Nonlinearity and Chaos in Engineering Dynamics

EDITED BY
J. M. T. THOMPSON
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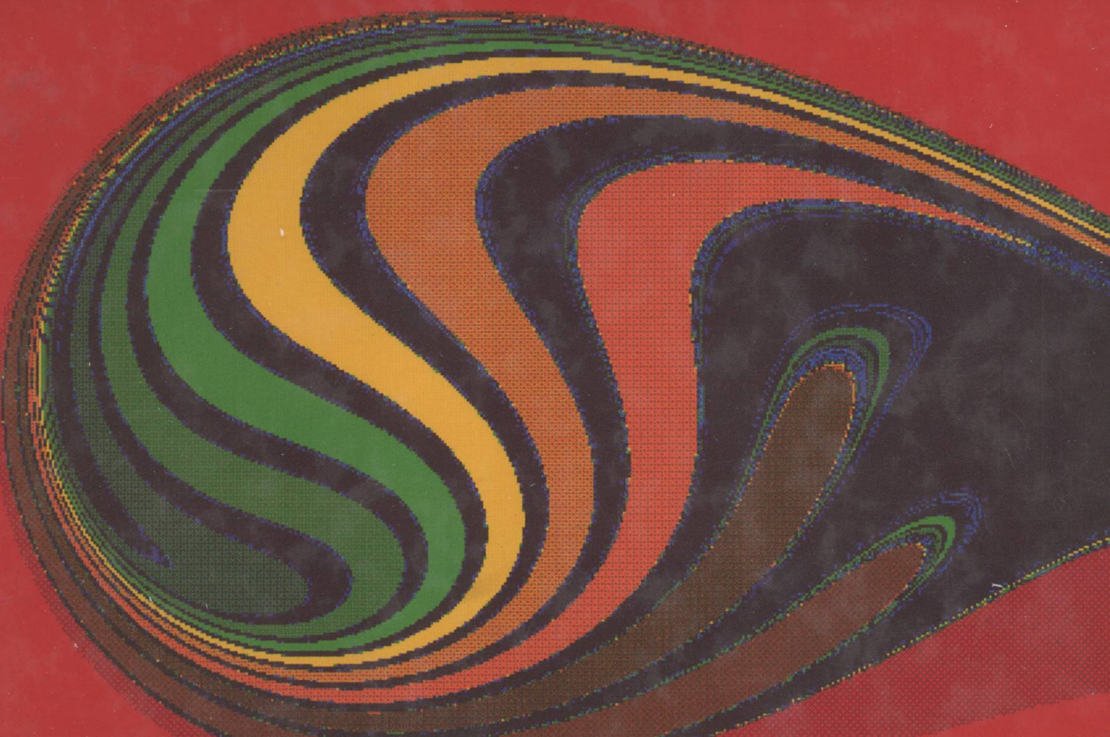

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PREFACE

Engineers and applied scientists are increasingly responding to the revolution in nonlinear dynamics which has uncovered the full complexity inherent in the equations of motion of macroscopic mechanical systems. The unpredictability implied by the chaotic motions and fractal basin boundaries of simple 'deterministic' systems is but one feature which nicely epitomizes the field. The ubiquitous new phenomena necessitate a change in emphasis away from the classical reliance on perturbation and averaging methods towards the use of computational techniques employing the powerful geometrical concepts developed by mathematicians.

This book gives a coherent overview of recent developments in the field, presenting in a structured fashion the latest research of leading international groups in theoretical and applied dynamics. Topics covered include: nonlinear problems of structural, mechanical, aerospace and naval engineering; topological and computer methods, cell mapping and global analysis; phenomenological studies of attractors, fractal basins, bifurcations and escape; control of chaos; experimental studies; piecewise-linear, stick–slip and impacting systems; time series analysis, phase-space reconstruction, parametric identification; dynamics of cables, beams and structures under fluid loading; stochastic bifurcation and resonance under random excitation.

Particular emphasis is given to recent developments in knot theory, which can identify bifurcational precedences in driven oscillators. New ideas are presented on the control and potential use of chaos; and on the phase-space reconstruction of time-series data. Discontinuous systems, including, for example, sliding friction or impacts, are another important and developing area of study that is strongly represented. Other contributions present important experimental verifications of theoretical concepts; and novel engineering applications related to vehicle dynamics and the chaotic motions of machine tools.

These research contributions were presented by their authors at a Symposium on *Nonlinearity and Chaos in Engineering Dynamics*, sponsored by the International Union of Theoretical and Applied Mechanics (IUTAM). This was held on 19–23 July 1993, at University College London. As with all specialist IUTAM symposia, invitations to attend, and to present papers, were made by a Scientific Committee which was constituted as follows: J. M. T. Thompson (Chairman), S. Al-Athel (Saudi Arabia), S. T. Ariaratnam (Canada), S. Arimoto (Japan), D. H. van Campen (Netherlands), F. L. Chernousko (Russia), C. S. Hsu (USA), F. C. Moon (USA), W. Schiehlen (Germany), S. W. Shaw (USA), W. Szemplińska-Stupnicka (Poland) and H. Troger (Austria). The Symposium brought together a wide spectrum of theoretical and applied dynamicists, 78 participants from 23 countries; and promoted a vigorous exchange of ideas. The extensive discussions helped to establish, consolidate and direct the emerging body of topological, analytical and computational expertise that is needed to address the challenging practical problems of engineering dynamics. The proceedings began with an

Opening Address by Sir James Lighthill who was the President of IUTAM during his period as Provost of University College London; followed by the opening general lecture given by Philip Holmes of Cornell University. They ended with a comprehensive final discussion session which provided a valuable focus for identifying desirable developments and lines of future research.

Financial support for the Symposium was generously provided by IUTAM, and most major international publishers contributed to the display of scientific books and periodicals. The detailed organization was in the hands of the Local Steering Committee comprising the following researchers from the Centre for Nonlinear Dynamics and its Applications: S. R. Bishop (Chairman), M. E. Davies, S. Foale, P. G. Holborn, F. A. McRobie, J. Stark and J. M. T. Thompson. Thanks are also due to Derek Roberts, Provost of University College London, and to Jim Croll, Head of the Department of Civil and Environmental Engineering, for their encouragement and support; to Anne Power for her invaluable work on the local organization; and to Margaret Thompson for her cheerful and enthusiastic help with the social programme.

This book contains a specially written intoduction to the subject area (Chapter 1); the full texts of all the lectures presented at the Symposium; abstracts of the posters in Appendix I; the names and addresses of all participants in Appendix II; and a comprehensive index. We are pleased to have had it so efficiently and attractively produced by John Wiley & Sons Ltd.

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CONTENTS

1 BASIC CONCEPTS OF NONLINEAR DYNAMICS

J. M. T. Thompson

1.1 INTRODUCTION

Like any new scientific discipline, the new geometrical theory of *nonlinear dynamics and chaos* has spawned a multitude of specialized concept and terminologies. These can be a major obstacle to applied scientists and engineers wishing to apply the powerful new methods in their own fields. To help overcome this, we provide here an overview that aims to highlight the central concepts and ideas that will be of particular importance in practical applications.

Recent books which the reader may find helpful are those of Guckenheimer and Holmes (1983), Thompson and Stewart (1986), Moon (1987), Arrowsmith and Place (1990) and Abraham and Shaw (1992). Collections of modern applications are edited by Schiehlen (1990), Thompson and Gray (1990), Kim and Stringer (1992), Thompson and Schiehlen (1992), and Mullin (1993).

1.2 DYNAMICAL SYSTEMS AND THE POINCARÉ SECTION

The general type of continuous dynamical system that will concern us here is described by an *autonomous* set of n first-order ordinary differential equations (ODEs),

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{1}$$

giving a stationary *vector field* in the n-dimensional *phase space*, $\mathbb{R}^n$ say, spanned by the components of vector $\boldsymbol{x}$. Non-autonomous equations, in which time, t, appears explicitly, can be rendered autonomous by identifying t as an extra phase coordinate governed by the dummy equation $\dot{t} = 1$. A driven mechanical oscillator can be put into the required form by identifying the velocity as a second phase coordinate and the time as a third. In a typical phase space the vectors vary smoothly with position, and trajectories are everywhere tangent to them. This leads naturally to the *Euler* time integration scheme. For a small time step, Δt,

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we can write $\Delta \boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x})\Delta t$ allowing us to make a small finite step from point i to the next point $i + 1$ using

$$\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \boldsymbol{f}(\boldsymbol{x}_i)\Delta t. \tag{2}$$

An improvement of this basic scheme is the *Runge–Kutta* method which uses an Euler-type prediction followed by corrections to achieve higher-order accuracy. The trajectories fill the phase space to form a *phase portrait*. In a dissipative system this portrait will show the structure of the attractors and basins, and is sometimes called the attractor–basin phase portrait.

A discrete dynamical system is described by the iterated *map* (or *mapping*),

$$\boldsymbol{x}_{i+1} = \boldsymbol{F}(\boldsymbol{x}_i). \tag{3}$$

The Euler time integration of a continuous system does, for example, generate a discrete system of this type. More globally, continuous dynamical systems on $\mathbb{R}^n$ are formally reduced to a mapping of dimension n-1 by the use of a Poincaré section.

Such a section transverse to the flow of a continuous system on $\mathbb{R}^n$ generates a Poincaré mapping on $\mathbb{R}^{n-1}$, taking a point in the surface of section to its image upon first return to the section. The value of such a mapping lies in the fact that it captures the (attractor-basin) dynamics of the system, and has the same general stability properties as the flow.

For a mechanical oscillator driven by a periodic excitation of period T, Poincaré sections can be defined, most simply, by the planes $t = iT$ where $i = 1, 2, 3, \ldots$. This corresponds to the *stroboscopic sampling* of the velocity and displacement. It should be emphasized, however, that alternative Poincaré sections are often advantageous: in impacting systems, for example, it can be useful to work with the 2D impact map whose coordinates are the phase and velocity sampled at impact (Foale and Bishop, 1992).

The Poincaré mapping of a smooth continuous dynamical system will typically be a *diffeomorphism*, namely a smooth differentiable one-to-one mapping with a unique and smooth differentiable inverse.

1.3 DIVERGENCE, DISSIPATION AND RECURRENT BEHAVIOUR

The non-crossing trajectories of a continuous system give a fluid-like *flow* in the phase space. Writing the set of n first-order ODEs in scalar form as

$$\dot{x}_i = f_i(x_j) \tag{4}$$

we have the important scalar divergence,

$$\operatorname{div}(x_i) = \partial f_1/\partial x_1 + \partial f_2/\partial x_2 + \cdots + \partial f_n/\partial x_n. \tag{5}$$

The rate of change of a small volume, V, of the phase 'fluid' is given by

$$\dot{V}(t)/V(t) = \operatorname{div}(x_i). \tag{6}$$

The analogous result for the two-dimensional mapping

$$x_{i+1} = G(x_i, y_i), \quad y_{i+1} = H(x_i, y_i) \tag{7}$$

gives us the ratio of small areas

$$A_{i+1}/A_i = D = (\partial G/\partial x)(\partial H/\partial y) - (\partial G/\partial y)(\partial H/\partial x) \tag{8}$$

where D is the Jacobian determinant.

A conservative, autonomous mechanical system with no energy dissipation is called a *Hamiltonian* system. Its equations of motion can be written in terms of the Hamiltonian function, $\mathscr{H}$, (numerically equal to the sum of the kinetic and potential energies) as

$$\dot{q}_i = \partial\mathscr{H}/\partial p_i, \quad \dot{p}_i = -\partial\mathscr{H}/\partial q_i \tag{9}$$

where q_i are the r generalized coordinates and p_i the generalized momenta. This canonical form shows immediately that a Hamiltonian system has an identically zero divergence function on $\mathbb{R}^{2r}$, this result being known as *Liouville's theorem.* The Hamiltonian flow is thus akin to that of an incompressible fluid. In the wider context of non-mechanical systems, not all systems with an identically zero divergence can be reduced to this classical canonical form: we can refer to these, more generally, as *volume-preserving* systems.

Dissipation of energy tends to give a negative divergence to the flow. Consider for example the driven oscillator

$$\ddot{x} + b(\dot{x}) + c(x) = F\sin(\omega t) \tag{10}$$

which we reduce to the first-order form

$$\dot{x} = y, \quad \dot{y} = -c(x) - b(y) + F\sin\theta, \quad \dot{\theta} = \omega \tag{11}$$

to obtain for the (x, y, θ) phase space the divergence function

$$\operatorname{div}(x, y, \theta) = -\mathrm{d}b/\mathrm{d}y = -b_y(y). \tag{12}$$

We see that the divergence is just a function of y, and governed only by the dissipation function $b(y)$: the sinusoidal forcing does not appear in it; nor does the restoring force, so that even in the vicinity of an unstable hilltop, with for example $c(x) = -x$, the sign of the divergence depends only on the form of $b(y)$. The (x, y, θ) phase space of such a periodically driven oscillator can be usefully viewed in the toroidal space $\mathbb{R}^2 \times S^1$, product of the plane $\mathbb{R}^2$ and the circle S^1.

We use the adjective *dissipative* to describe any system that does not have an identically zero divergence. Often a system so described will be totally dissipative, in the sense that the divergence function is everywhere negative. This would be the case with *Duffing's equation,* describing a driven oscillator with a cubic or polynomial restoring force, but with simple, positive linear damping corresponding to an energy sink. But we also encounter systems in which the phase space might have regimes of positive divergence, containing for example a repellor. This arises in the *van der Pol equation* of an oscillator with a nonlinear damping characteristic such that the autonomous system is capable of sustained self-excited oscillation in a limit cycle. The energy source for such behaviour is typically provided by a fluid flowing over an elastic structure. Similar results and subdivisions according to the divergence properties apply to iterated mappings, and we shall focus most of our attention on dissipative flows and maps.

In a phase space regime of negative divergence, a cloud or ensemble of starts will shrink asymptotically onto an attracting set of zero volume. Setting $\mathrm{div}(x_i)$ equal to a constant, $-k$, in equation (6) gives, for example,

$$V(t) = V(0)\exp(-kt) \tag{13}$$

A typical start within this cloud will experience a *transient* before settling asymptotically onto a stable *steady-state* solution, called an *attractor*. Such a post-transient set can be a point attractor, a periodic or quasi-periodic attractor, or a chaotic attractor. Generically each attractor is entirely surrounded in phase space by its own basin of attraction. All transients initialized in a small neighbourhood around the attractor move back to it, making it *asymptotically stable* in the local sense of Lyapunov. All the above attractor types can appear alternatively as unstable steady states giving the saddles and repellors that we discuss later.

To distinguish these steady-state attractors, saddles and repellors from transients, geometrical dynamics uses the concept of a *recurrent state*. A particular state of a dynamical system is deemed recurrent if, after sufficient time, the system returns arbitrarily close to the state. The relaxation of the definition away from precise repetition is here used to embrace quasi-periodic and chaotic motion as recurrent. An ensemble of recurrent states linked together by a single trajectory constitutes recurrent behaviour. A further relaxation is to the *non-wandering* state which is one that has arbitrarily close states that return arbitrarily close. This generalization of a recurrent state (any recurrent state is non-wandering, but not vice versa) is needed to embrace a homoclinic orbit.

1.4 POINT, PERIODIC AND QUASI-PERIODIC ATTRACTORS

An equilibrium or fixed point, $\mathbf{x}_e$, of (1) is characterized by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}_e) = 0. \tag{14}$$

It can be stable or unstable, and is the first (trivial) form of recurrent behaviour. If it is asymptotically stable it is a *point attractor*.

A local stability analysis of any fixed point starts with the linearized equations describing small variations about the point. For a fixed point of a flow, stability hinges on the signs of the real parts of the eigenvalues. Typically the point will be hyperbolic (non-critical) with no zero real parts and we then have: the necessary and sufficient condition for stability is that all signs be negative; the necessary and sufficient condition for instability is that at least one sign be positive.

A fixed point of a flow (or map) that has all its linear eigenvalues in the stable or unstable domains is called *hyperbolic*. There are then no critical eigenvalues corresponding to neutral stability, and the phase portrait around the fixed point is structurally stable against perturbations of the system. (We should note that the term *hyperbolic point* is used differently in the literature on Hamiltonian systems to mean a saddle, near which trajectories follow a roughly hyperbolic shape.) Fixed points of a Hamiltonian system (or any volume-preserving system) can be at most neutrally stable with all local trajectories staying close, though not returning to, the point.

In the phase space of a flow, a closed orbit satisfying recurrence by returning precisely to its starting point after its periodic time T, is called a *periodic motion*. Such a motion (not