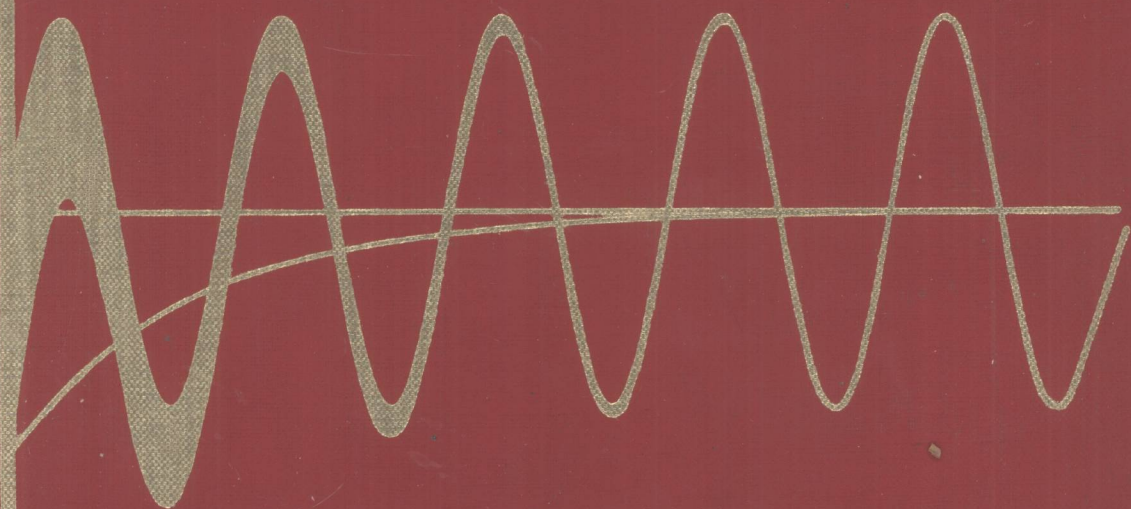


**DIGITAL
FILTERING
and
SIGNAL
PROCESSING**



Childers and Durling

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DIGITAL FILTERING and SIGNAL PROCESSING

DONALD CHILDERS AND ALLEN DURLING

UNIVERSITY OF FLORIDA



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DIGITAL FILTERING
AND
SIGNAL PROCESSING

DEDICATED TO OUR STUDENTS,
WHEREVER THEIR MINDS MAY BE

preface

This book is an outgrowth of a course which has been taught four times as a three quarter-hour course and three times as a five quarter-hour course over a five year interval. It is our hope and intention that the material presented here will be offered to senior students or first year graduate students. The text requires no statistical background although statistical topics are from time-to-time mentioned and the instructor should feel free to supplement this material according to the level of his class. The background material assumed is a basic undergraduate course in linear systems.

The text is design oriented with many computer projects. While the software is copiously documented, hardware considerations are also presented. The reader is assumed to be familiar with Fortran although this too is not required.

We have tried to make the text useful to the practicing engineer who is unfamiliar with modern digital filtering and signal processing techniques. The approach taken has been to relate continuous-time (or analog) systems to discrete-time systems via various transform methods.

To the beginning engineer we can say that this book will provide him with the necessary background to perform a competent and innovative engineering job in digital filtering and signal processing. The book is intended to be used as both a text and as a handbook, and includes the recommended IEEE standards in digital filtering and signal processing.

viii Preface

It is our belief that the student should interact with the material presented in the text as much as possible and that the students themselves should lead discussion topics. We, therefore, recommend that at least two major computer projects be done by each student (possibly working in a team of two), one on digital filtering and one on signal processing, for example, spectrum analysis. Typical computer projects are provided beginning in Chapter 3, and one from that chapter should be assigned as early in the course as possible to be finished approximately at the half-way point.

The text organization is not particularly sacred except that Chapters 1 and 2 are introductory and background review. We have found, however, that the student finds it very satisfying to be able to design a digital filter, although elementary, by the time Chapter 2 is completed. This was our chief motivation for covering digital filtering before the discrete Fourier transform and the fast Fourier transform (FFT) algorithm.

The two most likely paths to follow in teaching from this text are to cover the chapters as presented or in the order 1, 2, 5, 3, 4, 6.

We also hope the text will prove useful to disciplines other than electrical engineering, particularly those interested in digital filtering and discrete spectrum analysis using FFT techniques.

Donald Childers
Allen Durling
Gainesville, Florida

acknowledgments

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It is with special gratitude we acknowledge the contributions of numerous colleagues who have contributed to the field whose ideas we hope we have done credit to. In particular we thank Jim Kaiser for permission to use some of the figures he and his colleagues at Bell Telephone Laboratories have published on their hardware design of digital filters; Walt Elden, for his summaries of the speech process and multiplexing; E. M. Hofstetter and his colleagues of M.I.T. Lincoln Laboratories for his high order filter frequency response characteristics; R. P. Chambers of Bell Telephone Laboratories for his figures illustrating psuedo-random sequences; J. W. Cooley and his colleagues at IBM for their example of the effects of aliasing; J. D. Markel of the Speech

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We note with special thanks the permission granted by Benjamin Leon and Stephen Bass to reproduce in Appendix 2 their article on the effects of finite arithmetic on digital filter design.

We sincerely apologize to those we may have unintentionally overlooked.

Finally, any errors which may remain are solely those of the authors and should in no way reflect on those mentioned above.

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1 introduction

While digital signals and systems can be designed without reference to continuous systems, it is intuitively appealing and frequently an aid to understanding to build a theory of digital signal processing on the foundation of continuous signals and systems which most engineers already understand. Many applications of digital systems have continuous input signals which are converted to sampled data through analog-to-digital converters (ADC or A/D); on the other hand a digital system output might be used as the input to a continuous system through the use of digital-to-analog converters (DAC or D/A).

With these considerations in mind, Chapter 2 assists the reader familiar with continuous system analysis to make the transition to the now popular digital filtering and signal processing techniques. For those readers with a background in digital signal analysis Chapter 2 serves as more than review material for it provides a concise, interwoven integration of analysis procedures used for both continuous and digital systems.

2 Introduction [Ch. 1]

The primary emphasis of this book is directed toward the design of digital filters by various methods, the spectral analysis of discrete data from numerous sources, and other selected signal processing procedures. Therefore, it seems appropriate that we devote some of this chapter to a discussion of the broad terms which occur repeatedly throughout the text. Where possible we follow the IEEE standard terminology [1].

Discrete filters are composed of two classes: *sampled-data filters* and *digital filters*.

A *sampled-data filter* is an algorithm which acts upon an input which is a continuum of amplitude values quantized in the independent variable (usually time). The filter transforms the input sampled-data signal into another (output) sampled-data signal.

A *digital filter* is a device or program that performs a prescribed manipulation or algorithm on an input sequence of numbers resulting in the desired output sequence of numbers. The numbers are limited to a finite precision. For this text we consider only time-invariant, linear digital filters.

Historically, digital filters have been designed to perform operations analogous to analog or continuous filters. With fast, low-cost, digital integrated circuits it has been possible to implement accurate digital filters which are small in size even for operation in low frequency ranges where component size is a limitation for analog or continuous filters. Although digital filters have some disadvantages, such as the limited set of coefficient values available, once set, the values do not drift and they can be programmatically changed to alter the filter either for time sharing or for other applications.

Since several methods for the design of digital filters depend intimately on their analog or continuous counterparts it is important to distinguish among analog, continuous, discrete, quantized, and digital signals.

An *analog signal* is a (possibly discontinuous) function of the independent, continuous variable time which may take on values in a continuous range. The terms analog and continuous are frequently used

synonymously, but more recently the term continuous is being used in the context of *continuous-time* to imply that the independent variable in an analog waveform (or filter) takes on a continuous range of values. It is now recommended that the term continuous-time waveform be used in place of analog waveform.

Discrete signals are limited in the range of the independent and/or dependent variables. A *discrete-time signal* is a function defined on only a discrete set of time values t_0, t_1, t_2, \dots . If a signal is discretized in the range of values of the signal it is said to be *quantized*. Discrete-time signals that take on a continuum of values are known as *sampled-data signals*.

A *digital signal* is a discrete-time signal which is quantized in both time and amplitude. A digital signal is a *sequence of numbers*, perhaps quantized values of a sampled-data signal, produced in actual physical situations such as in digital circuitry hardware or computer programs.

To help clarify these definitions consider Figure 1.1 which illustrates quantization in amplitude. The quantization step size S can be made as small as is practical. Note that in this case the quantizer steps are very large and the quantized signal is not a good representation of the continuous-time waveform. Further, the quantization levels do not occur at uniformly spaced time intervals.

Figure 1.2 shows how a digital signal is generated for uniform sampling instants. Here an arbitrary unique coding is assigned to each quantization level. If a binary representation for the code is used, the number of quantization levels is selected as a power of 2.

From Figures 1.1 and 1.2 it is observed that the quantized signal and the original continuous-time signal differ. Further this difference or error is not predictable, but occurs randomly. Usually this error is modeled as noise which arises as a result of the quantization process and is referred to as *quantization noise*.

An expression for the error can be derived from statistical considerations. The peak-to-peak range of the signal is divided into N equal intervals each with

4 Introduction [Ch. 1]

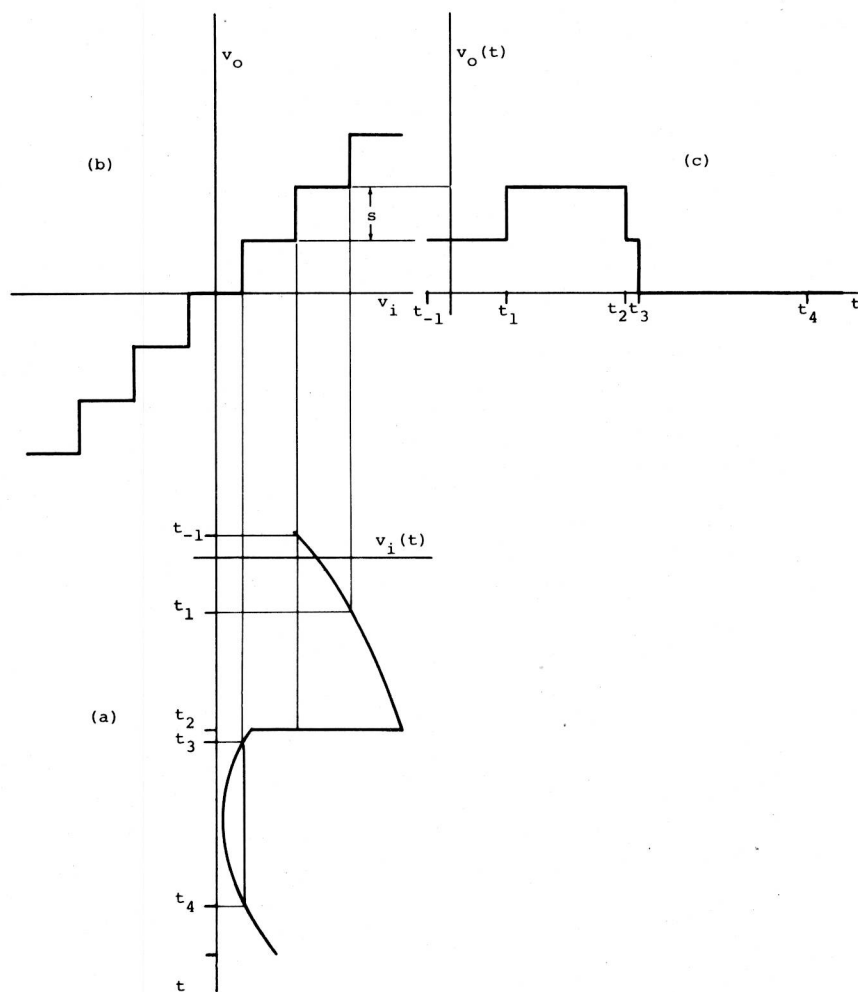


FIGURE 1.1. Amplitude Quantization: (a) Continuous-time waveform, (b) quantizer input-output characteristics, and (c) quantizer output - continuous-time waveform.

magnitude S as illustrated in Figures 1.1 and 1.2. The assumption is then frequently made that when the signal is located within a particular quantization interval the error or difference between the continuous-time signal and the particular quantization level is uniformly distributed over the quantization interval in question. In other words the error is equally-likely to occur over the interval. One can