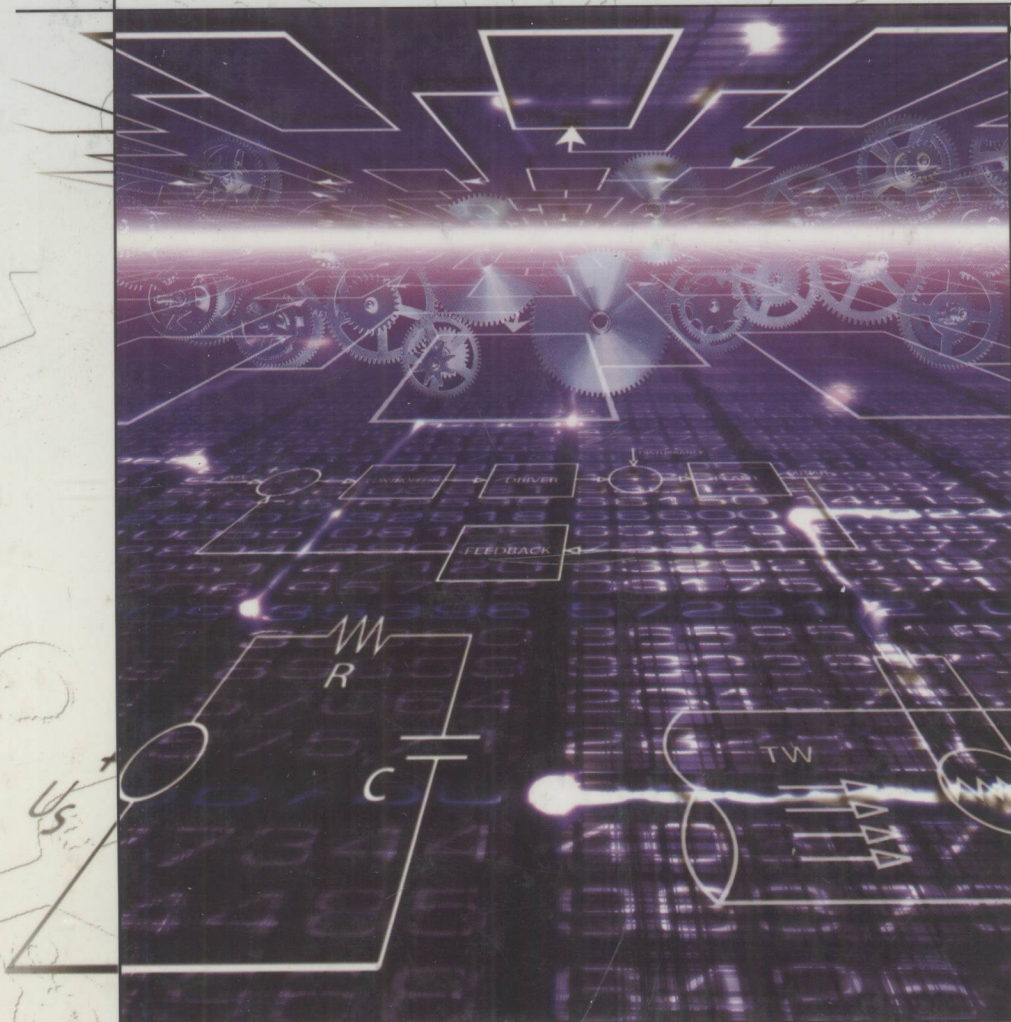


# Modeling & Control of Dynamic Systems

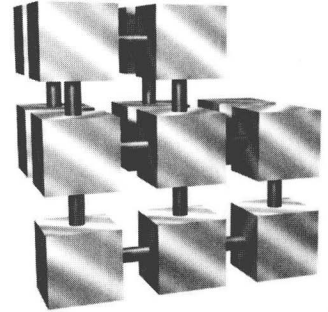


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& Thaler

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# Modeling and Control of Dynamic Systems



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**Narciso F. Macia**  
**George J. Thaler**



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*This book is dedicated to the late George J. Thaler,  
who originally was going to be the first author of this book, and who  
exhibited commendable self-control in his life: by keeping the proper  
balance in sharing his life among God, family, physical activity (tennis) and  
his academic profession. Without his contribution this book would not exist.*

*And to my wonderful family,  
Donna, Maria, Graciela, Rebeca, Melinda and Cisito who have supported me  
whole heartily in this effort.*

# Preface

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## Introduction

This book is intended as a first text in dynamic systems and their control, appropriate for undergraduate engineering technology and engineering students. It can also be used as a graduate text if additional assignments are given to graduate students.

What type of student should take such a dynamic system course? In my opinion all engineering technology and engineering students, regardless of their area of specialization, because a good grasp of dynamic behavior will open doors to a better understanding of real-world problems. Critical elements in the formulation of a solution to problems are often overlooked if only the static or steady-state conditions are considered. This limited perspective produces solutions that are short lived or do not solve the problem for its entire range of operation. Every effort has been made in writing this book to equip the reader with the tools necessary for assessing the issues associated with the dynamic aspects of the problem. Needless to say, we have also encouraged order-of-magnitude analysis, based on static conditions to ‘bracket’ the solution.

Today’s students find curriculums that are over packed with subjects. Such curriculum takes students on an endless introduction of many subject areas and often they fail to grasp the beautiful harmony that exists among the various disciplines. I am a strong believer that studying dynamic systems has the potential of galvanizing the underlying concepts found in the many subjects taken by students, and allows them to apply these concepts to the real world. It also helps students transform a bunch of apparently disconnected ideas and principles into a beautifully interrelated framework. We have made every effort to assist the student in this endeavor. It is a great experience to see students discover this harmony in the behavior of dynamic systems, and help them become more confident in the subjects they have already learned.

This book uses simple, every-day examples that most students have previously experienced, as vehicles for teaching dynamic system principles. It provides a step-by-step explanation of how to obtain a “model schematic” for the physical system, a graphical representation that captures the essence of the dynamic system under question. It shows how to transform this model schematic into a mathematical model (differential equation or transfer function), which then can be simulated (used to obtain a time response) with MALTAB/SIMULINK.

The book immerses students in one of the most powerful and easy-to-use software environments ever developed: MATLAB/SIMULINK. The SIMULINK environment provides a similar environment to that used by engineering students 25 years ago: solving differential equations and dynamic systems with an analog

computer. However, today's students, by using MATLAB/SIMULINK, possess a much richer set of displaying and troubleshooting tools than their predecessors did. In addition, today's students have at their disposal a large number of nonlinear blocks that permit the evaluation of their effect on the response. Similarly, students can easily see the effect of changing a time constant or a gain.

### **Implementation**

In Chapter 11, Implementing the Controls Scheme with Hardware: PLCs, students are exposed to the reality that there are many ways of realizing the control task. For instance, the engineer could use a mainframe computer, a micro-controller, or a PLC (Programmable Logic Controller) to mention just a few possibilities. It offers a taste of the process that an engineer has to go through in making this selection, by bringing into the picture issues such as cost, development time, computational power, size, and so on. The book assumes that, for the most part, the PLC will be the most likely vehicle for the reader to implement his control scheme. For this reason, it offers an introduction of the discrete-event controlling capability of the PLC and then goes into detail of how to use the PLC for implementing a closed-loop PID (Proportional + Integral + Derivative) loop.

### **DC Solenoid**

One of the instruments I use to facilitate a deepening sense of inquiry is a series of exercises and experiments that focus on one specific piece of hardware: a DC-solenoid, position control system. This allows students to develop depth with at least one dynamic system and its controller. This goal is embodied in Chapter 13 and also the accompanying Lab Manual. It is my intent that at the end of the semester students will feel very comfortable with the drivers, sensors, and hardware, and with the underlying control methods used in this particular control system's characterization, analysis, and design.

### **Idea Portfolio**

In addition, I use a suggestion made by a former graduate student, Sapto Susilo, to increase the interest and perceived relevance of dynamic systems and control. Each student picks an idea of their own, and relates it to all the tools and methods presented in the control class. The intention here is that this idea, which is of particular interest to the student, might serve as an effective motivator for him or her to invest additional time exploring this field of study. The implementation of this suggestion is what I call the Idea Portfolio. It represents a series of questions at the end of each chapter that will hopefully motivate students to engage in an investigative





process where they produce sketches, estimate model parameters, and perform simulations as they investigate their ideas from the perspective of a controls engineer. Appendix G contains additional suggestions and provides a useful form that can guide students through this creative process.

### **Implementation Platforms**

One goal of this class is to help students gain an appreciation for the various platforms for performing the control action. There are many ways of performing the control action, as discussed in Chapter 11. However, after reflection and experimentation regarding which piece of hardware would be the most practical as a teaching tool, I chose the Programmable Logic Controller (PLC). This takes into consideration the fact that I have students in the dynamic systems and control class that come from electronics, computing, mechanical, and manufacturing backgrounds. The PLC is a controller platform that all engineering students can use, regardless of whether they are familiar with electronics or programming. The PLC is also the backbone of the manufacturing operation and is used in the testing of many systems due to the ease in which it can be programmed.

### **Supplements**

**Lab Manual** includes over 20 hands-on projects and experiments.

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(ISBN#: 140189254X)

### **About the Authors**

This book integrates dynamic systems and their control, two fields that are closely intertwined. I have been involved (and fascinated) with dynamic systems since my sophomore year as a mechanical engineering student. I have nurtured this interest on a consistent basis as a graduate student, an engineer employed in the industry, and as a university professor. I have also been involved with the analysis, design, and implementation of controllers ranging from fluidic circuits to Programmable Logic Controllers (PLC). In the course of this journey, I have come to appreciate the strengths and weaknesses of these platforms. My late co-author, the distinguished professor George J. Thaler, dedicated his professional life to the development and teaching of control theory. He authored and co-authored numerous books and articles on the

subject. His contribution regarding how to design a control system will prove extremely valuable for any control engineer. Without any doubt, this book would not have been possible without his contribution.

## Acknowledgments

I am extremely grateful to those individuals who encouraged me in the pursuit of writing a book. I also would like to acknowledge those teachers and professors who believed in me or imparted to me the knowledge that has allowed me to engage in this monumental (for me) writing task. Among them are Linda Darnell, Charles C. Blackwell, Robert L. Woods, David Hullender, Robert White, Walter Higgings and William J. Dorson.

The team at Delmar has been extremely supportive and patient: Michelle Ruelos Cannistraci has provided a steady and politely firm hand trying to keep this book on schedule. It has also been a pleasure working with her collaborators Linda DeMasi, Kathleen Vonk, Jon Duff, and Stacy Masucci. Other individuals have been a source of encouragement in this pursuit: Mimi Villaca, Charles Okonkwo, Antonio Rodriguez, and Jose Manuel Campoy. I would like to convey my gratitude to the many reviewers who reviewed the manuscript. Their suggestions have made this book several orders of magnitude more clear. Thank you to the following reviewers:

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I am also extremely grateful to the many students who corrected errors, suggested control problems, or expressed a better way to convey a principle. They also gave me the opportunity to collaborate on their thesis or applied projects. We both learned, who learned more I am not sure.

I am extremely eager to improve this effort and welcome any suggestions or corrections that you might have.

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Arizona State University Polytechnic  
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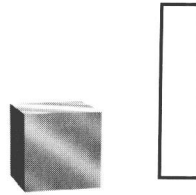


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# Introduction

## Learning Objectives

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In this chapter, you will learn how to:

- Recognize dynamic systems
  - Characterize closed-loop systems
  - Differentiate between open- and closed-loop systems
  - Identify uses of open and closed-loop systems
  - Be aware of unstable feedback systems
  - Name the tests a control engineer performs to identify models
- 

## 1.1 Introduction

This book is concerned with systems. A system is a device, or a collection of devices, for accomplishing some task. This definition implies that a system must have at least one input and one output and that there is a cause-and-effect relationship between the two (i.e., a change in the input causes a change in the output).

In this book, we are concerned with dynamic systems—systems for which the input causes the output to vary with time. For example, let the dynamic system be a simple integrator, and let the input be a positive constant. For this type of input, the output of the integrator increases linearly with time (e.g., the output is a ramp). Dynamic systems are encountered in every engineering discipline, and it might even