ELECTRICAL SCIENCE SERIES

Frequency Domain Criteria for Absolute Stability

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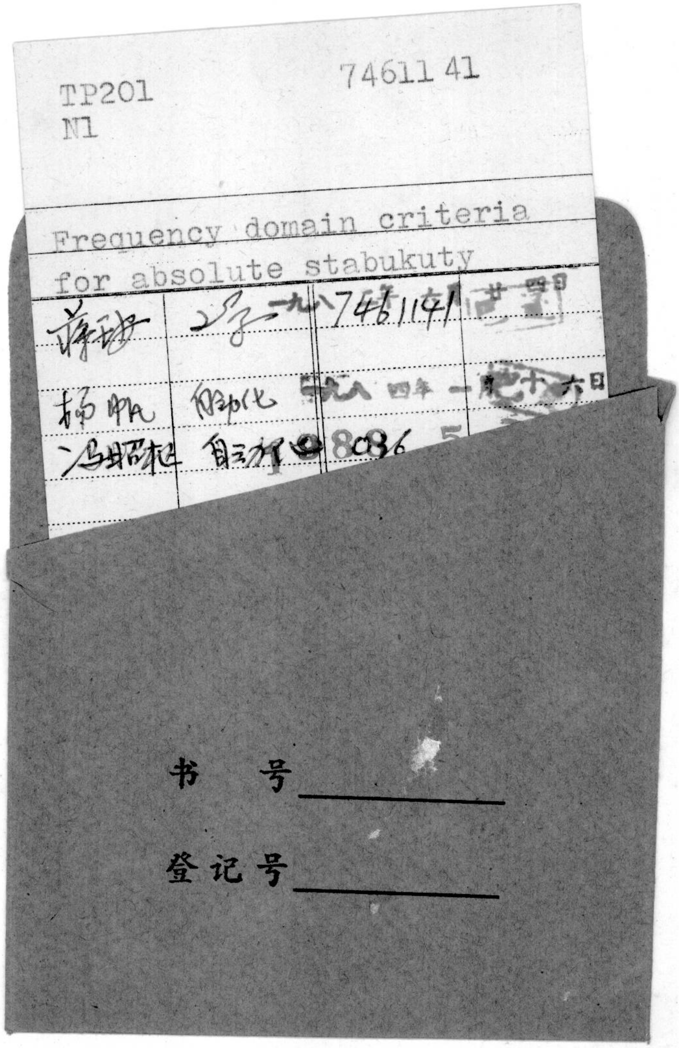

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To
BARBARA AND ANNE-MARIE

FOREWORD

To write a highly interesting book in a field in which there exists a rich literature is certainly a difficult task. The present book fully realizes this performance. Its success is due mainly to the great care with which the authors selected the material included in the book, with the obvious aim of giving the reader a deep and broad understanding of the subject.

While the main theorems are recent ones, the authors show very clearly the strong connections of the theory with the classical results of Hurwitz, Lyapunov, Nyquist, etc. Their reappraisal of the traditional engineering methods of control theory—including the daring but often successful technique of "describing functions"—provides an opportunity to point out another important source of the ideas that generated the contemporary view of the problem. While developing the theory with care for rigor and generality, the authors also show much concern for concrete examples and often illustrate the general theory by significant and illuminative applications, treated in detail.

These qualities make the book very useful even for persons who have little or no previous knowledge of the subject. These people will find this book an excellent introduction to the field. On the other hand, those already familiar with the subject will find a detailed exposition of some of the most advanced results which are harder to find elsewhere and which are due mainly to the outstanding research done in the field by the authors themselves.

Books which successfully cover such a broad range of interests are rare. They are also very much needed, because they are bound to produce a favorable influence upon research.

V. M. Popov

PREFACE

This book presents some recent generalizations of the well-known Popov solution to the absolute stability problem proposed by Lur'e and Postnikov in 1944. The Popov frequency domain stability criterion and the results of several earlier approaches to the Lur'e–Postnikov problem are presented in detail in the excellent books of Lefschetz and of Aizerman and Gantmacher; the work that led to the formulation of the absolute stability problem and the first solutions to it are not considered here.

The success of Popov's elegant criterion inspired many extensions of the basic Lur'e–Postnikov problem. Studies of these related questions gave rise to a great number of stability criteria, derived using both the direct method of Lyapunov and the positive operator concept of functional analysis. The great interest in this area has resulted in a continuing state of rapid development. The generation of this type of frequency domain stability criteria has now reached a relative state of completeness. It is also notable that the two seemingly disparate analytic approaches have led to stability criteria that are equivalent in most respects, and thus it is possible to present a unified picture of the recent research in this area using only Lyapunov's direct method. In each of the two fundamental approaches there are several points of view which have been used to good effect by various groups of researchers. It should thus be noted that this book is founded on a single set of techniques based on the direct method of Lyapunov and developed first at Harvard University and then at Yale University and the Indian Institute of Science (Bangalore, India). This makes the book rather specialized in its overall scope, but the techniques are found to be applicable to a wide range of important questions regarding the stability of nonlinear systems.

In view of the approach taken, several important results derived using a functional analysis viewpoint have either been omitted entirely or only mentioned in passing. Since the emphasis is on the application of Lyapunov's direct method to generate frequency domain criteria for stability, many fine results related to other aspects of the stability problem are omitted. The bibliography is by no means complete for this reason and contains only works directly related to the problems discussed.

Continuous-time systems are considered here, although many similar results already exist for discrete systems. In the first eight chapters, systems with a single nonlinear function or time-varying parameter are treated. Systems with multiple nonlinearities or time-varying gains are considered in Chapter IX; some criteria are derived in detail while others are presented in outline form as an indication of the state of current research.

This book can serve very well as a reference for research courses concerning stability problems related to the absolute stability problem of Lur'e and Postnikov. Engineers and applied mathematicians should also find the results contained herein, particularly the geometric stability criteria, of use in practical applications. Because of the diversity of the audience being addressed, rigorous theory is developed with what we hope can be considered a minimum of mathematical formalism. Certain sections contain some quite condensed technical material required as a foundation for the derivations; these may be omitted by those whose interest is limited to applications.

It is assumed that the reader is familiar with matrix operations that are utilized in dealing with the state vector representation of dynamic systems. All definitions and theorems are developed as needed so that the derivations are independent of other works; some acquaintance with the basic concepts of stability and Lyapunov's direct method would be helpful. The historical development of the work associated with the Lur'e–Postnikov problem has been strongly linked to the theory of automatic control, so control systems terminology is used sparingly wherever it is reasonable to expect that the meaning is clear to all readers.

ACKNOWLEDGMENTS

The authors are indebted to Roger M. Goldwyn, Charles P. Neuman, and Yo-Sung Cho (who were graduate students under the guidance of the first-named author), and M. A. L. Thathachar and M. D. Srinath, all of whose work forms an important basis for the material presented here. An early review of this effort by V. M. Popov was also significant; his suggestions and generous comments provided both encouragement and improvement in completing the final version of this work.

It is a pleasure for the authors to acknowledge colleagues and graduate students who have generously given their assistance both in general discussions and in recommending specific changes in the manuscript. The efforts of R. Viswanathan, M. D. Srinath, N. Viswanadham, and S. Rajaram are especially appreciated in this regard. Finally, an important factor in the completion of this effort was the able assistance provided by Mrs. Coralie Wilson, Mrs. Jean Gemmell, and Mrs. Anne-Marie Taylor, who typed many drafts and corrections.

The support of various institutions has also been invaluable. The second-named author would like to acknowledge the support received from Yale University while a graduate student and also the generosity of the Indian Institute of Science where he was recently a Visiting Assistant Professor.

One of the authors was in Bangalore, India while the other was in New Haven, Connecticut during the entire period of preparation of this book. That the sequence of corrections, additions, and revisions carried across several continents finally converged is in itself an achievement in the eyes of the authors. It may be safely said that without the patience, understanding, and encouragement of our wives, Barbara Narendra and Anne-Marie Taylor, this book would not have been completed.

SPECIAL NOTATION

Throughout this book the following symbol conventions are generally adhered to:

(i) Scalars are denoted by lower case Greek characters ($\rho, \tau, \sigma_0 = h^T x + \rho\tau$ etc.). The principal exception is the independent variable t (time).

(ii) Column vectors and explicit functions are denoted by lower case Latin characters ($x, h; f(\sigma_0)$, etc.). The notation $x = 0$ signifies that all elements x_i of the vector are zero.

(iii) Matrices, transfer functions, function classes, function bounds, and n-dimensional Euclidean spaces are denoted by capital Latin characters ($A = [a_{ij}]$; $G(s)$; $f(\sigma_0) \in \{F\}$; $\underline{K} \leqslant k(t) \leqslant \bar{K}$; $x \in X$). Upper and lower bounds are distinguished by bars above and below, respectively. $A = \underline{0}$ denotes the null matrix ($a_{ij} = 0$, all i and j) and I is the unit matrix ($I = \text{diag}\,(1, 1, \ldots, 1)$).

(iv) Function ranges (as in (iii) above) may be specified by the notation $k(t) \in [\underline{K}, \bar{K}]$. A bracket indicates a closed interval, whereas a parenthesis indicates an open interval. Thus $k(t) \in (0, \bar{K}]$ signifies that $0 < k(t) \leqslant \bar{K}$.

(v) The transpose of a vector or matrix is designated by a superscript T ($x^T = [x_1, x_2, \ldots, x_n]$; $A^T = [a_{ji}]$), and the inverse of a non-singular matrix is denoted by a superscript -1 [$(sI - A)^{-1}$].

(vi) The notation $G(s) \in \{Z_N\}$ denotes the membership of $G(s)$ in some class $\{Z_N\}$ of transfer functions. In particular, {PR} is the class of all positive real functions and {SPR} the class of all strictly positive real functions (Chapter III, Section 5).

(vii) Systems may be classified as being
LTI (linear time-invariant)
LTV (linear time-varying)
NLTI (nonlinear time-invariant)
or NLTV (nonlinear time-varying).

(viii) The complete notational designation of nonlinear time-varying gain functions ($g(\sigma_0, t) \in \{G_i[N, T]\}$) is detailed in Chapter II, Section 1.

(ix) The classes of matrices $\{A_1\}$ and $\{A_0\}$ are defined in Chapter II, Section 1.

(x) The superscript * denotes the complex conjugate of a scalar or the complex conjugate transpose of a vector or matrix.

(xi) An open square (□) indicates the end of a theorem, lemma, definition, or proof.

(xii) In all theorems and lemmas, $Z(s)^{\pm 1}$ implies that either $Z(s)$ or $Z^{-1}(s)$ can be used in satisfying the indicated condition.

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CONTENTS

IX. Absolute Stability of Systems with Multiple Nonlinear Time-Varying Gains