Andreas Dress
Yinfeng Xu
Binhai Zhu (Eds.)

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Combinatorial Optimization and Applications

First International Conference, COCOA 2007
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Preface

The papers in this volume were presented at the 1st International Conference on Combinatorial Optimization and Applications (COCOA 2007), held August 12-15, 2007, in Xi'an, China. The topics cover most areas in combinatorial optimization and applications.

Submissions to the conference this year were conducted electronically. A total of 114 papers were submitted, of which 29 were accepted. The papers were evaluated by an International Program Committee consisting of Tetsuo Asano, Kyung-Yong Chwa, Bill Chen, Bo Chen, Andreas Dress, Pater Eades, Omer Egecioglu, Rudolf Fleischer, Bin Fu, Mordecai Golin, Ron Graham, Pavol Hell, Xiao-Dong Hu, Marek Karpinski, Minghui Jiang, Michael Langston, Hanno Lefmann, Ko-Wei Lih, Andy Mirzaian, Brendan Mumey, Mauricio G.C. Resende, Takao Nishizeki, Mike Steel, Zheng Sun, My T. Thai, Kanliang Wang, Michael Waterman, Gerhard Woeginger, Yinfeng Xu, Boting Yang, Wenan Zang, Alex Zelikovsky and Binhai Zhu. It is expected that most of the accepted papers will appear in a more complete form in scientific journals.

The submitted papers are from Australia, Canada, China, France, Germany, Greece, Hong Kong, Japan, Korea, Mexico, Poland, Romania, Russia, Switzerland, Tunisia, Turkey and USA. Each paper was evaluated by at least two Program Committee members (and in some cases by as many as seven Program Committee members), assisted in some cases by subreferees. In addition to selected papers, the conference also included two invited presentations, by Bailin Hao and Kurt Mehlhorn, and eight invited papers.

We thank all the people who made this meeting possible: the authors for submitting papers, the Program Committee members and external referees (listed in the proceedings) for their excellent work, and the two invited speakers. Finally, we thank Xi'an Jiaotong University and NSF of China for the support and local organizers and colleagues for their assistance.

August 2007

Andreas Dress
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Contributed Papers

Matchings in Graphs Variations of the Problem

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Many real-life optimization problems are naturally formulated as questions about matchings in (bipartite) graphs.

- We have a bipartite graph. The edge set is partitioned into classes E_1, E_2, ..., E_r. For a matching M, let s_i be the number of edges in $M \cap E_i$. A *rank-maximal matching* maximizes the vector $(s_1, s_2, \ldots, s_r)$. We show how to compute a rank-maximal matching in time $O(r\sqrt{n}m)$ [IKM+06].
- We have a bipartite graph. The vertices on one side of the graph rank the vertices on the other side; there are no ties. We call a matching M more popular than a matching N if the number of nodes preferring M over N is larger than the number of nodes preferring N over M. We call a matching *popular*, if there is no matching which is more popular. We characterize the instances with a popular matching, decide the existence of a popular matching, and compute a popular matching (if one exists) in time $O(\sqrt{n}m)$ [AIKM05].
- We have a bipartite graph. The vertices on both sides rank the edges incident to them with ties allowed. A matching M is *stable* if there is no pair $(a, b) \in E \setminus M$ such that a prefers b over her mate in M and b prefers a over his mate in M or is indifferent between a and his mate. We show how to compute stable matchings in time $O(nm)$ [KMMP04].
- In a random graph, edges are present with probability p independent of other edges. We show that for $p \geq c_0/n$ and c_0 a suitable constant, every non-maximal matching has a logarithmic length augmenting path. As a consequence the *average running time of matching algorithms on random graphs* is $O(m \log n)$ [BMSH05].

References

[AIKM05] Abraham, D., Irving, R., Kavitha, T., Mehlhorn, K.: Popular Matchings. SODA, pp. 424–432 (2005) (http://www.mpi-sb.mpg.de/~mehlhorn/ftp/PopularMatchings.ps)

[BMSH05] Bast, H., Mehlhorn, K., Schäfer, G., Tamaki, H.: Matching Algorithms are Fast in Sparse Random Graphs. Theory of Computing Systems 31(1), 3–14 (2005) preliminary version. In: Diekert, V., Habib, M. (eds.) STACS 2004. LNCS, vol. 2996, pp. 81–92. Springer, Heidelberg (2004)

A. Dress, Y. Xu, and B. Zhu (Eds.): COCOA 2007, LNCS 4616, pp. 1–2, 2007.

[IKM+06] Irving, R., Kavitha, T., Mehlhorn, K., Michail, D., Paluch, K.: Rank-Maximal Matchings. ACM Transactions on Algorithms 2(4), 1–9 (2006), A preliminary version appeared in SODA 2004, pp. 68–75, (2004) (`http://www.mpi-sb.mpg.de/~mehlhorn/ftp/RankMaximalMatchings`)

[KMMP04] Kavitha, T., Mehlhorn, K., Michail, D., Paluch, K.: Strongly Stable Matchings in Time $O(nm)$ and Extensions to the Hospitals-Residents Problem. In: Diekert, V., Habib, M. (eds.) STACS 2004. LNCS, vol. 2996, pp. 222–233. Springer, Heidelberg (2004), full version to appear in TALG (`http://www.mpi-sb.mpg.de/~mehlhorn/ftp/StableMatchings.ps`)

Combinatorics from Bacterial Genomes

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By visualizing bacterial genome data we have encountered a few neat mathematical problems. The first problem concerns the number of longer missing strings (of length $K + i$, $i \geq 1$) taken away by the absence of one or more K-strings. The exact solution of the problem may be obtained by using the Golden-Jackson cluster method in combinatorics and by making use of a special kind of formal languages, namely, the factorizable language. The second problem consists in explaining the fine structure observed in one-dimensional K-string histograms of some randomized genomes. The third problem is the uniqueness of reconstructing a protein sequence from its constituent K-peptides. The latter problem has a natural connection with the number of Eulerian loops in a graph. To tell whether a protein sequence has a unique reconstruction at a given K the factorizable language again comes to our help.

References

1. Hao, B., Xie, H., Yu, Z., Chen, G.: A combinatorial problem related to avoided strings in bacterial complete genomes. Ann. Combin. 4, 247-255 (2000)
2. Xie, H., Hao, B.: Visualization of K-tuple distribution in prokaryote complete genomes and their randomized counterparts. In: Bioinformatics. CSB2002 Proceedings, pp. 31-42. IEEE Computer Society, Los Alamitos, California (2002)
3. Shen, J., Zhang, S., Lee, H.-C., Hao, B.: SeeDNA: a visualization tool for K-string content of long DNA sequences and their randomized counterparts. Genomics, Proteomics and Bioinformatics 2, 192-196 (2004)
4. Shi, X., Xie, H., Zhang, S., Hao, B.: Decomposition and reconstruction of protein sequences: the problem of uniqueness and factorizable language. J. Korean Phys. Soc. 50, 118-123 (2007)

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An Algorithm for Computing Virtual Cut Points in Finite Metric Spaces

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Abstract. In this note, we consider algorithms for computing **virtual cut points** in finite metric spaces and explain how these points can be used to study **compatible decompositions** of metrics generalizing the well-known decomposition of a **tree metric** into a sum of **pairwise compatible split metrics**.

Mathematics Subject Classification codes: 05C05, 05C12, 92B10.

1 Terminology

A **metric** D defined on a set X is a map $D : X^2 \to \mathbb{R} : (x, y) \mapsto xy$ from the set $X^2 := \{(x, y) : x, y \in X\}$ of all (ordered) pairs of elements from X into the real number field $\mathbb{R}$ such that $xx = 0$ and $xy \leq xz + yz$ (and, therefore, also $0 \leq xy = yx$) holds for all $x, y, z \in X$. A metric D is called a **proper** metric if $xy \neq 0$ holds for any two distinct points $x, y \in X$. Further, given X and D as above, we denote

(i) by $\sim_D$ the binary relation defined on X by putting $x \sim_D y \iff xy = 0$ which, in view of the fact that $xy = 0 \iff \forall_{a \in X}\, xa = ya$ holds for all $x, y \in X$, is clearly an equivalence relation,
(ii) by x/D the equivalence class of x relative to this equivalence relation, and
(iii) by X/D the set $\{x/D : x \in X\}$ of all such equivalence classes.

In case D is proper, the pair $M = M_D := (X, D)$ is also called a **metric space**, X is called the **point set** of that space – and every element $x \in X$ a **point** of M.

Further, given any metric space $M = (X, D)$, we denote

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(D1) by $[x, y]$, for all $x, y \in X$, the **interval** between x and y, i.e., the set

$$[x, y] := \{z \in X : xy = xz + zy\},$$

(D2) by $\text{Prox}(M) = (X, E(M))$ the (abstract) **proximity graph** of M (sometimes also called **the underlying graph** of M, cf. [11]), that is, the graph with vertex set X and edge set

$$E(M) := \Big\{\{u, v\} \in \binom{X}{2} : [u, v] = \{u, v\}\Big\}.$$

(D3) by $C_x(y)$, in case x and y are two distinct points in X, the connected component of the induced graph

$$\text{Prox}(M|x) := \text{Prox}(M)|_{X-\{x\}} := \Big(X - \{x\}, E(M) \cap \binom{X - \{x\}}{2}\Big)$$

containing y, and by $\overline{C_x}(y) := C_x(y) \cup \{x\}$ the "augmented" connected component of $\text{Prox}(M|x)$ containing y, i.e., the union of $C_x(y)$ and the one-point set $\{x\}$,

(D4) and we denote by

$$\pi_x := \{C_x(y) : y \in X - \{x\}\}$$

the collection of connected components of $\text{Prox}(M|x)$, and by

$$\overline{\pi}_x := \{\overline{C_x}(y) : y \in X - \{x\}\}$$

the corresponding collection of "augmented" connected components of $\text{Prox}(M|x)$.

Note that

(i) $[x, z] \subseteq [x, y]$ holds for all $x, y, z \in X$ with $z \in [x, y]$,

and that, in case X is finite,

(ii) $\text{Prox}(M) = (X, E(M))$ is connected,
(iii) $C_x(y) = C_x(y')$ holds for all $y, y' \in X - \{x\}$ with $x \notin [y, y']$,
(iv) and $C_x(y) \cup C_y(x) = X$ holds for any two distinct $x, y \in X$ $\big($indeed, $z \in X - C_x(y)$ implies $zx = zx + (xy - xy) = zy - xy < zy + yx$ and, hence, $y \notin [x, z]$ which in turn $\big($cf. (iii)$\big)$ implies $C_y(z) = C_y(x)\big)$.

2 Cut Points of Metric Spaces

Given a metric space $M = (X, D)$, let $Cut(M)$ denote the set of all **cut points** of M, i.e., the set of all points $x \in M$ for which two subsets A, B of X with $A \cup B = X$ and $A \cap B = \{x\}$ of cardinality at least 2 exist such that $x \in [a, b]$ holds for all $a \in A$ and $b \in B$. Concerning cut points, one has: