Advanced Physics Problems

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Advanced Problems in Physics

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Preface

In this collection, which is intended as a sequel to *Physics Problems*, the questions are grouped into five main sections which roughly correspond to the conventional Advanced level syllabus divisions. Although the old demarcations are changing, if not vanishing, this still seemed the easiest classification for use in a book. Since many years of teaching have led to the accumulation of this material it is impossible even to identify all the first sources. Of course we must acknowledge our indebtedness, both directly and indirectly, to several examining boards for ideas – and for the form of many old questions that have returned translated into SI. There are many items too which we think, but cannot positively claim, to be original.

There is some overlap in standard between the more difficult questions in *Physics Problems* and the easier ones (coded *I*) in the present book. This breadth may commend itself particularly to the wide range of interest in sixth form colleges. We hope that the collection will help its users to think for themselves and to understand, and that it will also be found to contain some stimulating teaching material.

B. K. Harris and G. R. Noakes.

Standards of difficulty are indicated by code letters ; *I* (introductory about 34 per cent), *A* (A level), and *S* (S level and Oxbridge entry and scholarship about 13 per cent). Constants and values, if not given in a question, are usually given at the start of the section or sub-section to which they apply. In a few questions the pupil is required to obtain the necessary data for himself in which the phrase 'data required' is used and in some, guidance in the method to be used is given. A complete set of answers is given at the end of the book.

Contents

1 Mechanics and general physics

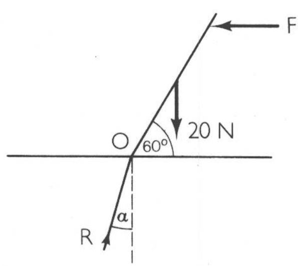
ELEMENTARY MECHANICS *Statics – moments, resolution and triangle of forces; Dynamics; Conservation of linear momentum; Conservation of energy; Power; Linear mechanics – more difficult problems*

Elementary problems in mechanics are intended to give practice in the use of the basic principles of statics and linear dynamics.

Take g to equal 9.8 m s^{-2} unless otherwise stated

Statics – moments, resolution and triangle of forces

1.1 *I* A uniform rod of mass 2 kg is pivoted at one end and supported at an angle of 60° to the horizontal by a horizontal force acting at the other end as shown in the diagram. The weight of the rod is 20 N acting through its centre of gravity and R is the reaction acting at the pivot at an angle α to the vertical.



(a) Take moments about O to find F.
(b) Resolve vertically and horizontally to find R.
(c) Check the results by the triangle of forces using the fact that when a body acted on by three forces is in equilibrium their lines of action pass through one point. (This may be done by drawing.)

1.2 *I* A concrete block of mass 200 kg is being pulled steadily up a slope which is at an angle of 30° to the horizontal by a rope attached to its leading bottom edge, which acts at an angle of 20° to the slope. The coefficient of friction between block and surface being 0.3, what is the tension T in the rope?

Method Draw a diagram putting in mg, the weight of the block, T, R the normal reaction and μR the force acting along the slope where μ is the coefficient of friction. Resolve perpendicular and parallel to the slope and hence find T. (The most important part of the exercise is the resolution.)

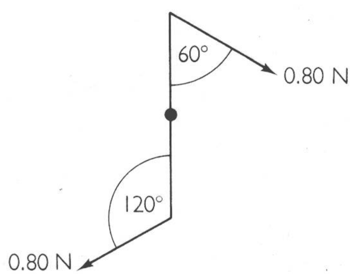
1.3 *I* A cylinder of length 50 mm and diameter 30 mm rests with its base on a rough plane which can be tilted. Find the angle of the slope with the horizontal at the point when it topples, assuming that it does not slip.

1.4 *I* The beam of a balance in which the three knife-edges are collinear has a pointer which is 0.18 m long, the total mass of beam and pointer being 0.06 kg. If the centre of gravity is 0.50 mm below the central pivot and the distance between consecutive knife-edges is 0.12 m, through what distance will the end of the pointer move when a mass of 10 mg is placed on one scale pan?

1.5 *I* The diagram shows a rod of length 0.25 m pivoted at its centre with two equal forces of 0.80 N acting at its ends as shown. Calculate

(a) the moment of the couple acting on the rod,

(b) the force acting on the pivot.



Dynamics

Take g as 10 m s^{-2} in the next three exercises

1.6 *I* A golf ball when hit along a horizontal fairway travels a distance of 140 m before hitting the ground, the maximum height reached being 20 m. Air resistance is negligible. Find

(a) the time of flight,

(b) the vertical component of the initial velocity,

(c) the horizontal component of the initial velocity,
(d) the initial velocity and angle of projection,
(e) the height reached 1.0 s after being hit.

1.7 *I* A mass of 0.50 kg on a smooth table is attached by a string to a mass m which hangs vertically over the smooth edge of the table. What is the value of m if it descends with an acceleration of 4.0 m s^{-2} and what is the tension in the string? What would be the acceleration for the same value of m if there was a frictional force of 1.5 N between the 0.50 kg mass and the table?

1.8 *I* A lift starts moving vertically upwards with an acceleration of 2.0 m s^{-2} after which it moves at a uniform speed and finally stops with a retardation of 2.0 m s^{-2}. A mass of 70 kg rests on a spring balance in the lift which is calibrated in newtons. What does the balance read (a) at the start, (b) in the middle, (c) at the end of the journey?

1.9 *I* A jet rocket ejects a mass m of fuel per second at a velocity v relative to the rocket. Show that the thrust is mv.

A landing module of mass 500 kg is moving at a constant velocity of 9 m s^{-1} directly towards the Moon at a point near its surface where the acceleration of free fall is 1.6 m s^{-2}. The constant velocity is maintained by a jet of velocity 640 m s^{-1} relative to the rocket which is directed vertically downwards. What mass of fuel is used per second? In order to reduce the velocity to 1 m s^{-1} for landing, the rate at which the fuel is burnt is doubled. For how long must this burst continue? Assume that the total mass of the module remains constant.

1.10 *I* A wind blowing normally on to a vertical board produces a pressure of 276 Pa on it. Assuming that the air after striking the board has zero velocity in a direction normal to the board, calculate the velocity of the wind, taking the density of the air to be 1.2 kg m^{-3}.

Method Write down an expression for the mass of the air impinging on a square metre of the board per second in terms of its velocity v. Then use the fact that force equals change of momentum per second, or mass per second multiplied by change of velocity.

1.11 *I* A ball of mass 0.15 kg has a velocity of 6.0 m s^{-1} when it strikes a bat. It is deflected through a right angle and leaves the bat with velocity 8.0 m s^{-1}. Calculate the impulse given by the bat to the ball in magnitude and direction.

Method If F is the force and δt the time for which it acts then $F\,\delta t$ is the impulse and this equals the change of momentum. Find the change of velocity by means of a triangle of velocities and hence the change in momentum.

Conservation of linear momentum

1.12 *I* One surface of a block of mass 1.5 kg standing on a table is hit by a ball of mass 0.20 kg travelling with a velocity of 5.0 m s^{-1} along a normal to the surface which bounces off with a velocity of 1.0 m s^{-1} along the same normal. What is the initial velocity of the block, assuming that it is free to move?

1.13 *I* A mass of 4.0 kg with a velocity of 3.0 m s^{-1} collides with a mass of 2.0 kg moving with a velocity of 1.0 m s^{-1} in the opposite direction. After the collision the 4.0 kg mass continues to move with a velocity of 0.5 m s^{-1} in the same direction. What is the velocity of the second mass?

1.14 *I* A trolley of mass 500 kg carries a man of mass 80 kg. When the trolley is travelling along horizontal lines with a velocity of 2.0 m s^{-1} the man jumps from its back and lands on the ground with zero velocity. What is the subsequent velocity of the trolley?

Conservation of energy

1.15 *I* A simple pendulum has a bob of mass *m* and a length of 0.80 m. The bob is pulled back keeping the thread taut until the thread is horizontal and then released. Use the conservation of energy to find the speed of the bob (a) at its lowest point, (b) when it has risen through an angle of 45° from its lowest point.

1.16 *I* A railway truck of mass 1000 kg moving horizontally with a velocity of 0.80 m s^{-1} collides with a stationary truck of mass 3000 kg. If after the collision the trucks move together calculate (a) the initial common velocity, (b) the energy lost in the collision. What has happened to this energy?

1.17 *I* A mass of 0.10 kg falls vertically through the atmosphere a distance of 5.0 m in 1.1 s. Calculate:

(a) the potential energy lost,
(b) the final velocity assuming uniform acceleration,
(c) the kinetic energy gained,
(d) the work done against air resistance,
(e) the magnitude of the average resistance.

Power

1.18 *A* (a) A railway truck of mass 2000 kg which runs on horizontal rails is accelerated from rest by a force of 500 N acting through its coupling. What power is being used to accelerate the truck at the end of 3.0 s?

(b) A lift is powered by a 20 kW motor. Calculate the total mass of the lift and load if it is travelling vertically upwards at a constant speed of $2.5\ \mathrm{m\ s^{-1}}$ and 20 per cent of the power is being used in work done against friction.
(c) Gravel falls vertically at the rate of $12\ \mathrm{kg\ s^{-1}}$ on to a horizontal conveyor belt moving at the rate of $0.30\ \mathrm{m\ s^{-1}}$. Calculate the gain of momentum per second of the gravel and the power required. Why is the power greater than the kinetic energy gained per second by the gravel?

Linear mechanics – more difficult problems

1.19 *A* A ballistic pendulum consists of a block of wood of mass 0.88 kg suspended by cords so that it is free to swing. A bullet of mass 0.020 kg is fired horizontally into the wood causing the block to swing and so rise through a vertical distance of 0.032 m with the bullet embedded in it.
Calculate the speed of the bullet just before it hits the block.

1.20 *A* A pile driver of mass 200 kg falls vertically through 3.2 m on to a pile of mass 600 kg which it drives into the ground a distance of 0.080 m. If the pile driver remains in contact with the pile after the collision what is the average resistance of the ground? Take g to equal $10\ \mathrm{m\ s^{-2}}$. (*Hint* The pile and driver both lose potential energy as well as kinetic energy when the ground is penetrated.)

1.21 *A* The spring of an air-gun requires a force of 4.0 N to compress it by 10 mm. When loaded the spring is compressed by 70 mm. Calculate the potential energy stored in the spring. Assuming that when the gun fires a pellet of mass 7.0×10^{-4} kg, 40 per cent of this energy is transferred to the pellet in the form of kinetic energy, calculate its muzzle velocity.

1.22 *S* A rocket, 75 per cent of whose mass is fuel burns the fuel at a steady rate, the gases being ejected at a velocity u relative to the rocket. Show that when all the fuel is burnt the velocity of the rocket is $1.39\,u$ if the rocket starts from rest. Neglect any work done against gravity during the acceleration period.

CIRCULAR MOTION

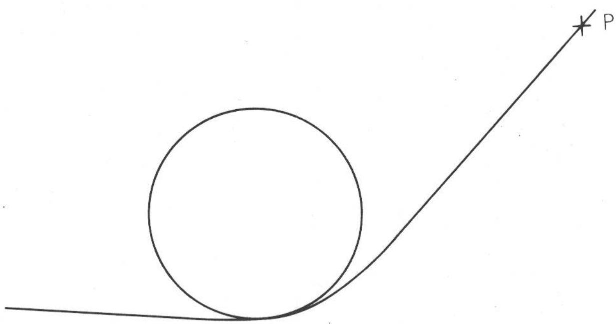
1.23 *I* A skater of mass 70 kg is skating along an outside edge in an arc of a circle. If the radius of the arc described by the centre of mass of the skater is 1.5 m and the angle between the vertical and the line passing through the edge and centre of mass of the skater is 10° calculate (a) the speed of the skater, (b) the horizontal force exerted by the ice on the skate edge.

1.24 *I* A cylinder of diameter 12 mm and height 18 mm is placed on the rotating table of a gramophone which revolves at 78 revolutions per minute with one flat end on the table. What is the maximum distance from the centre of the table at which it can be placed if it is not to topple, assuming that it does not slip?

1.25 *A* An aeroplane makes a tight turn of radius 600 m when travelling at a speed of 150 m s^{-1}. What is (a) the correct angle of banking, (b) the ratio of the normal reaction of the aeroplane on the pilot to the normal reaction when the aeroplane is stationary on the ground?

1.26 *A* It is possible for a motorcyclist to ride round the inside of a hollow vertical cylinder. Assuming that the centre of mass of man and cycle travels round a horizontal circle of diameter 8.0 m at a speed of 12 m s^{-1} calculate (a) the angle machine and rider make with the horizontal, (b) the minimum value of the coefficient of friction between tyres and wall if there is to be no slipping.

1.27 *A* A small body of mass m is held at a point P on a smooth loop-the-loop track as shown in the diagram, the radius of the circular loop being r.



Find in terms of r (a) the minimum height of P above the lowest point if the body, when released, is to complete the loop, (b) the reaction of the track on the body at the lowest point after it is released from P, (c) the reaction when the body has risen through a vertical height of $r/2$ from the lowest point after release from P.

1.28 *A* Estimate the difference in the observed acceleration of free fall at the poles and the equator due to the rotation of the earth. The radius of the earth is 6.4×10^6 m.

1.29 *S* A horizontal metal hoop of diameter 0.80 m and mass 1.5 kg is rotated in a horizontal plane about a vertical axis passing through its centre.

What is the greatest angular velocity if the tension in the hoop is not to exceed 10^3 N?

MOMENT OF INERTIA

1.30 *I* A wheel rotating on its bearings has a moment of inertia of 1.2×10^{-2} kg m^2. If it takes 15 s for the angular velocity of the wheel to drop from 25 rad s^{-1} to 5.0 rad s^{-1} what is the frictional couple acting?

1.31 *I* A force F acts tangentially on the rim of a disc of radius r to produce a torque $\boldsymbol{T}$. Find the work done per revolution by the force and hence show that the work done per revolution by the torque is $2\pi \boldsymbol{T}$.

The wheel in 1.30 is accelerated from rest by a couple of 2×10^{-2} N m in excess of that required to overcome friction. Use the conservation of energy to find the angular velocity of the wheel after it has made 8 revolutions. What power is being used at this angular velocity to accelerate the wheel?

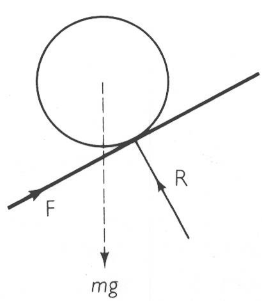
1.32 *I* A hoop, disc, sphere and a wheel of radius of gyration equal to half its radius all have the same mass and radius, and roll with the same velocity. Compare their kinetic energies. The moment of inertia of a disc about an axis through its centre and perpendicular to its plane is $mr^2/2$ and that of a sphere about its centre is $2/5mr^2$.

1.33 *A* A flywheel of diameter 0.10 m free to rotate about a horizontal axis has a thin cord attached to its rim. The cord is wound round the rim and to its free end is attached a mass of 0.20 kg which hangs vertically. When released the mass descends a distance of 0.80 m in 1.6 s. Calculate the moment of inertia of the wheel by the following methods:

(a) equating the potential energy lost to the gain of kinetic energy of wheel and mass,

(b) calculating the acceleration of the mass and the tension in the cord.

1.34 *A* A sphere starting from rest rolls without slipping down an incline of 1 in 8. Use the following two methods to find its linear velocity after it has travelled a distance of 1.6 m. The moment of inertia of a sphere about its centre is $2/5\ mr^2$. (See over for method.)



Method 1 Let m be the mass and r the radius of the sphere and apply the conservation of energy.
Method 2 Show that the moment of inertia of the sphere about the point of contact with the slope (instantaneous centre of rotation) is $7/5\ mr^2$ and obtain an expression for the angular acceleration and hence obtain the linear acceleration from the torque taken about this point (see diagram).

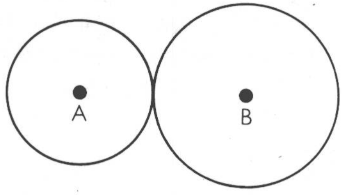
1.35 *A* A uniform rod of length 0.49 m which is pivoted at one end so that it hangs freely is raised at the other end until it is horizontal, and then released. Calculate (a) the angular velocity, (b) the angular acceleration at the instant that the angle between the rod and the horizontal is 60°.

The moment of inertia of a rod about its end is $\frac{1}{3}ml^2$ where m is the mass of the rod and l its length. (*Hint* Use E_k and E_p for first part.)

ANGULAR MOMENTUM

1.36 *A* A mass of 10^{-2} kg on a smooth table is attached to a thread which passes through a hole in the middle of the table and is held taut beneath. The mass is set in motion so that it describes a circle of radius 0.30 m with an angular velocity of 4.0 rad s^{-1} at the end of the thread. The end of the thread beneath the table is then pulled vertically downwards until the radius of the circle described by the mass is reduced to 0.20 m. Calculate (a) the final angular velocity, (b) the initial and final velocities, (c) the work done by the force pulling the thread down.

1.37 *S* Two similar disc wheels A and B can rotate freely on parallel axles. Both are made of the same material and have the same thickness, but the radius of B is $\sqrt{2}$ times that of A. If the moment of inertia of A is I, what is that of B?



Wheel A is made to rotate at 20 rad s^{-1} and its rough rim is brought into contact with that of the wheel B which is at rest (see diagram). What will be the subsequent angular velocity of the wheel B? (*Hint* Let p be the impulse between the two rims. Changes in the angular momentum of each wheel can then be considered. Conservation of angular momentum cannot be directly applied. Why is this?)

1.38 *S* A billiard ball at rest is given an impulse by a cue in a horizontal direction at a height equal to the radius of the ball above the table. The ball skids on the table before it starts to roll. If v is the velocity of the ball immediately after it is hit, what will be its velocity when it rolls?

What would happen to the same ball if it was rolling on the table with velocity v and then made a perfectly elastic collision with a table cushion which it hit normally? What would be the velocity of the ball when it started to roll? The moment of inertia of a sphere about its centre is $2/5\,mr^2$.

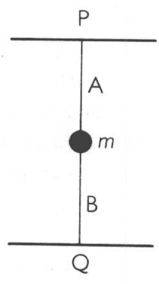
SIMPLE HARMONIC MOTION

1.39 *I* A mass of 10^{-2} kg oscillates with simple harmonic motion of period 2.0 s and amplitude 5.0×10^{-2} m. Calculate

(a) the maximum accelerating force exerted on the mass,
(b) its acceleration when the displacement is 3.0×10^{-2} m,
(c) the total energy of the motion (*Hint* There is one point where the energy is all kinetic.)
(d) the velocity when the displacement is 4.0×10^{-2} m,
(e) the shortest time taken for the mass to pass between points distant 2.0×10^{-2} m and 4.0×10^{-2} m from the centre.

1.40 *I* A particle moving with simple harmonic motion has a velocity of $0.15\ \mathrm{m\,s^{-1}}$ and an acceleration of $0.36\ \mathrm{m\,s^{-2}}$ when its displacement from the mean position is 4.0×10^{-2} m. Calculate the period and amplitude of the motion.

1.41 *A*



A mass m when suspended from the lower end of a piece of elastic which is fixed at its top extends it by 0.060 m. The unstretched elastic is cut into two halves A and B and attached to the mass and to two fixed points P and Q which are in a vertical line as shown in the diagram. The mass m is displaced vertically through a short distance and released. Assuming that both pieces of elastic are in tension throughout show that the subsequent motion is simple harmonic. If its period is 0.25 s calculate a value for the acceleration of free fall.

What would be the period of vibration if the unstretched elastic had been cut so that B had twice the length of A?

1.42 *A* A barometer tube is filled with mercury and a finger placed over the open end before it is inverted in a bowl of mercury with the open end just below the surface. When the finger is removed the mercury descends and oscillates about its eventual equilibrium position. Show that when these oscillations are of small amplitude they are of a simple harmonic nature and estimate their period for a normal barometer reading. Why must the vibrations be small?

Assume that the area of the open surface of mercury is sufficiently large for changes in its level to be negligible.

1.43 *A* A horizontal metal plate which has sand thinly spread on it vibrates with simple harmonic motion of amplitude 9.0×10^{-2} m in a horizontal plane. What is the maximum frequency of the oscillations if the sand is not to move relative to the plate? The coefficient of friction between sand and table is 0.4.

1.44 *A* A spiral spring which has a force constant of σ per unit extension of the spring hangs vertically with a mass m attached to the lower end. The spring is unstretched initially and the mass when released performs vertical simple harmonic oscillations of amplitude d. The mass of the spring itself is negligible. Find expressions for

(a) the period of oscillation and hence the kinetic energy at the mid-point,

(b) the relation between d and σ,

(c) the potential energy of the mass at the mid-point and the lowest point in terms of σ and d taking the zero of potential energy at the top point,

(d) the elastic potential energy stored at the mid and bottom points.

Hence show that the total energy is zero at all three points. Eventually the mass comes to rest. What happens to the kinetic energy lost?

1.45 *A*