Introduction to Fluid Mechanics

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Preface

This book is intended for use in an introductory course in fluid mechanics. The student is expected to have completed two years of college mathematics and to be familiar with ordinary differential equations, partial differentiation, multiple integrals, Taylor series, and the basic elements of vector analysis.

The book is based primarily on a common core fluid mechanics course in which the author participated while at Northwestern University. There, each department (with the exception of Electrical Engineering) offered advanced undergraduate courses in fluid mechanics, and it was necessary to provide a rigorous foundation in the common core course. The first eight chapters are the result of experience in teaching that course. They can be covered satisfactorily in approximately 40 lectures, and thus can be used in either a three-unit semester course or a four-unit quarter course.

Chapters 9, 10, and 11 were added to provide flexibility for those persons who may wish to use the book in a terminal course. Under these circumstances it may be necessary to delete some material in Chapters 1–8; however, certain sections must be covered if subsequent material is to be understood. These sections are marked with an asterisk.

Vector notation is used freely throughout the text, not because it leads to elegance or rigor but simply because fundamental concepts are best expressed in a form which attempts to connect them with reality.

A variety of people contributed in innumerable ways to the completion of this text; they have the author's thanks. Special appreciation is due Professor John C. Slattery of Northwestern University, for the origin of the text rests largely on an endless series of conversations with him regarding the problems of teaching fluid mechanics to undergraduates.

Davis, California STEPHEN WHITAKER

Nomenclature†

Roman Letters

a	acceleration vector (82)
A	cross-sectional area, portion of a closed surface (33)
$A_e(t)$	area of entrances and exits (214)
$A_s(t)$	area of solid moving surfaces (214)
A_s	area of solid fixed surfaces (214)
A^*	characteristic area (287)
$\mathscr{A}$	area of a closed surface fixed in space (33)
$\mathscr{A}_a(t)$	area of an arbitrary closed surface moving in space (33)
$\mathscr{A}_m(t)$	area of a closed material surface (33)
b	width (47)
b	arbitrary constant vector (86)
C	constant of integration (41)
C_D	drag coefficient (304)
C_d	discharge coefficient (333)
C_c	contraction coefficient (334)
c	wave speed (372), velocity of sound (399)
c_p	constant pressure heat capacity per unit mass (402)
c_v	constant volume heat capacity per unit mass (402)
D	tube diameter (5)
d	rate of strain tensor in Gibbs notation (133)
d_{ij}	rate of strain tensor in index notation (133)
e	internal energy per unit mass (392)
$\mathbf{e}_{(i)}$	unit base vectors for rectangular, Cartesian coordinate system (26)

† Page number in parentheses indicates where the symbol is first defined.

$\dot{E}_v$	rate of viscous dissipation (227)
E_{sp}	specific energy (360)
f	friction factor (287)
F_D	drag force (286)
$\mathbf{F}$	force vector (10)
$\mathbf{g}$	gravity vector (34)
g	magnitude of gravity vector (41)
g_c	gravitational constant (11)
h	fluid depth (43), enthalpy per unit mass (394)
h_f	friction head loss (308)
h_m	minor head loss (308)
H_w	change in head caused by a pump or turbine (308)
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit base vectors for rectangular, Cartesian coordinate system (24)
$\mathbf{I}$	unit tensor (129)
k	thermal conductivity (404)
K	head loss coefficient (311)
KE^*	characteristic kinetic energy per unit volume (287)
ℓ	length (47), Prandtl mixing length (201)
L	length (42)
L_e	entrance length (171)
m	mass (402), Ostwald-de Wael model parameter (20)
$\dot{m}$	mass flow rate (265)
M	mass (10), Mach number (403)
MW	molecular weight (13)
n	number of moles (9), Ostwald-de Wael model parameter (20), Manning roughness factor (352)
$\mathbf{n}$	outwardly directed unit normal (35)
N_{Re}	Reynolds number (5)
$N_{Re,x}$	length Reynolds number (431)
N_{Fr}	Froude number (165)
N_{ca}	cavitation number (22)
p	absolute pressure (38)
p_{vp}	vapor pressure (22)
p_g	gauge pressure (45)
p°	stagnation pressure (407)
p_0	ambient or atmospheric pressure (43)
P	dimensionless pressure (160)
$\mathscr{P}$	dimensionless pressure which includes the body force term (160)
q	volumetric flow rate per unit width (67)
$\mathbf{q}$	heat flux vector (392)
Q	volumetric flow rate (61)
$\dot{Q}$	rate of heat transfer (392)
r, θ, z	cylindrical coordinates (94)
r, θ, ϕ	spherical coordinates (94)
$\mathbf{r}$	spatial position vector (76)
$\bar{\mathbf{r}}$	position vector locating the center of stress (49)
$\mathbf{R}$	material position vector (76)
R	dimensionless radius (162), gas constant (402)
R_h	hydraulic radius (159)
$\mathscr{R}$	universal gas constant (402), dimensionless ratio of like quantities (162)
s	entropy per unit mass (397), arc length (98)
S	wetted perimeter (351)
$\mathbb{S}$	scalar function (93)
t	time (10)
$\mathbf{t}_{(\mathbf{n})}$	stress vector (35)
$\mathbf{t}^*_{(\mathbf{n})}$	net stress vector (257)
T_0	Bingham model yield stress (19)
T	absolute temperature (9)
T°	stagnation temperature (407)
$\mathbf{T}$	total stress tensor in Gibbs notation (112)

T_{ij} total stress tensor in index notation (115)
$\mathscr{T}$ torque vector (48)
u_0 characteristic velocity (159)
u_∞ velocity far removed from an immersed body (430)
$\mathbf{v}$ fluid velocity vector (77)
$\mathbf{U}$ dimensionless fluid velocity vector (160)
v_x, v_y, v_z scalar components of $\mathbf{v}$ in rectangular Cartesian coordinates (24)
v magnitude of fluid velocity vector (222)
v^+ dimensionless fluid velocity (206)
$\mathbf{v}_r$ relative fluid velocity vector (156)
V volume (54)
$\mathscr{V}$ control volume fixed in space (33)
$\mathscr{V}_a(t)$ arbitrary volume moving in space (33)
$\mathscr{V}_m(t)$ material volume (33)
$\mathbf{w}$ arbitrary velocity vector (79)
$\dot{W}$ rate of work (227)
x, y, z rectangular, Cartesian coordinates (24)
X, Y, Z dimensionless rectangular, Cartesian coordinates (164)
y^+ dimensionless distance (206)

Greek Letters

α_{ij} direction cosine (118)
β coefficient of expansion (13)
γ specific gravity (55), ratio of specific heats (402)
δ boundary layer thickness (426)
δ_{ij} Kronecker delta (130)
ε/D relative roughness (293)
η length (47)
θ angle (47)
Θ dimensionless time (160)
κ compressibility (13), bulk coefficient of viscosity (133)
λ wave length (374)
$\boldsymbol{\lambda}$ unit tangent vector (99)
μ shear coefficient of viscosity (14)
μ_{app} apparent viscosity (19)
μ_0 Bingham model viscosity (19)
$\mu^{(t)}$ eddy viscosity (200)
ν kinematic viscosity (16)
π 3.1416. . .
ρ density (2)
σ surface tension (21)
$\boldsymbol{\tau}$ viscous stress tensor in Gibbs notation (130)
τ_{ij} viscous stress tensor in index notation (130)
$\bar{\boldsymbol{\tau}}^{(t)}$ turbulent stress tensor (194)
τ_0 wall shear stress (205)
ϕ gravitational potential function (40)
Φ viscous dissipation function (223)
ψ stream function (102)
ω angular velocity (178)
$\boldsymbol{\omega}$ vorticity vector (152)
$\boldsymbol{\Omega}$ vorticity tensor in Gibbs notation (152)
Ω_{ij} vorticity tensor in index notation (138)

Mathematical Symbols

∇ "del" vector operator (40)
∇^2 the Laplacian (154)
$\frac{D}{Dt}$ material derivative (78)
$\frac{d}{dt}$ total derivative (77)
$\frac{\partial}{\partial t}$ partial derivative (79)
$\langle\ \rangle$ area or volume average (109)
— time average (187)

Contents

* Sections marked with an asterisk must be covered if subsequent material is to be understood.

† The general form of Newton's law of viscosity is listed in the previous section.
‡ This section requires supplementary discussion if it is presented independently of the previous section.

Introduction

1

This chapter is devoted to a brief discussion of the fundamental postulates governing the motion of fluids, the types of flow that are to be investigated, the physical properties of fluids, and vector notation. The first five sections are qualitative and may be read quickly; however, Sec. 1.6 must be studied carefully for we will draw upon that material throughout the remaining chapters.

*1.1 The Continuum Postulate

The object of this text is to formulate the equations governing the motion of a continuum and apply them to the problem of fluid motion. In treating a fluid as a continuum we postulate that functions such as velocity, pressure, density, etc. are continuous point functions. In actual fact this is not true, for the materials we wish to study are made up of molecules. We may speak of the velocity of a molecule with some assurance that this quantity is well defined; however, the velocity at a fixed location in space is rather meaningless from the molecular point of view. We need not be concerned with this dilemma, because the cases we wish to study represent a class of problems for which the distance between molecules is so small that they represent a continuous system.

The density of a fluid may be defined as

$$\rho = \lim_{\Delta V \to 0} \left(\frac{\Delta M}{\Delta V} \right) \tag{1.1-1}$$

where ΔM is the mass contained in a small volume ΔV. As defined by Eq. 1.1-1, the density ρ might be represented by the curve shown in Fig. 1.1-1.

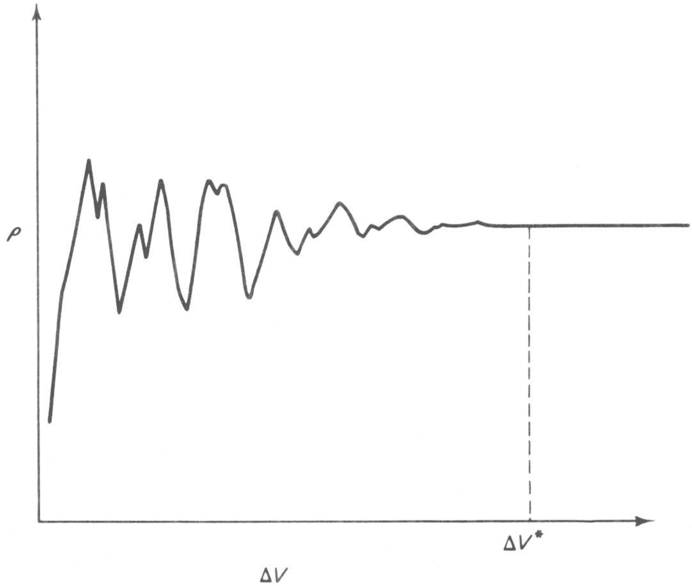

Fig. 1.1-1. Density as a function of volume.

The volume ΔV^* is the same order of magnitude as the cube of the mean free path† for gases, and is comparable to the volume of a molecule for liquids. For the continuum approach to hold, we must be dealing with systems that have dimensions much larger than either the mean free path or the molecular diameter. Since both these quantities are generally quite small, we can expect satisfactory results for a great many practical situations. Two examples where the continuum approach must be used with caution are the following: the motion of a spacecraft through the upper atmosphere; the motion of a gas through the pores of a catalyst pellet such as those currently used in petroleum refining processes. In the first case, the pressure is very low; thus, the mean free path of the gaseous molecules is large (on the order of 1 ft at 70 mi from

† The mean free path is the average distance traveled by molecules between collisions.

the earth).[1] In the second case, the mean free path of the molecules may be very small (on the order of 10^{-6} cm), yet the pore size in the catalyst pellet may also be extremely small and comparable to the mean free path.[2] In both of these examples, the velocity is *not* a continuous function, and we consider these to be cases of *slip flow*. We use the term "slip flow" because there is not intimate contacting of the fluid molecules with the solid surfaces, and the velocity of the fluid at the solid-fluid interface need not be zero.

*1.2 Types of Flow

We shall examine several types of flow in this text, and the boundaries which divide them into various classes are not always clear. Some distinct differences exist, however, and it will be helpful to discuss them.

Compressible and incompressible flow

Very often a fluid is considered incompressible if its density undergoes "negligible" changes for "appreciable" changes in temperature and pressure. The words negligible and appreciable are rather vague, and they have meaning only in terms of our experience. Thus, the density of water changes by less than 5 per cent in 100°C and less than 1 per cent in 100 atm, and we are inclined to consider water as incompressible. In actual fact water is taken to be an incompressible fluid simply because the types of flows which *generally* occur with water are satisfactorily treated by the incompressible form of the equations of motion. However, if we heat a pan of water on the stove, we note that circulation patterns are set up. They occur because warm water at the bottom of the pan is less dense than the cooler water at the surface, and buoyancy effects give rise to convective flows. Although it is the nonuniform density of the fluid which causes the flow, such a flow is usually not termed "compressible," a term which general usage reserves for flows where the fluid velocity approaches or exceeds sonic velocity (i.e., the speed of sound). This situation is more likely to occur in gases where the sonic velocity is about 1100 ft/sec at normal temperatures and pressures. The "sonic boom" caused by high-speed jet aircraft is an obvious example of the rapid changes in density (and pressure) that occur at a shock wave.

Sonic velocity in water is about 4700 ft/sec; thus, we might expect that compressible flows are less likely to occur. However, the common phenomenon of "water hammer" that occurs when a valve is suddenly closed in a water

1. E. J. Opik, *Physics of Meteor Flight in the Atmosphere* (New York: Interscience Publishers, Inc., 1958), p. 13.

2. P. Emmett, ed., *Catalysis* (New York: Reinhold Publishing Corp., 1955), Vol. 2, p. 126.

line is a case of compressible flow in liquids. The point here is that we must consider whether a given *flow* may be treated as incompressible, *not* whether the *fluid* is incompressible. Compressible flows are treated in Chap. 10.

Laminar and turbulent flow

The distinction between laminar and turbulent flows is somewhat easier to make than the distinction between compressible and incompressible flows. Laminar flow is characterized by smooth motion of one lamina of fluid past another, while turbulent flow is characterized by an irregular and nearly random motion superimposed on the main motion of the fluid. The two types of flow can be observed in the trail of smoke leaving the burning cigarette shown in Fig. 1.2-1. The smoke rises from the cigarette in a smooth, laminar stream for perhaps 1 or 2 in.; however, at that point it generally becomes unstable and a transition to turbulent flow takes place. This is characterized by whirls and a more random motion of the smoke stream as it rises into the air.

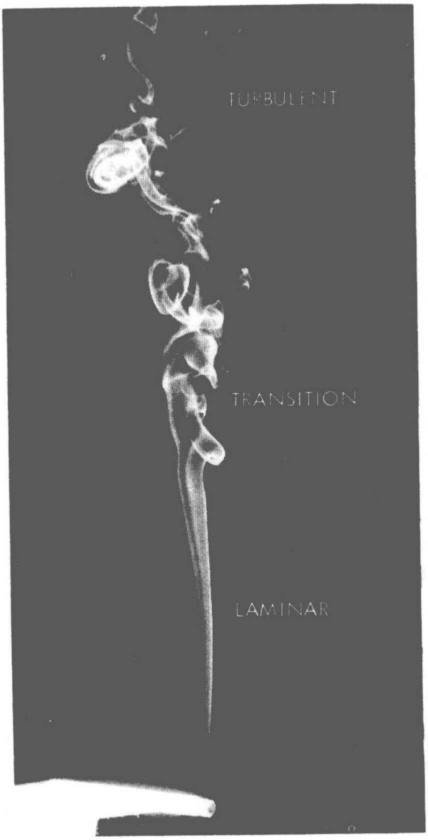

Fig. 1.2-1. Laminar and turbulent flow from a burning cigarette.

The transition from laminar to turbulent flow in tubes was first investigated by Osborne Reynolds,[3] and a sketch of the apparatus used by Reynolds is illustrated in Fig. 1.2-2. The system consisted essentially of a bell-mouthed glass tube into which a dye streak was injected with the water that entered the tube from a reservoir. Reynolds observed two distinct types of flow. In the first, the dye streak maintained its identity and remained in the center of the tube, although it spread slowly

3. O. Reynolds, "An Experimental Investigation of the Circumstances which Determine whether the Motion of Water Shall be Direct or Sinuous and the Law of Resistance in Parallel Channels." *Phil. Trans. Roy. Soc. (London) Ser. A*, 1883, 174: 935.

because of molecular diffusion. In the second, the dye streak was soon dispersed throughout the tube when the laminar flow that existed at the entrance of the tube underwent the transition to turbulent flow. The dispersion of the dye streak is similar in some respects to the dispersion of the thin stream of smoke given off by the cigarette. Reynolds found that the

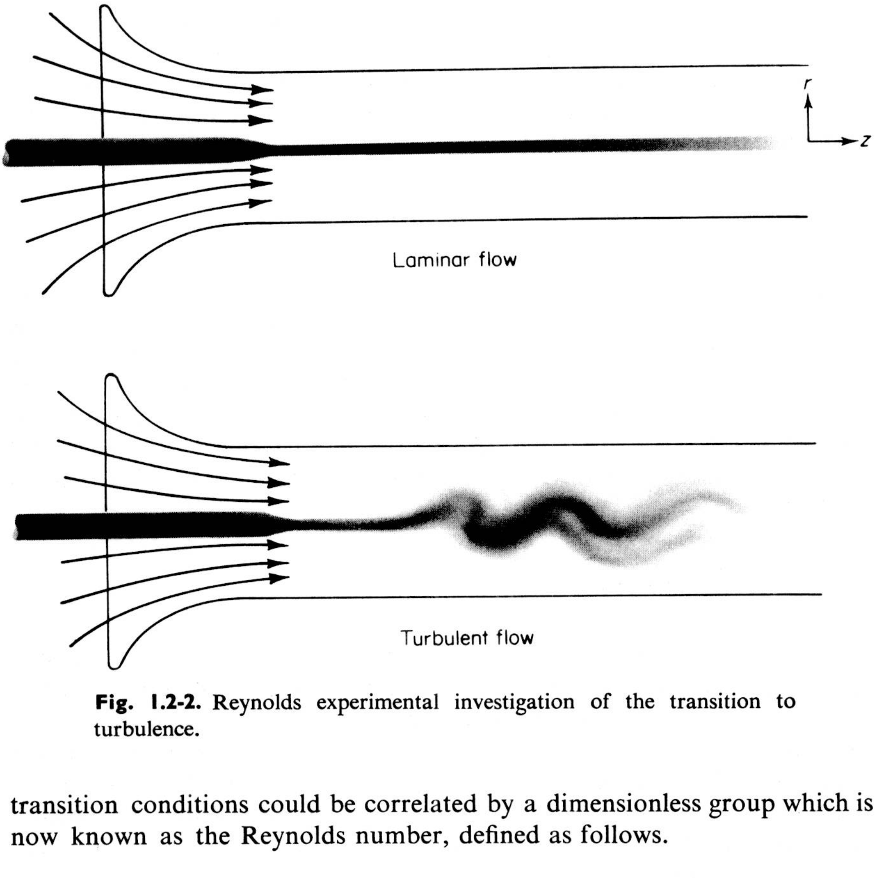

Fig. 1.2-2. Reynolds experimental investigation of the transition to turbulence.

transition conditions could be correlated by a dimensionless group which is now known as the Reynolds number, defined as follows.

$$N_{\mathrm{Re}} = \frac{\rho \langle v_z \rangle D}{\mu} \tag{1.2-1}$$

where ρ = density
$\langle v_z \rangle$ = average velocity in the z-direction
D = tube diameter
μ = viscosity

Reynolds found that the transition took place for values of N_{Re} of about 2100, regardless of the specific values of ρ, $\langle v_z \rangle$, D, and μ. In Chap. 5, we will be able to prove that the Reynolds number is, indeed, the governing parameter for the transition to turbulent flow.

The notation in Eq. 1.2-1 deserves some comment, for it will be used consistently throughout the text. Dimensionless numbers or groups will always be denoted by the letter N with a subscript appropriate to the name of the number; area and volume averages will be denoted by angular brackets, $\langle\, \rangle$; and the scalar components of a vector will be denoted by either an alphabetical or a numerical subscript. Thus v_z represents the scalar component of $\mathbf{v}$ in the z-direction, and not the derivative of v with respect to z. The latter interpretation is commonly encountered in mathematics texts, but rarely found in books on mechanics.

Steady and unsteady flow

These two designations are fairly obvious, and we only need to clarify their meaning in the case of turbulent flow. If a laminar flow is steady, the three components of the velocity—v_x, v_y, and v_z—and the pressure p are independent of time t. Turbulent flows are naturally unsteady; however, we shall refer to a turbulent flow as steady if the time-averaged components of velocity and pressure—$\bar{v}_x$, $\bar{v}_y$, $\bar{v}_z$, and $\bar{p}$—are independent of time. A careful treatment of the time-averaged equations of motion for turbulent flow appears in Chap. 6.

One-dimensional flow

By one-dimensional flow we mean that the velocity $\mathbf{v}$ is a function of only one spatial coordinate. One-dimensional turbulent flow, of course, implies that the time-averaged velocity $\bar{\mathbf{v}}$ is a function of only one spatial coordinate. The flow in the Reynolds' apparatus is one-dimensional some distance downstream from the entrance (i.e., $\mathbf{v}$ is only a function of r), but at the entrance, $\mathbf{v}$ is a function of r and z and the flow is two-dimensional. Often we approximate two- and three-dimensional flows by one-dimensional models, because the velocity field is easily determined for a one-dimensional flow.

*1.3 The Solution of Flow Problems

In attempting to formulate the equations of fluid motion, we need a clear understanding of the fundamental postulates governing this motion. The student has already made use of Newton's second law to solve problems in statics and dynamics, and it would seem natural to include it as one of the