter 4, *"Linear Time-Frequency Analysis II: Wavelet-Type Representations"*, discusses "multi-resolution" or "multi-scale" methods that are based on the notion of scale rather than frequency. Starting with the continuous wavelet transform, the chapter presents orthogonal wavelet decompositions and multi-resolution analyses. It also studies generalizations such as multi-wavelets and wavelet packets, and presents some applications (compression and noise reduction, image alignment).

Quadratic (or bilinear) time-frequency methods are the subject of the three following chapters. Chapter 5, *"Quadratic Time-Frequency Analysis I: Cohen's Class"*, provides a unified treatment of the principal elements of Cohen's class and their main characteristics. This discussion is helpful for selecting the Cohen's class time-frequency representation best suited for a given application. The characteristics studied concern theoretical properties as well as interference terms that may cause practical problems. This chapter constitutes an important basis for several of the methods described in subsequent chapters.

Chapter 6, *"Quadratic Time-Frequency Analysis II: Discretization of Cohen's Class"*, considers the time-frequency analysis of sampled signals and presents algorithms allowing a discrete-time implementation of Cohen's class representations. An approach based on the signal's sampling equation is developed and compared to other discretization methods. Subsequently, some properties of the discrete-time version of Cohen's class are studied.

The first part of this book ends with Chapter 7, *"Quadratic Time-Frequency Analysis III: The Affine Class and Other Covariant Classes"*. This chapter studies quadratic time-frequency representations with covariance properties different from those of Cohen's class. Its emphasis is placed on the affine class, which is covariant to time translations and contractions-dilations, similarly to the wavelet transform in the linear domain. Other covariant classes (hyperbolic class, power classes) are then considered, and the role of certain mathematical concepts (groups, operators, unitary equivalence) is highlighted.

The second part of the book begins with Chapter 8, *"Higher-Order Time-Frequency Representations"*, which explores multilinear time-frequency analysis. The class of time-*multifrequency* representations that are covariant to time and frequency translations is presented. Time-(mono)frequency representations ideally concentrated on polynomial modulation laws are studied, and the corresponding covariant class is presented. Finally, an opening towards multilinear affine representations is proposed.

Chapter 9, *"Reassignment"*, describes a technique that is aimed at improving the localization of time-frequency representations, in order to enable a better interpretation by a human operator or a better use in an automated processing scheme. The reassignment technique is formulated for Cohen's class and for the affine class, and its properties and results are studied. Two recent extensions – supervised reassignment and differential reassignment – are then presented and applied to noise reduction and component extraction problems.

The two following chapters adopt a statistial approach to non-stationarity and time-frequency analysis. Various definitions of a non-parametric "time-frequency spectrum" for non-stationary random processes are presented in Chapter 10, *"Time-Frequency Methods for Non-stationary Statistical Signal Processing"*. It is demonstrated that these different spectra are effectively equivalent for a subclass of processes referred to as "underspread". Subsequently, a method for the estimation of time-frequency spectra is proposed, and finally the use of these spectra for the estimation and detection of underspread processes is discussed.

Chapter 11, *"Non-stationary Parametric Modeling"*, considers non-stationary random processes within a parametric framework. Several different methods for non-stationary parametric modeling are presented, and a classification of these methods is proposed. The development of such a method is usually based on a parametric model for stationary processes, whose extension to the non-stationary case is obtained by means of a sliding window, adaptivity, parameter evolution or non-stationarity of a filter input.

The two chapters concluding this book are dedicated to the application of time-frequency analysis to measurement, detection, and classification tasks. Chapter 12, *"Time-Frequency Representations in Biomedical Signal Processing"*, provides a well-documented review of the contribution of time-frequency methods to the analysis of neurological, cardiovascular and muscular signals. This review demonstrates the high potential of time-frequency analysis in the biomedical domain. This potential can be explained by the fact that diagnostically relevant information is often carried by the non-stationarities of biomedical signals.

Finally, Chapter 13, *"Application of Time-Frequency Techniques to Sound Signals: Recognition and Diagnosis"*, proposes a time-frequency technique for supervised non-parametric decision. Two different applications are considered, i.e., the classification of loudspeakers and speaker verification. The decision is obtained by minimizing a distance between a time-frequency representation of the observed signal and a reference time-frequency function. The kernel of the time-frequency representation and the distance are optimized during the training phase.

As the above outline shows, this book provides a fairly extensive survey of the theoretical and practical aspects of time-frequency analysis. We hope that it will contribute to a deepened understanding and appreciation of this fascinating subject, which is still witnessing considerable developments.

We would like to thank J.-P. Ovarlez for important contributions during the initial phase of this work, G. Matz for helpful assistance and advice, and, above all, P. Flandrin for his eminent role in animating research on time-frequency analysis.

Vienna and Saint Nazaire, June 2008.