

# Practical Computer Analysis of

# Switch Mode Power Supplies

**JOHNNY C. BENNETT**



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# **Practical Computer Analysis of Switch Mode Power Supplies**

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## *Preface*

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For many years prior to the 1970s, engineers designed and built switch mode power supplies (SMPSs) using methods based largely on intuitive and experimentally derived techniques. In general, these power supplies were able to achieve their primary goal of high-efficiency power conversion; unfortunately, due to the lack of adequate theoretical analysis techniques, many of these power supplies only marginally met their desired performance requirements. In many cases, they were considered to be unreliable. Although they appeared to be very simple in concept, these switching regulators exhibited phenomena that were not understood and certainly could not be analyzed.

Things began to improve, however, in the early 1970s, when Dr. R.D. Middlebrook and his group of students at the California Institute of Technology developed the powerful circuit-averaging techniques, thus opening the door for the application of conventional linear circuit analysis methods. With these tools brought to bear, the many subtle complexities of the conceptually “simple” switching regulator were soon understood, allowing engineers to design SMPSs with improved performance and higher reliability. At this point, the power electronics field began to expand rapidly as better components were developed, power conversion technology advancements were made, and sophisticated computer-aided design and analysis methods were utilized.

Having been in the power electronics field for many years, I have had the good fortune to be involved with the design and analysis of many different types of power supplies. Here in this book, one of my goals is to provide the reader with a good understanding of the essential requirements for analyzing the switching regulated power supply performance characteristics. Another goal is to further demonstrate the power of the circuit-averaging technique by using computer circuit simulation programs to provide the desired performance analyses. At this point, I would like to reference the very important work of Dr. Vincent Bello, who, in his seminal paper,<sup>9</sup> pointed the way to using the SPICE-based computer circuit simulator to perform linear small signal analysis and nonlinear large signal transient performance analysis as well. The simulation techniques presented in this book are based almost entirely on Dr. Bello’s approach. There have been several theoretical and practical contributors to the advancement of the circuit-averaging techniques over the years and I hope that the information presented here can help to further these advancements.

- Chapter 1 is a refresher of the basics of SMPS fundamentals and circuit-averaging modeling. This may also be a primer for the newcomer, but it is recommended that the beginner read the referenced works to obtain a more complete understanding.

- Chapter 2 provides information on the general analysis requirements of a power supply. This is deemed necessary because it is equally important to know what questions to ask as it is to provide the answers.
- Chapter 3 gives information on how to develop the general types of SMPS models and demonstrates the analysis approach using a SPICE-based circuit simulator.
- Chapter 4 looks, in a practical way, at most of the basic first-order types of analysis generally associated with SMPS performance.
- Chapter 5 provides more practical and detailed information on developing an SMPS and SMPS component models.
- In Chapter 6, three power supplies are analyzed in practical detail. In these examples, emphasis is placed on using the circuit-averaging macromodel of the integrated circuit PWM controller. This is felt to simplify and expedite the analysis of a particular design that uses these commercially available controllers. As circuits and systems become larger and more complex, the macromodel approach will continue to increase in importance in almost all areas of electronic circuit analysis. The PWM macromodeling effort presented here will hopefully lead to the future development of many more such macromodels for commercially available PWM controllers, as has been the case with macromodels for transistors, op amps, etc.
- Appendix A deals with the optimal design of SMPS input filters. This is included here simply because of the fundamental importance of this subject to any power supply.
- Appendix B provides the first-order approach used in developing the macromodel for two commercially available PWM controllers. As was stated earlier, this is only a first step and hopefully will lead to the advancement and further development of these macros.

Although they are very important aspects of any switch mode power supply, the analyses of actual switching circuits per se are not specifically addressed in this book. For our purposes, these switching circuits are considered to be in the realm of conventional electronic circuit transient analysis and not implicitly related to the performance of an SMPS. There may be exceptions to this, of course.

I would like to express my thanks and appreciation to all the wonderful people with whom I have worked over the years who have shared their invaluable knowledge and experiences. I would also like to expressly thank Col. William T. McLyman of the Jet Propulsion Laboratory for his encouragement and for being instrumental in producing this book.

**Johnny C. Bennett**

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## *Author*

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# 1

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## *Review of Switch Mode Power Supply Fundamentals*

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A switch mode power supply (SMPS) may in general be defined as any type of electronic circuit that converts and/or regulates voltage or current by utilizing switching circuits and energy storage elements (capacitors and inductors). These circuits are ideally lossless with 100% energy transfer. This beginning chapter will provide a review of the most basic power converter topologies in their simplest form and an explanation of all their various modes of operation and control.

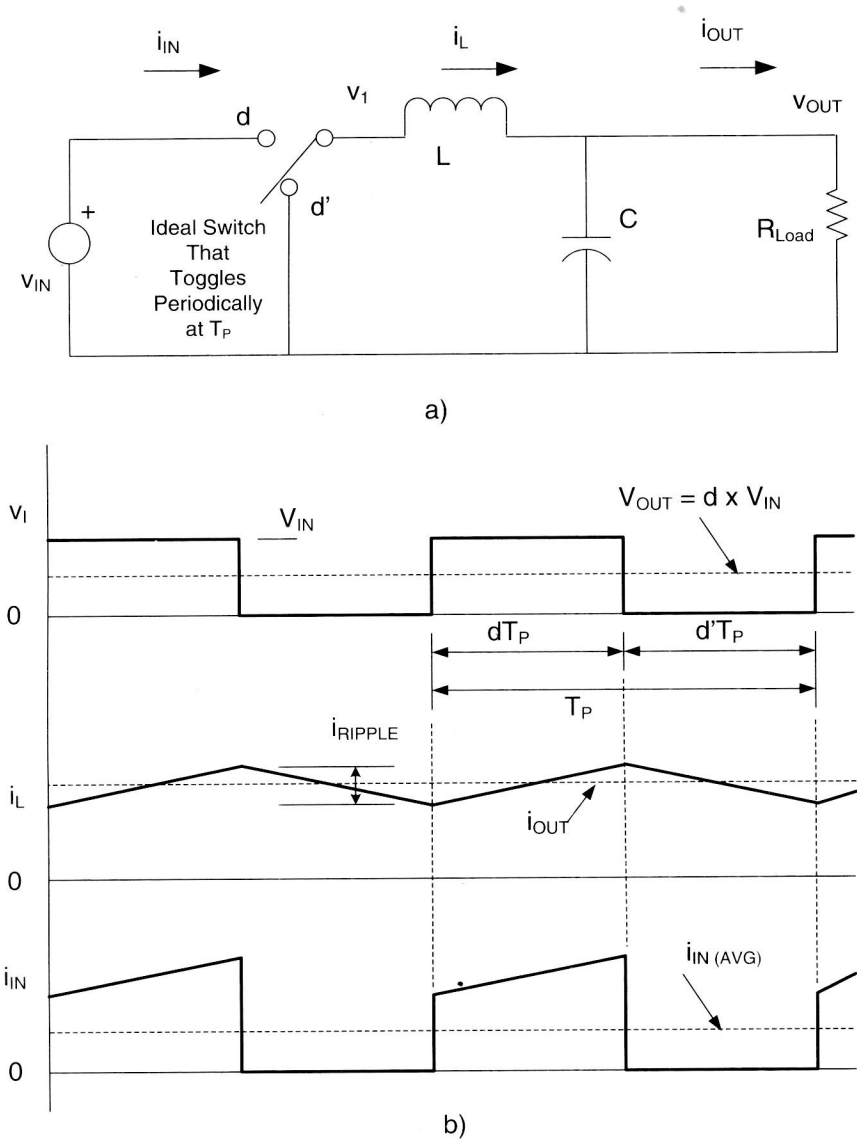
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### **1.1 Basic Topologies**

There are basically three fundamentally defined switch mode power supply converter topologies: the buck; the boost, and the combined form generally referred to as the buck-boost converter. Each of these converters has its unique properties and, in general, is applied in a complementary manner to each of the others. Also, each may have the capability of operating in one of two fundamental modes: the continuous mode or the discontinuous mode. These will be discussed in detail in the following sections.

#### **1.1.1 Buck Converter — Continuous Mode**

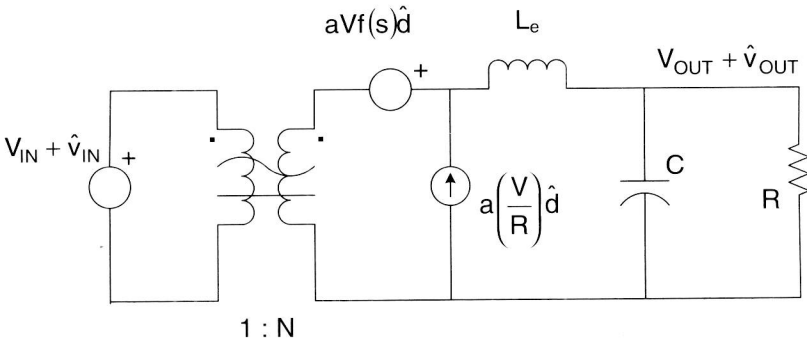
As an immediate initiation to the fundamentals, consider an illustration using the simplest example of all: the elementary buck converter. (Presumably, the defining word “buck” is deduced from the fact that the input voltage is bucked, or attenuated, in amplitude and a lower amplitude voltage appears at the output.) Figure 1.1a shows the circuit topology and Figure 1.1b shows the defining current and voltage waveforms. The switch positions  $d$  and  $d'$  represent the fraction of time that the periodically toggling switch remains in each position. (The period  $dT_p$  is generally referred to as the converter ON time and the period  $d'T_p$  is called the converter OFF time.) By assigning a period duration of unity, it can then be seen that  $d + d' = 1$ . The switching period,  $dT_p$ , occurs at a frequency that is much greater than the cut-off



**FIGURE 1.1**  
Basic buck converter: a) topology; b) continuous mode waveforms.

frequency of the LC low-pass filter; thus, it provides the average or DC component of the switched or “chopped” input voltage,  $v_{IN}$ , to the load with an attenuated and desired very low AC ripple component. The large signal nonlinear transfer function of this converter is indicated in Equation 1.1.

$$v_{OUT} = dv_{IN} \tag{1.1}$$



$$d = D + \hat{d} \quad (\text{DC} + \text{incremental quantities})$$

$$v = V_{IN} + \hat{v}_{IN}$$

$$v = V_{OUT} + \hat{v}_{OUT}$$

	N	a	f(s)	L <sub>e</sub>
Buck	D	$\frac{1}{D}$	1	L
Boost	$\frac{1}{1-D}$	$\frac{1}{1-D}$	$1 - s \left( \frac{L_e}{R} \right)$	$\frac{L}{(1-D)^2}$
Buck Boost	$\frac{D}{1-D}$	$\frac{D}{D(1-D)}$	$1 - s \left( \frac{DL_e}{R} \right)$	$\frac{L}{(1-D)^2}$

**FIGURE 1.2**  
Basic SMPS continuous conduction mode canonical model.

A small signal linear model developed by Middlebrook and Cuk<sup>1,2</sup> that has achieved wide acceptance is shown in Figure 1.2. This canonical model is applicable to all three basic topologies operating in the continuous mode; the different circuit component parameters are noted in the table in this figure. The different facets of each topology will be discussed in the following corresponding sections. This linearized model is used to allow all of the linear circuit analysis techniques developed over the years to be applied when analyzing a power supply at a particular DC operating point. Note the dependent voltage and current generators. Some important observations are immediately noted.

First, when a feedback control from the output is used to control  $\hat{d}$ , it is immediately obvious that the two-pole LC filter presents a “sticky” AC stability concern that must be dealt with. Next, with an ideal input voltage source — that is, with zero source impedance — the dependent current generator is essentially

shorted and has no effect on the output control; the dependent voltage generator provides output voltage control through the two-pole LC filter. It is then recognized that when real-world power sources with finite source impedances are used, the control,  $\hat{d}$ , to output,  $v$ , will be affected by this dependent current generator.

### 1.1.2 Buck Converter — Discontinuous Mode

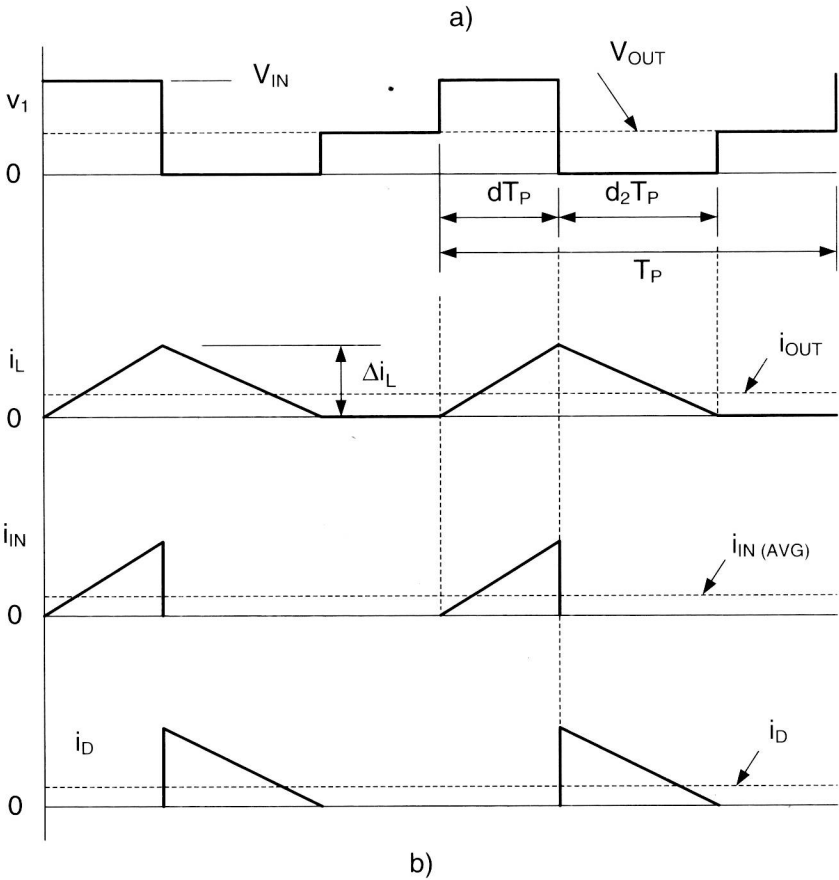
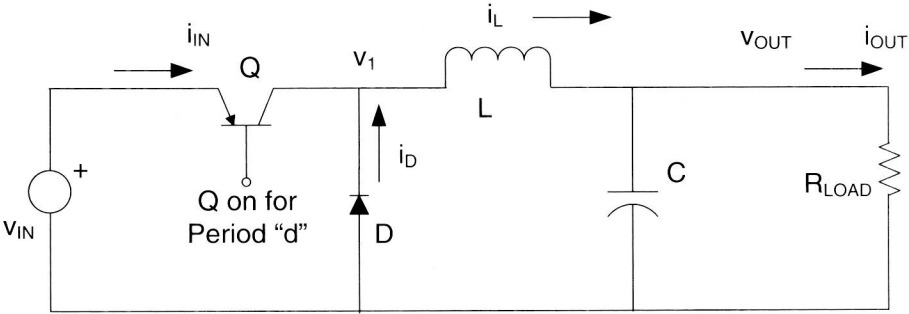
Figure 1.3a shows a more realistic representation of the buck converter in which the ideal switch is replaced by the transistor switch and a catch diode combination. When the load current,  $i_{OUT}$ , is reduced to a value that causes the average inductor current to be less than one-half the inductor ripple current,  $\Delta i_L$ , the inductor current wants to flow negatively through the inductor. However, with the transistor switch off and the catch diode reversed biased, the negative inductor current has no path along which to flow. At this point, no current will flow in the inductor for the remainder of the converter OFF time. With this discontinuity in the inductor current, this mode of operation is generally referred to as the discontinuous mode. (Sometimes this zero inductor current condition is referred to as “the inductor running dry.”)

Figure 1.3b shows the defining current and voltage waveforms with obvious differences noted between those of the continuous mode in Figure 1.1b. The converter OFF time for this mode of operation is generally designated in a different way. The portion of the OFF time during which inductor current is still flowing is designated as  $d_2 t_p$ , and it is now recognized that  $d + d_2 < 1$ . The convention for this was established in Cuk and Middlebrook<sup>3</sup> in the course of their pioneering work in the development of analytical power converter models. The transfer function for the discontinuous mode is noted in Equation 1.2 and is not as simple a relationship as that of the continuous mode. It is now a function of  $d$ , output load current,  $i_{OUT}$ , and the ratio of  $L/t_p$ . Middlebrook<sup>3</sup> defines a “conduction parameter,”  $k = 2L/Rt_p$ , that denotes the boundary between continuous and discontinuous modes. For the buck converter,  $k = D'$  at this boundary. This relationship can be derived very easily from the waveforms shown in Figure 1.1b and Figure 1.3b.

$$v_{OUT} = v_{IN} \left( \frac{2}{1 + \sqrt{1 + \frac{4k}{d^2}}} \right) \quad (1.2)$$

where

$$k = \frac{2L}{RT_p} \quad (1.2a)$$



**FIGURE 1.3**  
Basic buck converter: a) topology; b) discontinuous mode waveforms.

and

$$R \equiv \frac{V_{OUT}}{I_{OUT}} \tag{1.2.b}$$

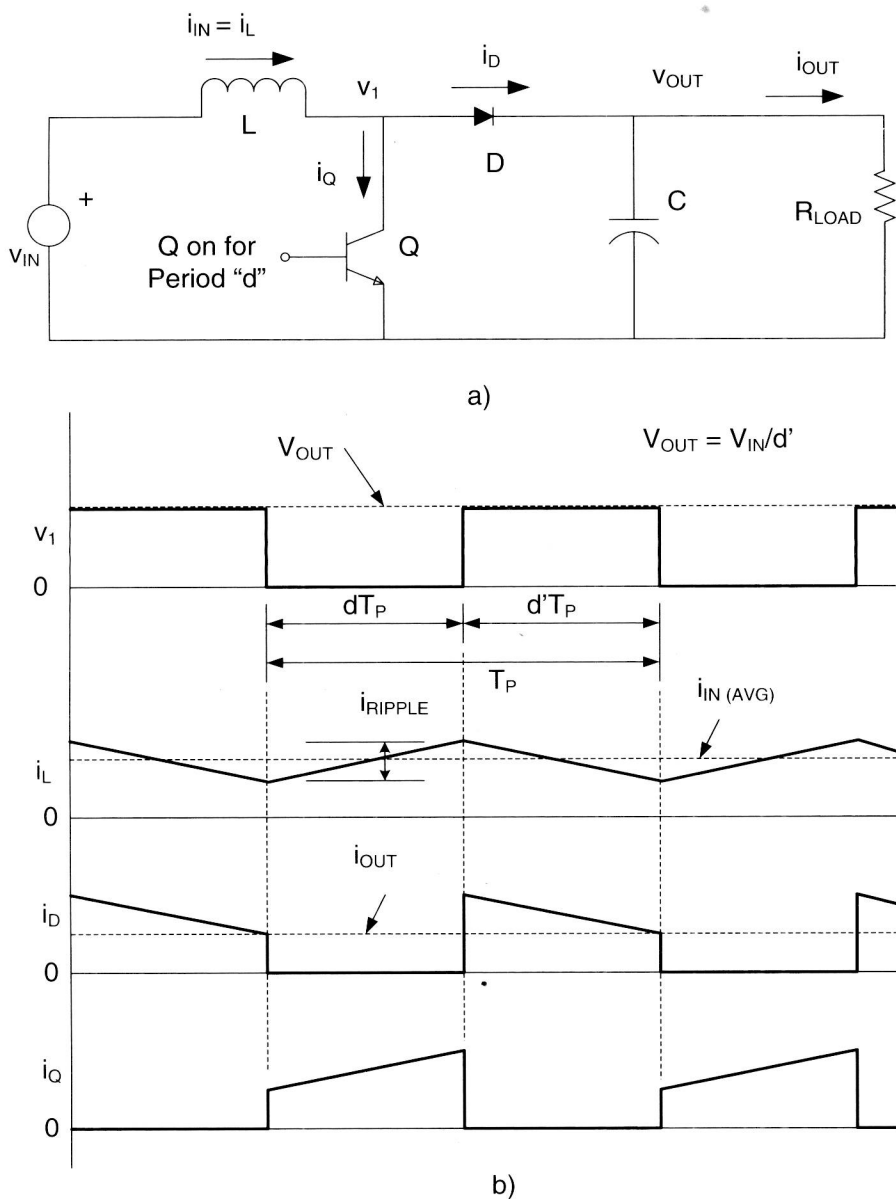


FIGURE 1.4

Basic boost converter: a) topology; b) continuous mode waveforms.

### 1.1.3 Boost Converter — Continuous Mode

The simple boost converter is shown in Figure 1.4a; as its name implies, it steps up or “boosts” the input voltage to a level higher than that of the input voltage. This topology is considered the “dual” or complement of the buck