

**Yang Hao ■ Raj Mittra**

# **FDTD**

## **MODELING OF METAMATERIALS**

**THEORY AND APPLICATIONS**

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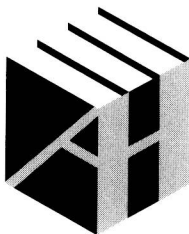
# FDTD Modeling of Metamaterials

Theory and Applications

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# **FDTD Modeling of Metamaterials**

**Theory and Applications**

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# Preface

Metamaterials can be generally defined as a class of “artificial” media, possessing extraordinary electromagnetic properties that cannot be found in natural ones. The subject of metamaterials has drawn considerable attention from both the physics and engineering communities worldwide and has received generous support from the funding agencies during recent years. The popularity of this topic has been adequately demonstrated by a rapid surge in the number of publications, special sessions at international conferences, research networks, and launching of new journals on the subject. Metamaterials are periodic electromagnetic structures that are not altogether dissimilar from frequency selective surfaces (FSSs), bianisotropic materials, and optical gratings, all of which have been around in the electromagnetic and optical communities for quite some time. Although there has been much hype recently about the extraordinary performance of devices containing metamaterials, recent studies have indicated that there are a number of fundamental issues, such as high losses and narrow bandwidth characteristics, that must be addressed before these materials can find widespread use in practical applications. Nevertheless, the study of metamaterials has engendered, perhaps for the first time, a widespread interest on the part of physicists, electronic engineers and material scientists, in pursuing collaborative and multidisciplinary efforts, with the common goal of developing an understanding of the fundamental physics of metamaterials, which, in turn, has the potential of achieving new breakthroughs in science and engineering. Research into metamaterials at Queen Mary College, London, was initiated in 2000 and has been supported by several grants from the United Kingdom’s Engineering and Physical Science Research Council (EPSRC). A range of computational techniques, including the finite-difference time-domain (FDTD) method, detailed in this book, have been developed for the modeling of metamaterials including electromagnetic bandgap (EBG) structures; left-handed materials (LHMs); artificial dielectrics; plasmonic waveguides; electromagnetic cloaking structures; and, a number of other devices designed for related applications of metamaterials. These computer codes have then been utilized for designing metamaterials and for gaining a physical insight into their electromagnetic characteristics. The FDTD has been widely accepted as one of the most efficient numerical techniques in computational electromagnetics and has been applied to periodic structures including the frequency selective surfaces (FSSs), which have previously found applications mainly as high-performance radomes and spatial filters, but are now finding new applications in metamaterial devices. The Electromagnetic Communication Laboratory of Pennsylvania State University has been engaged in the development of very high-performance computational electromagnetics (CEM) solvers capable of handling upward of  $10E+9$  unknowns. The GEMS code developed in this lab has



played a pivotal role in rigorously analyzing complex electromagnetic structures with multiscale features that often characterize metamaterials.

This book introduces the basics of the FDTD method, especially when it is used to model metamaterials. It shows how to compute the dispersion diagrams, deal with the material dispersion properties, and verify the left-handedness, among other things. Some metamaterials possess unique properties that require special treatments in the numerical code when we analyze them. This book explains how to properly define their material parameters and to characterize the interface of metamaterial slabs and quantify their spatial as well as frequency dispersion characteristics. There has been much recent interest in novel applications of metamaterials to antennas and microwaves and to various devices that have applications in optical engineering. In view of this, the book dedicates an entire chapter solely to this topic. It is shown how these structures can be modeled by using either the effective medium representation or the FDTD code. Though the latter is highly computer-intensive, we have argued that modeling the physical structure numerically and rigorously is the only way to obtain reliable results when attempting to predict the performance of metamaterial devices, because the rigorous results often disagree with those derived by using simplified models based on the effective medium approach. For this reason, we have devoted a substantial amount of space in this book to modeling the problem of the physical structures of metamaterials, instead of using their effective medium representations. In addition, we have analyzed the fundamental limits of metamaterials made from resonant particles, with the hope that the readers will get a true picture of the real-world metamaterials after going through these analyses. We view this book as a complement to a wide array of publications on the FDTD method that have preceded it, and we hope that colleagues in computational electromagnetics will benefit from recent advances in numerical techniques, especially the FDTD, when dealing with the problem of designing metamaterials.

# Acknowledgments

First and foremost, we would like to thank our former and current doctoral students for their many contributions to this work. They have been working and contributing diligently for more than five years to enrich and infuse new excitement into the study of FDTD and metamaterials. Without the results of their intensive studies, it would not be possible for us to put together the material for such a comprehensive book. In particular, we would like to thank Dr. Yan Zhao of Queen Mary College, London, for his contributions to the development of a spatially dispersive FDTD (Chapters 8 and 9) and the modeling of plasmonic wave guide and electromagnetic cloaks (Chapter 10); Dr. Wei Song of Queen Mary College for her work on conformal FDTD modeling (Chapters 2 and 3); Dr. Lai-Ching Ma of Penn State University for her work on parameter extractions for left-handed metamaterials (Chapter 7); Yoonjae Lee of Queen Mary College for his work on figure of merit studies on metamaterials (Chapters 5–7); Atiqur Rehman for the development of composite right-left handed transmission lines (CRLH-TL) and their applications to antenna engineering; and Christos Argyropoulos for his input to the modeling of silver lens and electromagnetic cloaks (Chapter 10).

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# Contents

Preface	<i>xi</i>
Acknowledgments	<i>xiii</i>
<b>CHAPTER 1</b>	
Introduction	1
1.1 What Are Electromagnetic Metamaterials?	1
1.2 A Historical Overview of Electromagnetic Metamaterials	2
1.2.1 Artificial Dielectrics	4
1.2.2 Artificial Magnetic Materials	8
1.2.3 Bianisotropic Composites	8
1.2.4 Double-Negative and Indefinite Media	9
1.2.5 Photonic and Electromagnetic Crystals	11
1.3 Numerical Modeling of Electromagnetic Metamaterials	15
1.4 Layout of the Book	17
References	18
<b>CHAPTER 2</b>	
Fundamentals and Applications of Electromagnetic Bandgap Structures	25
2.1 Introduction	25
2.2 Bloch's Theorem and the Dispersion Diagram	25
2.2.1 Translational Symmetry	26
2.2.2 Bloch's Theorem and Periodic Boundary Condition (PBC)	27
2.2.3 Brillouin Zone	29
2.2.4 Dispersion Diagram and EBG	30
2.3 An Overview of Numerical Methods for Modeling EBG Structures	33
2.3.1 The Generalized Rayleigh's Identity Method and the Korringa-Kohn-Rostoker (KKR) Method	33
2.3.2 Plane-Wave Expansion Method	35
2.3.3 The Transfer-Matrix Method	36
2.3.4 The Finite-Difference Time-Domain (FDTD) Method	39
2.4 An Overview of EBG Applications	41
2.4.1 In-Phase Reflection	41
2.4.2 Suppression of Surface Waves	45
2.4.3 EBGs Operating in Defect Modes	46
2.4.4 Subwavelength Imaging from the Passband of the EBGs	58

2.5	Summary	61
	References	61

### CHAPTER 3

	A Brief Introduction to the FDTD Method for Modeling Metamaterials	67
3.1	Introduction	67
3.2	Formulations of the Yee's FDTD Algorithm	67
3.2.1	Maxwell's Equations	67
3.2.2	Yee's Orthogonal Mesh	69
3.2.3	Time Domain Discretization: The Leapfrog Scheme and the Courant Stability Condition (CFL Condition)	70
3.3	Other Spatial Domain Discretization Schemes	72
3.3.1	Subgridding Mesh	72
3.3.2	Nonorthogonal Mesh	75
3.3.3	Hybrid FDTD Meshes	76
3.4	Boundary Conditions	78
3.4.1	Mur's Absorbing Boundary Conditions (ABCs)	78
3.4.2	Perfect Matched Layers (PMLs)	80
3.4.3	Periodic Boundary Condition (PBC)	81
3.5	Bandgap Calculation	83
3.5.1	Source Excitation	84
3.5.2	Dispersion Diagram Calculation	84
3.5.3	Transmission and Reflection Coefficient Calculation	85
3.6	Summary	87
	References	88

### CHAPTER 4

	FDTD Modeling of EBGs and Their Applications	91
4.1	Introduction	91
4.2	FDTD Modeling of Infinite Electromagnetic Bandgap Structures	91
4.2.1	Physical Model of EBG Structures	91
4.2.2	Mesh Generation and Simulation Parameters in FDTD Modeling	93
4.2.3	Simulation Results of Infinite EBGs Using the Conformal and Yee's FDTD	94
4.3	Conformal FDTD Modeling of (Semi-)Finite EBG Structures	102
4.3.1	FDTD Model and Simulation Results	102
4.4	Design and Modeling of Millimeter-Wave EBG Antennas	105
4.4.1	Introduction	105
4.4.2	Design and Modeling of Woodpile EBG	108
4.4.3	A Millimeter-Wave EBG Antenna Based on a Woodpile Structure	115
4.4.4	Experimental Results	117
4.5	Conclusions	121
	References	121

**CHAPTER 5**

<b>Left-Handed Metamaterials (LHMs) and Their Applications</b>	<b>123</b>
5.1 Introduction	123
5.2 Effective Medium Theory and Left-Handed Metamaterials	123
5.2.1 A Composite Medium of Metallic Wires and Split Ring Resonators	124
5.2.2 Isotropic Three-Dimensional Left-Handed Metamaterials	125
5.2.3 Left-Handed Metamaterials Using Simple Short Wire Pairs	126
5.3 Applications of Left-Handed Metamaterials	127
5.3.1 Imaging by a Perfect LHM Lens	127
5.3.2 Transmission Line Structures of Left-Handed Metamaterials	128
5.3.3 Directive Electromagnetic Scattering by an Infinite Conducting Cylinder Coated with LHMs	142
5.3.4 Negative Index Materials (NIM) for Selective Angular Separation of Microwave by Polarization	144
References	145

**CHAPTER 6**

<b>Numerical Modeling of Left-Handed Material (LHM) Using a Dispersive FDTD Method</b>	<b>147</b>
6.1 Introduction	147
6.2 The Effective Medium of Left-Handed Materials (LHMs)	148
6.3 Modeling of Left-Handed Metamaterials Using a Dispersive FDTD Method	156
6.3.1 Two-Dimensional Dispersive FDTD with Auxiliary Differential Equations (ADEs)	156
6.3.2 Phase Compensation Through Layered LHM Structures	160
6.3.3 Conjugate Dielectric and Metamaterial Slab as Radomes	161
6.3.4 Numerical Results	163
6.4 Conclusions	169
References	169

**CHAPTER 7**

<b>FDTD Modeling and Figure-of-Merit (FOM) Analysis of Practical Metamaterials</b>	<b>173</b>
7.1 Introduction	173
7.2 EM Response of the Infinite, Doubly Periodic DNG Slab with Plane Wave Illumination	174
7.2.1 Model Description of the Array Comprising of Split-Ring Resonators and Wires	174
7.2.2 Scattering Parameters Measurements Obtained from the PBC/FDTD Code	174

7.2.3	Phase Data Inside the DNG Slab	175
7.3	Retrieval of Effective Material Constitutive Parameters Using the Inversion Approach	182
7.3.1	Review of the Inversion Approach	182
7.3.2	Retrieval of the Effective Material Parameters from the Numerical S-Parameters Obtained from FDTD Simulations of Metamaterials	186
7.3.3	Summary of the Difficulties Encountered Using the Inversion Approach for Effective Medium Characterization	207
7.4	EM Response of a Finite Artificial-DNG Slab with Localized Beam Illumination	208
7.4.1	Slab with Localized Beam Illumination	209
7.4.2	FDTD Model	209
7.4.3	Total Transmission and Reflection Power Under Gaussian Beam Illumination	210
7.4.4	EM Response of the Artificial-DNG Slab at Normal Incidence with Ey Polarization	213
7.4.5	EM Response of the Artificial-DNG Slab at Oblique $TM_z$ Incidence Coming from $(\theta = 150^\circ, \phi = 90^\circ)$ with Hx Polarization	219
7.4.6	EM Response of the Artificial-DNG Slab at Oblique $TE_z$ Incidence Coming from $\theta = 150^\circ, \phi = 0^\circ$ with Ey Polarization	223
7.4.7	EM Response of a Finite Artificial-DNG Slab Excited by Small Dipole	226
7.5	Figure-of-Merit (FOM) Analysis	228
7.5.1	Loss and Bandwidth of Metamaterials with Different Electrical Sizes and Particle Densities	229
7.5.2	Figure-of-Merit Analysis by Numerical Experiments	232
7.6	Conclusions	235
	References	236

## CHAPTER 8

	Accurate FDTD Modeling of a Perfect Lens	239
8.1	Introduction	239
8.2	Dispersive FDTD Modeling of LHMs with Spatial Averaging at the Boundaries	241
8.2.1	The (E, D, H, B) Scheme	242
8.2.2	The (E, J, H, M) Scheme	244
8.2.3	The Spatial Averaging Methods	245
8.3	Numerical Implementation	250
8.4	Effects of Material Parameters on the Accuracy of Numerical Simulation	255
8.5	Effects of Switching Time	258
8.6	Effects of Transverse Dimensions on Image Quality	260

8.7 Modeling of Subwavelength Imaging	262
8.8 Conclusions	264
References	264

## **CHAPTER 9**

Spatially Dispersive FDTD Modeling of Wire Medium	267
9.1 Introduction	267
9.2 Spatial Dispersion in the Wire Medium	269
9.3 Spatially Dispersive FDTD Formulations	270
9.4 Stability and Numerical Dispersion Analysis	274
9.5 Perfectly Matched Layer for Wire Medium Slabs	279
9.6 Numerical Thickness of Wire Medium Slabs	282
9.7 Two-Dimensional FDTD Simulations	286
9.8 Three-Dimensional FDTD Simulations	294
9.9 Experimental Verifications	297
9.10 Internal Imaging by Wire Medium Slabs	299
9.11 Conclusions	303
References	304

## **CHAPTER 10**

FDTD Modeling of Metamaterials for Optics	307
10.1 Introduction	307
10.2 Dispersive FDTD Modeling of Silver-Dielectric Layered Structures for Subwavelength Imaging	307
10.2.1 Introduction	307
10.2.2 FDTD Modeling of the Silver-Dielectric Layered Structure	310
10.2.3 Numerical Results and Discussions	311
10.3 A Metamaterial Scanning Near-Field Optical Microscope	316
10.3.1 Introduction	316
10.3.2 Theory	317
10.3.3 Simulation	317
10.4 FDTD Study of Guided Modes in Nanoplasmonic Waveguides	321
10.4.1 Conformal Dispersive FDTD Method Using Effective Permittivities (EPs)	322
10.5 FDTD Calculation of Dispersion Diagrams	326
10.5.1 Wave Propagation in Plasmonic Waveguides Formed by Finite Number of Elements	331
10.6 FDTD Modeling of Electromagnetic Cloaking Structures	333
10.6.1 Dispersive FDTD Modeling of the Cloaking Structure	335
10.6.2 Numerical Results and Discussion	341
References	346

## **CHAPTER 11**

Overviews and Final Remarks	353
11.1 Introduction	353

11.2 Overview of Advantages and Disadvantages of the FDTD Method in Modeling Metamaterials	353
11.3 Overview of Metamaterial Applications and Final Remarks	354
11.3.1 Small Antennas Enclosed by an ENG Shell	357
11.3.2 Focusing and Superlensing Effects	361
11.3.3 Performance Enhancement of Planar Antennas	370
11.3.4 Electromagnetic Cloaks	370
References	371
List of Abbreviations	373
About the Authors	375
Index	377



# Introduction

## 1.1 What Are Electromagnetic Metamaterials?

There have been various definitions of *electromagnetic metamaterials* [1–114], where “meta” is a prefix in English meaning “beyond; transcending; more comprehensive.” In 2001, Walser [1] from the University of Texas at Austin, coined the term “metamaterial” to refer to artificial composites that “...*achieve material performance beyond the limitations of conventional composites.*” The definition was subsequently expanded by Browning and Wolf of Defense Advanced Research Projects Agency (DARPA) in the context of the DARPA Metamaterials program started also in 2001:

Metamaterials are a new class of ordered composites that exhibit exceptional properties not readily observed in nature. These properties arise from qualitatively new response functions that are: (1) not observed in the constituent materials and (2) result from the inclusion of artificially fabricated, extrinsic, low dimensional inhomogeneities.

Metamorphose, the European Network of Excellence [2], terms the metamaterials as:

Artificial electromagnetic (multi-)functional materials engineered to satisfy the prescribed requirements. Superior properties as compared to what can be found in nature are often underlying in the spelling of metamaterial. These new properties emerge due to specific interactions with electromagnetic fields or due to external electrical control. The metamaterials provide a conceptually new range of radio, microwave, and optical technologies.

Sometimes, metamaterials are specifically referred to as a class of artificial materials that have simultaneous negative permittivity and permeability and are also known as *left-handed materials* (LHMs). Present researchers have a tendency to expand the concept of metamaterials so as to make it as broad as possible. The editorial board of IEICE Transactions [3] even questions whether or not artificial materials such as CdS, GaAs, or InGaAs should have been classified as metamaterials. One popular classification of metamaterials is:

As an ordinary material is made of natural molecules, an artificial material is made of artificial molecules. Due to Maxwell equations’ macroscopic property, small particles made of typically metal and dielectric can be considered molecules when put together. The variation of each shape and total alignment makes macroscopically single negative, double negative, or double positive materials.

However, there exists a number of artificial electromagnetic structures, especially at microwave frequencies [e.g., electromagnetic crystals, high-impedance surfaces (HISs), and frequency selective surfaces (FSSs)]. Although these are made of small ordered metallic/dielectric inclusions, they cannot be homogenized by using conventional approaches and described in terms of constitutive parameters such as permittivity and permeability. Smith [4] at Duke University prefers to use the term metamaterials as artificial structures that display properties beyond those available in naturally occurring materials. This definition is a general one, and it may also include artificial dielectrics, artificial magnetics, and bianisotropic materials, which were the subject of extensive research back in the 1960s, 1970s, and 1990s. Most of the concepts in metamaterials originate from solid state physics that deal with the lattice structure of crystals, which is inherently periodic. Indeed periodic structures in nature have fascinating characteristics, which have frequently inspired scientists and engineers alike to think of novel applications of them. Periodic structures have had a long history in electromagnetics dating back to the 1900s, and they can be found as integral parts of microwave filters, traveling wave tubes (TWTs), antenna arrays, leaky wave antennas (LWAs), and FSSs, to name just a few. The role of the periodic structure has been the manipulation of the spectral and spatial spectrum, the selection of spatial harmonics to control the radiation of forward and backward waves, and the control of the phase and group velocities in slow wave structures. Periodic structures are also very popular among optical engineers and are widely used in the design of lasers. For example, a distributed Bragg reflector (DBR) is a structure composed of alternating layers of materials with varying refractive indices, or with periodically varying characteristics, such as height of a dielectric waveguide, which induces a periodic variation in the effective refractive index of the guide. Each layer interface induces a partial reflection of optical waves, at a wavelength for which many reflections undergo a constructive interference, thereby forming a high-quality reflector. The idea has been further extended to two and three dimensions by Yablonovitch [5] and John [6], who have described structures that are now broadly classified as photonic bandgaps (PBGs).

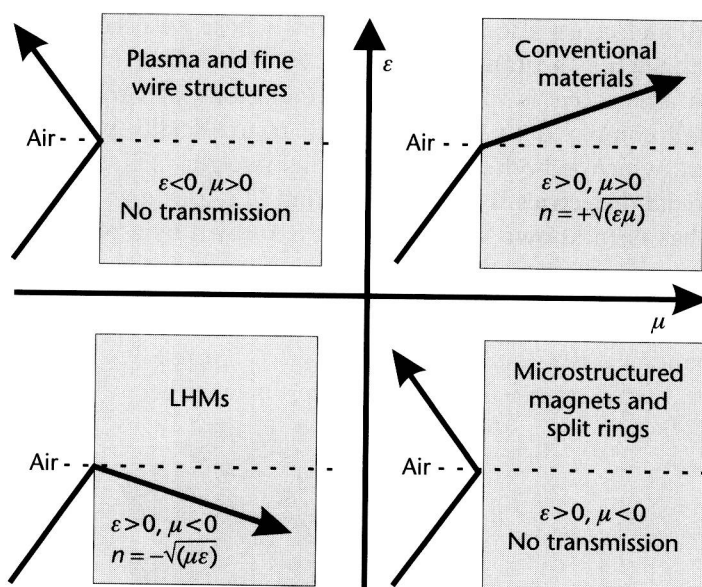
Earlier work of Yablonovitch on PBGs was carried out at microwave frequencies, by using a small dipole in a Fabry-Perot cavity formed by PBGs. They conjectured that such configurations will give a rise to increased directivity of small antennas by focusing their beam [5]. This has engendered new interests in the community of antenna and microwave engineering, and now the so-called electromagnetic bandgap (EBG) structures (in contrast to the PBGs that are their counterparts at optical frequencies) are finding usages in enhancing the performance of antennas and microwave devices.

## 1.2 A Historical Overview of Electromagnetic Metamaterials

Permittivity ( $\epsilon$ ) and permeability ( $\mu$ ) are two parameters used to characterize the electric and magnetic properties of materials interacting with electromagnetic fields. The *permittivity* is a measure of how much a medium changes to absorb electrical energy when subjected to an electric field [7, 8]. It is defined as a ratio of  $\vec{D}$  and  $\vec{E}$ , where  $\vec{D}$  is the electric displacement by the medium and  $\vec{E}$  is the electric field

strength. The common term dielectric constant is the ratio of permittivity of the material to that of free space ( $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ ). It is also termed as the relative permittivity. *Permeability* is a constant of proportionality that exists between magnetic induction and magnetic field intensity. Free-space permeability ( $\mu_0$ ) is approximately  $1.257 \times 10^{-6} \text{ H/m}$ . Recently, Ziolkowski [9, 10] has categorized metamaterials by their constitutive parameters as follows (Figure 1.1). Most of the materials in nature have positive permittivity and permeability, and hence, they are referred to as “double-positive (DPS)” media. In contrast, if both of these quantities are negative, they are called “double-negative (DNG)” and are also referred to as LHMs by others. Finally, materials with one negative parameter are named “single-negative (SNG)” and are further classified into two subcategories, namely, “epsilon-negative (ENG)” and “mu-negative (MNG).” Interestingly, natural materials such as cold plasma and silver exhibit negative permittivities at microwave and optical frequencies, respectively, and ferromagnetic materials exhibit a negative permeability behavior in the VHF and UHF regimes. However, to date, no materials that exhibit simultaneous negative permittivity and permeability have been found in nature, and hence, they must be created artificially.

The first comprehensive review of the history of negative refraction and metamaterials was given by Moroz [11]. He indicated that some of metamaterial research started long before Veselago’s work and went back to as far as 1905, when Lamb [12] suggested the existence of backward waves, which are associated with



**Figure 1.1** A diagram showing the possible domains of electromagnetic materials and wave refraction or reflection directions based on the signs of permittivity and permeability. The arrows represent wave vector directions in each medium. There is wave transmission only when both parameters have the same sign. Waves are refracted positively in conventional materials and negatively in LHMs. (From: [10]. © 2001 IEEE. Reprinted with permission.)