

A PRACTICAL APPROACH

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Montclair State College



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MATHEMATICS: A PRACTICAL APPROACH

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To our spouses, Judi Kalmanson and Fred Chichester

and our children,

Andrew and Matthew Kalmanson

Lori and Edward Kenschaft

Preface

Mathematics is rapidly becoming a part of the language of business, economics, biology, the social sciences, and even, to a lesser degree, the liberal arts. More and more colleges and graduate schools are requiring a practical knowledge of mathematics as a prerequisite for study in these fields, and students are finding mathematics increasingly useful in their intermediate-level courses.

This text is designed for such students. Numerous applications from business and economics appear throughout the book; they can be used to motivate discussion and to furnish a pattern of illustrations for the mathematical techniques used in these fields. Applications from biology and the social sciences are also included in each chapter.

Throughout the book we have tried to keep the writing practical and straightforward. For example, the phrase “if it exists” has been omitted in several places where it would have been included in a more rigorous exposition. Anything that is discussed is assumed to exist. The presentation of each concept is intuitive, but more detailed discussions (including proofs) are found in the appendixes.

Each section of the book is designed for one lesson. Nearly every section is followed by three exercise sets. Either exercise set A or set B constitutes a single homework assignment; the exercises in these sets are patterned on worked-out examples in the section. The C exercises are more challenging, require more originality, and, in addition, contain supplementary exercises in biology and the social sciences. Answers to the A and C exercises are in the text; answers to B exercises are given in the Instructor's Manual. All the exercises are graded in difficulty and involve a minimum of algebraic manipulation and numerical computation. Exercises where a hand calculator would be desirable are marked with an “HC” and can be included or omitted at the discretion of the instructor.

Each chapter is followed by a sample test, which can be used for self-testing, enabling students to pinpoint quickly any lack of understanding. Answers to the sample tests are given at the back of the book.

We assume that our readers have had two years of high school algebra or the equivalent. This does not mean they must remember all of elementary algebra, but rather that they be sufficiently familiar with it so they can relearn it with the help of a teacher. Chapter 0 contains a “man-

datory" review of linear equations and graphing, which may be covered just before Chapter 1 or Chapter 11, depending on the preference of the instructor. In addition, Chapter 10 (Sections 10.1 and 10.2) reviews exponents and logarithms. The diagnostic test on page 1 will help the reader decide whether or not the more extensive review of algebra provided at the back of the book is required.

The material in this book consists of three major sections: Linear Mathematics (Chapters 1–6); Probability, Statistics, and the Mathematics of Finance (Chapters 7–10); and Calculus (Chapters 11–15). There is enough material included for a one-year, 8-semester-hour course if the book is covered from beginning to end. For a 6-semester-hour course, one can delete a chapter or two (for example, Chapters 9 and 10), or perhaps several sections from different chapters. One can also rearrange the material as we do at Montclair State College, covering calculus before linear mathematics. The possible permutations depend only upon the chapter flowchart on page vii and the needs and ingenuity of the instructor. The text can also be used in a one-semester course offering 3 or 4 semester hours; some suggestions for such a course are on page viii.

We wish to express our special appreciation to the following people, all of whom read chapters and offered valuable suggestions: Robert Bixby, Northwestern University; Robert Canavan, Monmouth College; Richard Detmer, University of Tennessee, Chattanooga; Richard Gabriel, Seton Hall University; Charles Masiello, Pace University; Peter Rice, University of Georgia; Joseph Rosenstein, Rutgers University; Charles Sinclair, Portland State University; Neil Weiss, Arizona State University. Our colleagues at Montclair State College also have been consistently helpful, especially as they class-tested large portions of the book.

Others who offered helpful suggestions include Frank Warner, University of Pennsylvania; Calvin Butler, College of Southern Idaho; Suzanne Butschun, Tacoma Community College; Kenneth Rager, Metropolitan State College. We also wish to thank Kusum Jain and Clare May, both of whom checked the page proofs for mathematical and typesetting errors.

Finally, we are grateful to the many people at Worth Publishers who guided this book from an idea to a reality. Joe Zurla has supported our projects from the very beginning. Our editor, Sally Anderson, has been generous with good ideas, patience, and humor as we worked to improve the quality of the book. To these people and many others, too numerous to name, we express our gratitude.

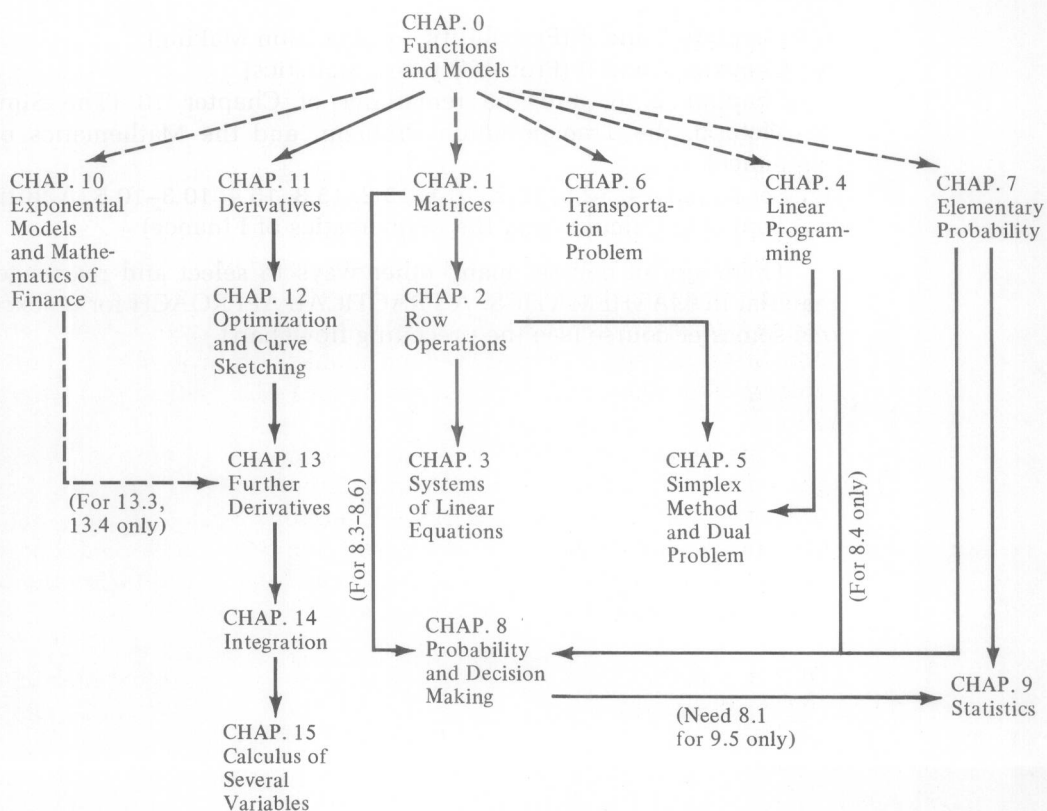
February 1978
Upper Montclair, N.J.

Kenneth Kalmanson
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How to Use the Book

Chapter Flowchart

The accompanying diagram shows which chapters are required for those that follow (a dashed line indicates that a chapter is desirable but not necessary for the one that follows).



Suggestions for One-Semester Courses

Topic	Sections	Number of Lessons
Functions and Models	0.1–0.5, 10.1, 10.2	7
Matrices	1.1–1.5	5
Linear Equations	2.1, 2.2, 2.4, 3.1, 3.2, 3.5	6
Linear Programming	4.1–4.4	4
Derivatives	11.1–11.5	5
Applications	12.1–12.4	4
More Derivatives	13.1, 13.3–13.5	4
Integration	14.1–14.6	6
		Total: 41

The remaining 4 class periods of a standard 3-credit/45-period course would normally be allotted to testing and/or review.

In a 4-credit/60-period course, the additional 15 periods could be allotted to:

1. Chapters 7 and 8 (Probability and Decision Making)
2. Chapters 7 and 9 (Probability and Statistics)
3. Chapters 5, 6, and the remainder of Chapter 10 (The Simplex Method, the Transportation Problem, and the Mathematics of Finance)
4. Sections 12.4, 12.5, 13.2, 16.1, 15.2, 15.3, 15.4, 10.3–10.5 (Additional Topics in Calculus and the Mathematics of Finance)

There are, of course, many other ways to select and rearrange the material in *MATHEMATICS: A PRACTICAL APPROACH* for an exciting one-semester course (see the preceding flowchart).

Applications

BIOLOGY AND ECOLOGY

Projecting the size of a bacteria culture

- Examples 0.2.10, 10.1.8
Exercises 10.1.A problem 10 B problem 10

Surface area of a horse

- Exercise 0.4.C problem 5

Human blood types

- Exercise 1.2B problem 4

Ecological systems, input-output matrices

- Example 1.3.10
Exercises 1.2.A problem 3 B problem 7

Bacteria surviving in a test tube

- Examples 2.1.3, 3.1.10
Exercise 2.1.C problem 1

Stocking a lake with fish totaling maximum weight

- Example 4.2.6
Exercise 4.2.B problem 4

Animals grazing

- Exercise 4.3.C problem 4

Blood types

- Exercises 7.1.B problem 4 7.2.B problem 8

Heredity and genes

- Example 7.2.6
Exercises 7.2.A problems 8, 9 B problems 8, 9 7.4.C problem 4
7.5.B problem 2 7.6.A problem 7

Lawn care

- Exercise 7.7.A problem 5

Pine tree deaths

- Example 8.1.9
Exercise 8.1.A problem 1

Enforcing antipollution laws

- Exercises 8.1.B problem 6 8.2.B problem 1

Planting strategies

- Example 8.2.4
Exercises 8.2.A problem 3 8.3.A problem 6 8.5.B problem 4

Rate of change of populations

- Example 11.3.9
Exercises 11.2.C problem 3 14.1.C problem 2

Pulmonary ventilation

- Exercise 11.3.B problem 6

Mink farm harvest

- Exercise 12.2.C problem 4

	Contraction of an amphibian's thigh muscle
Example	12.3.15
	Changing volumes of fruit
Exercise	13.5.C problem 1
	Total change of a bacteria culture or rabbit population
Example	14.4.10
Exercise	14.4.C problem 1
	Average number of rabbits
Exercise	14.5.C problem 4
	Rate of cricket chirping
Exercise	15.3.B problem 6
	PALEONTOLOGY
	Carbon dating (recent fossils and evolution)
Example	14.3.8
	Beryllium dating (deep-sea sediment)
Exercise	14.3.C problem 1
	Thorium dating (coral and shells)
Exercise	14.3.C problem 2
	MEDICINE
	Balancing a diet
Example	0.1.12
Exercise	0.1.C problem 4
	Sigmoid function (limited-growth curves)
Example	0.2.9
	Hemoglobin and oxygen combination in lungs
Example	0.3.14
	Poiseuille's law (blood flow in an artery)
Example	0.4.9
	Heart disease
Examples	7.1.6, 7.1.7, 7.2.12
Exercise	7.2.A problem 4
	Cancer and testing
Example	7.7.1
Exercise	7.7.B problem 5
	Selecting hospitals for a pilot study
Exercise	9.1.B problems 5, 6, 7
	Gestation periods
Exercise	9.5.B problem 4
	Rate of respiration
Exercises	11.2.C problem 4 11.3.A problem 6
	Concentration of drugs in body fluids
Example	12.1.13
Exercises	13.3.A problem 5 B problem 5 13.4.C problem 2
	Epidemic analysis and control
Exercises	8.2.B problem 2 8.3.B problem 6 8.4.B problem 4 12.1.C problem 1
	Reynolds number (aorta size)
Example	13.4.6
	Potassium isotope tracer
Exercises	13.3.A problem 6 B problem 6 14.4.C problem 2

- Example **Fick's law and diffusion through a cell membrane**
14.1.12
- Exercise **Average pollution in the air**
14.6.C problem 5
- Exercise **Diffusion equation of artery or vein**
15.1.C problem 1
- Exercise **Feeding experimental animals**
15.3.A problem 5
- Exercise **Death rates of American men due to lung cancer**
15.3.C problem 5

PSYCHOLOGY

- Exercise **College test scores**
1.2.A problem 4
- Example **Dominance in dogs**
7.6.10
- Exercise **Verifying the skill of wine tasters**
7.6.A problem 12
- Exercises **Movements of mice in a partitioned cage**
8.5.B problem 7 8.6.B problem 3
- Exercises **IQ distribution**
9.2.B problem 2 9.3.B problem 2 9.4.B problem 6
- Exercise **Mental retardation**
9.5.C problem 2
- Example **Loudness and intensity**
10.2.8
- Example **"Eureka" experience**
11.1.7
- Exercise **Fechner stimulus-response equation**
11.1.C problem 6
- Exercise **Learning-time models**
13.5.C problem 2
- Exercise **Learning-time models**
14.6.C problem 4

SOCIOLOGY

- Example **Population growth, limited**
0.2.9
- Example **Demographic input-output matrix**
1.1.12
- Exercise **Home owning and renting of American whites and nonwhites**
7.3.A problem 4
- Exercises **Death rates of American white and nonwhite babies**
7.3.B problem 4 8.1.A problem 5
- Example **Hiring practices and discrimination**
7.6.11
- Exercises **Birthdays problem**
7.6.A problem 11 B problem 12
- Exercise **Spread of rumors**
7.6.C problem 1
- Example **Spread of rumors**
8.6.5

- Exercises 8.6.A problem 6 B problem 6
Heights of American men
- Example 9.5.7
Frequency of letters in English
- Exercises 9.2.A problem 3 B problem 3
Rate of change of populations
- Example 11.2.8
Epidemic control
- Example 12.3.14
 Exercise 12.3.C problem 5
Doubling times of populations of specific countries
- Exercises 13.3.A problem 4 B problem 3
Projected population of earth
- Example 13.4.5
 Exercise 14.3.A problem 4
Population growth, unchecked
- Example 14.1.11
Growth rates of populations
- Exercises 14.3.A problem 4 B problem 4
Projected populations of United States
- Exercise 14.3.B problem 4
Life expectancy of American women
- Exercise 15.3.A problem 6
- ARCHAEOLOGY**
- Example **Carbon dating of artifacts**
 14.4.9
 (See also Paleontology)
- BUSINESS AND ECONOMICS**
- Example **Monetary conversion tables as functions**
 0.1.3
- Exercises 0.1.A problem 5 B problem 5
Linear cost-output model
- Examples 0.1.8, 0.1.9, 0.1.10, 0.1.11, 0.5.2
 Exercises 0.1.A problems 6, 7, 8, 9 B problems 6, 7, 8, 9 0.4.A problems 3, 4, 5 B problems 3, 4, 5 0.5.A problems 2, 3 B problems 2, 3
Demand function
- Examples 0.2.6, 6.1.3, 13.2.10
 Exercises 0.5.A problem 4 B problem 4 13.2.A problem 4 B problem 6
 15.1.A problem 5 B problem 5
Determining nonlinear and least-squares models
- Examples 0.4.8, 15.3.2, 15.3.3
 Exercises 0.4.C problems 2, 3 15.3.B problems 4, 5
Postage rates
- Example 1.1.6, 1.1.8, 1.3.8
Motorcycle tire prices
- Exercise 1.1.A problem 7
Steel shelving prices
- Exercise 1.1.B problem 7

	Parts-listing problems
Examples	1.2.1, 1.3.9, 2.6.2
Exercises	1.2.A problems 8, 9 B problems 8, 9 1.3.A problems 6, 7 B problems 6, 7 2.6.A problems 2, 3, 4 B problems 2, 3, 4
	Leontief matrices
Example	1.2.7
Exercises	1.2.A problems 1, 2 B problems 1, 2
	Stadium ticket sales
Example	1.3.6
Exercises	1.3.A problems 2, 3 B problems 2, 3
	Computing time needed for a series of jobs
Exercises	1.3.A problems 4, 5 B problems 4, 5
	Open Leontief models
Examples	1.4.1, 2.5.4
Exercises	1.4.A problem 3 B problem 3 C problem 1 Sample Test 1 problem 8 2.5.A, B, C
	Filling orders
Examples	2.1.1, 2.1.2, 2.2.1
Exercises	2.1.A problems 1, 2, 3, 4 B problems 1, 2, 3, 4 2.2.A problems 1, 2, 3, 4 B problems 1, 2, 3, 4 C problems 1, 2 Sample Test 2 problem 1
	Accounting model
Example	2.6.1
Exercises	2.6.A problem 1 B problem 1
	Closed Leontief models
Example	3.2.5
Exercise	3.2.C problem 1
	Maximizing profits
Examples	4.1.5, 4.2.2, 4.2.3, 4.2.7, 4.3.2, 4.3.3, 5.2.1
Exercises	4.2.A problems 2, 3, 4, 5 B problems 2, 3, 5 4.3.A problems 1, 2, 3, 4 B problems 1, 2, 3, 4
	Mutual funds
Example	5.3.2
	Minimizing shipping costs
Examples	6.1.1, 6.1.5, 6.4.1
Exercises	6.1.A problem 1 B problem 1
	Minimum cost of producing items on different machines
Example	6.1.7
Exercises	6.1.A problem 2 B problem 2
	Assigning projects to plants within a company
Exercises	6.1.A problem 4 B problem 4
	Glassblowers' defective bottles
Example	7.2.11
Exercise	7.2.B problem 4
	Life insurance rate
Example	8.1.4
	Commission income
Exercise	8.1.A problems 2, 3, 4
	Pay of construction workers
Exercise	8.1.B problems 2, 3, 4

	Pricing various products advantageously
Examples	8.3.10, 8.4.1
Exercises	8.3.A problem 4 8.4.A problem 4
	Credit card holders
Examples	8.5.7, 8.5.9, 8.9.10
	Choosing which product to produce
Example	8.6.4
Exercises	8.6.A problem 7 B problem 7
	Defective ball bearings and manufacturer
Example	9.5.6

Exercises Using Hand Calculators

	Chapter 0
Exercises	0.1.C problem 5 0.4.C problem 5
	Chapter 7
Exercise	7.6.C problem 1
	Chapter 9
Exercises	9.2.A problem 3 B problem 3 9.3.A problems 3, 4 B problems 3, 4 9.4.A problems 4, 5 B problem 5 C problem 4
	Chapter 10
Exercises	10.3.A problem 5 B problem 5 C problems 1, 2
	Chapter 11
Exercises	11.1.A problems 1, 6 B problems 1, 6 C problem 7 11.2.C problem 1 11.3.A problem 2 B problem 2
	Chapter 12
Exercise	12.2.C problem 3
	Chapter 13
Exercises	13.1.C problem 3 13.2.C problem 3 13.5.A problem 8 B problem 8
	Chapter 14
Exercises	14.6.A problem 6 B problem 6
	Chapter 15
Exercise	15.3.C problem 5

Contents

	DIAGNOSTIC TEST	1
Chapter 0	FUNCTIONS AND MODELS	3
0.1	Functions as Models	3
0.2	Graphs as Models	12
0.3	Slopes and Straight Lines	24
0.4	Determining Linear and Quadratic Models	33
0.5	Functions of Several Variables	42
Chapter 1	MATRICES: BASIC SKILLS AND APPLICATIONS	51
1.1	Definitions, Addition, Scalar Multiplication, and Notation	51
1.2	Parts-Listing and Input–Output Matrices; Triangular, Diagonal, and Symmetric Matrices	58
1.3	Matrix Multiplication and Vector Inner Products	70
1.4	Input–Output Models and Compact Notation	80
1.5	Identities and Inverses	86
Chapter 2	GAUSS–JORDAN ROW OPERATIONS	96
2.1	Linear Equations with a Unique Solution	96
2.2	Linear Equations with a Unique Solution (continued)	106
2.3	Elementary Matrices	112
2.4	Finding the Multiplicative Inverse of a Matrix	118
2.5	Using Inverses in Leontief Models	124
2.6	Parts-Listing Problem and Accounting Model	130
Chapter 3	SYSTEMS OF LINEAR EQUATIONS WITHOUT UNIQUE SOLUTIONS	136
3.1	Recognizing Nonunique Solutions	136
3.2	Finding Nonunique Solutions of a System of Linear Equations	144

3.3	Analysis of Traffic Flow Networks	155
3.4	Geometric Interpretations of Linear Equations	160
3.5	Classical Expansion of Determinants	169
Chapter 4	INTRODUCTION TO LINEAR PROGRAMMING	180
4.1	Graphing Linear Inequalities	180
4.2	Setting Up Linear Programming and the Graphical Approach	189
4.3	Tabular Solutions of Linear Programming Problems	200
4.4	Minimum Problems	209
Chapter 5	THE SIMPLEX ALGORITHM AND DUAL PROBLEMS	216
5.1	Solving Standard Linear Programming Problems Using the Simplex Algorithm	216
5.2	Why the Simplex Algorithm Works	225
5.3	Linear Programming Problems That Are Not Standard	234
5.4	Definition of Dual Problems and Economic Interpretation	247
5.5	Solving for the Independent Variables in a Dual Problem	253
Chapter 6	THE TRANSPORTATION PROBLEM	260
6.1	Northwest Corner Algorithm and Minimum Cell Algorithm	260
6.2	The Stepping-Stone Algorithm	275
Chapter 7	ELEMENTARY PROBABILITY	287
7.1	Sets	287
7.2	Sample Spaces, Events, and Probability	296
7.3	Joint, Marginal, and Conditional Probabilities	308
7.4	Independent Events and Repeated Trials	317
7.5	The Binomial Theorem and Counting	328
7.6	Permutations and Combinations	336
7.7	Bayes' Theorem	343
Chapter 8	PROBABILITY AND DECISION MAKING	349
8.1	Expected Value	349