Fast analytical techniques for ELECTRICAL and ELECTRONIC CIRCUITS

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Preface

The title of this book could easily have been called *Variations on a Theme by Middlebrook*, or *Applications of The Extra Element Theorem and its Extensions.* Neither title, however, would have captured the unique message of this book that one can solve very complicated linear circuits in symbolic form almost by inspection and obtain more than one meaningful analytical answer for any transfer function or impedance. The well-known and universally practiced method of nodal or loop analysis not only becomes intractable when applied to a complicated linear circuit in symbolic form, but also yields unintelligible answers consisting of a massive collection of symbols. In a meaningful analytical answer, the symbols must be grouped together in *low-entropy* form – a term coined by R. D. Middlebrook – clearly indicating series and parallel combination of circuit elements, and sums and products of time constants. The illustrative examples in Chapter 1 serve as a quick and informal introduction to the basic concepts behind the radically different approach to network analysis presented in this book.

Today, the only method of circuit analysis known to most engineers, students and professors is the method of nodal or loop analysis. Although this method is an excellent general tool for obtaining *numerical solutions*, it is almost useless for obtaining *analytical solutions* in all but the simplest cases. Anyone who has attempted inverting a matrix with symbolic entries – sometimes as low as second-order – knows how tedious the algebra can get and how ridiculous the resulting high-entropy expressions can look. The purpose of this book is not to eliminate the linear algebra approach to network analysis, but instead to provide additional new and efficient tools for obtaining analytical solutions with great ease and without letting the algebra run into a brick wall.

Among the most important techniques discussed in this book are the extra element theorem (EET) and its extension the N-extra element theorem (NEET). These two theorems are discussed in Chapters 3 and 4 after a brief and essential review of transfer functions given in Chapter 2. The EET and its proof were given by R. D. Middlebrook. The NEET was given without proof by Sarabjit Sabharwal, an undergraduate at Caltech in 1979. In Chapter 4, a completely original treatment of the NEET is given, where it is stated in its most general form using a new compact notation and, for the first time, proven directly using matrix analysis.

The subject of electronic feedback is treated in Chapter 5 using the EET for dependent sources, and another theorem by R. D. Middlebrook called simply "the feedback theorem". Both methods lead to a much more *natural* formulation of electronic feedback than the well-known block diagram approach found in most textbooks. Block diagrams are useful tools in linear system theory to help visualize abstract concepts, but they tend to be very awkward tools in network analysis. For instance, in an electronic feedback circuit neither the impedance loading nor the bi-directional transmission of the feedback network are easily captured by the single-loop feedback block diagram unless the feedback network and the amplifier circuit are both manipulated and *forced* to fit the block diagram. The fact is block diagrams bear little resemblance to circuits and their use in network analysis mainly results in loss of time and insight.

The examples presented in Chapters 6 and 7 are a *tour de force* in analysis of complicated circuits which demonstrate the efficacy of the fast analytical techniques developed in the previous chapters. Among the examples discussed in these chapters are higher-order passive filters and a MESFET amplifier. Some infinite networks, including fractal networks, are discussed in Chapter 7 where an interesting, and possibly new, result is presented. It is shown that a resistor, an inductor and a capacitor are all special cases of a single, two-terminal, linear element whose voltage and current are related by a fractional derivative or its inverse, the Riemann–Liouville fractional integral.

Pulse-width-modulated (PWM) switching dc-to-dc power converters are introduced in Chapter 8 to illustrate further the applications of the fast analytical techniques presented in this book. The analysis of PWM converters has been one of the hot topics of nearly every conference in power electronics since the early 1970s, and many specialized analytical techniques have been developed since. The simplest and fastest of these techniques is based on the equivalent circuit model of the PWM switch, which is introduced after a discussion of basic PWM converters. The PWM switch is a three-terminal nonlinear device which is solely responsible for the dc-to-dc conversion function inside a PWM converter. Hence, the PWM switch and its equivalent circuit model are to a PWM converter what the transistor and its equivalent circuit are to an amplifier. To analyze the dynamics of a PWM converter, one simply replaces the PWM switch with its equivalent circuit model and proceeds in exactly the same way as in an amplifier circuit analysis.

This book is based on my experience in electronic circuit analysis as a student, design engineer, teacher and researcher. The limitations of the "standard" circuit analysis I studied as an undergraduate soon became apparent on my first job as a power supply design engineer at Digital Equipment Corporation, Maynard, MA. I spent inordinate amounts of time deriving various small-signal transfer functions of switching converters in order to understand and improve their stability and dynamic behavior. Most of the senior engineers around me had acquired excellent design skills mostly by experience and did not rely too much on analysis. When I

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returned to graduate school at Caltech, I took Middlebrook's course which engendered a complete turn around: I learned how to handle complicated linear networks and obtain transfer functions, in low-entropy form, using very simple and elegant techniques. I gradually adopted these techniques in my seven years of teaching at Virginia Polytechnic Institute and State University confirming the adage, "the best way to learn something is to teach it."

Logically, Middlebrook's book, which is still in preparation, should have preceded mine. I began writing this book in the summer of 1996 with the intention of completing it by the winter of 1997. Clearly, I did not realize that writing a book at nights and on weekends would be considerably more difficult and time consuming than I had ever imagined. Fortunately, I had the constant support and encouragement of family, friends and colleagues. I would especially like to thank Gene Wester and Dave Rogers, both at the Jet Propulsion Laboratory, for their careful review and corrections of some of the chapters of this book. I would also like to thank my former supervisor Robert Detwiler; my current supervisor Mark Underwood; my colleagues Chris Stell, Tony Tang, Roman Gutierrez, Avo Demirjian, Dan Karmon, Mario Matal, Joseph Toczylowsky, Karl Yee, James Gittens, Mike Newell, David Hykes, Chuck Derksen and Tien Nguyen for making JPL an enjoyable place to work. Although this book is dedicated to my parents for their countless sacrifices, I would not have been able to write it without the enduring support, love and care of my favorite mezzo-soprano, best friend and wife Shoghig.

Vatché Vorpérian
June 2000

Contents

1 Introduction
The joys of network analysis

1.1 Fast analytical methods

The universally adopted method of teaching network theory is the formal and systematic method of nodal or loop analysis. Although the matrix algebra of formal network analysis is ideal for obtaining *numerical* answers by a computer, it fails hopelessly for obtaining *analytical* answers which provide physical insight into the operation of the circuit. It is not hard to see that, when numerical values of circuit components are not given, inverting a 3 × 3, or higher-order, matrix with symbolic entries can be very time consuming. This is only part of the problem of matrix analysis because even if one were to survive the algebra of inverting a matrix symbolically, the answer could be an unintelligible and lengthy symbolic expression. It is important to realize that an analytical answer is not merely a symbolic expression, but an expression in which various circuit elements are grouped together in one or more of the following ways:

(a) series and parallel combinations of resistances

Example: $R_1 + R_2 \parallel (R_3 + R_4)$

(b) ratios of resistances, time constants and gains

Example: $1 + \dfrac{R}{R_3 \parallel R_4}$, $1 + \dfrac{g_m R_L}{A_o}$, $A_m\left(1 + \dfrac{\tau_1}{\tau_2}\right)$

(c) polynomials in the frequency variable, s, with a unity leading term and coefficients in terms of sums and products of time constants

Example: $1 + s(\tau_1 + \tau_2) + s^2\tau_1\tau_3$

Such analytical expressions have been called low-entropy expressions by R. D. Middlebrook[1] because they reveal useful and *recognizable* information (low noise or entropy) about the performance of the circuit. Another extremely important advantage of low-entropy expressions is that they can be easily approximated into simpler expressions which are useful for design purposes. For instance, a series-parallel combination of resistances, as in (a), can be simplified by ignoring the smaller of two resistances in a series combination and the larger of two resistances

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in a parallel combination. When ratios are used as in (b), they can be simplified depending on their relative magnitude to unity. Depending on the relative magnitude of time constants, frequency response characteristics as in (c) can be simplified and either factored into two real roots, with simple analytical expressions, or remain as a complex quadratic factor.

In light of the above, the aim of fast analytical techniques can be stated as follows: fast derivation of low-entropy analytical expressions for electrical circuits. The following examples illustrate the power of this new approach to circuit analysis.

1.2 Input impedance of a bridge circuit

We will determine the input resistance, R_{in}, of the bridge circuit[2] in Fig.1.1 in a few simple steps using the extra element theorem (EET). The EET[3] and its extension, the N-extra element theorem[4] (NEET), are the main basic tools of fast network analysis discussed in this book. Both of these theorems will be introduced, derived and stated in their general form in later chapters, but since the EET for an impedance function is so trivial, we will use it now to obtain an early glimpse of what lies ahead.

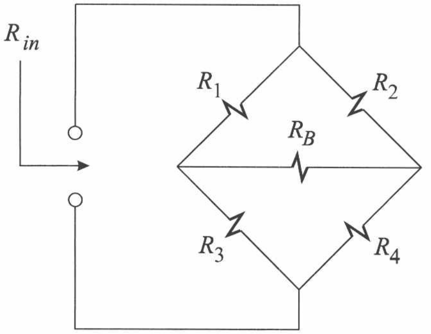

Figure 1.1

We see in Fig. 1.1 that if any one of the resistors of the bridge is zero or infinite, we can write R_{in} immediately by inspection. For instance, if we designate R_B as the extra element and let $R_B \to \infty$, as shown in Fig. 1.2*a*, we can immediately write:

$$R_{in}|_{R_B\to\infty} = (R_1 + R_3) \,\|\, (R_2 + R_4) \tag{1.1}$$

The EET now requires us to perform two additional calculations as shown in Figs. 1.2*b* and *c*. We denote the port across which the extra element is connected by (B).

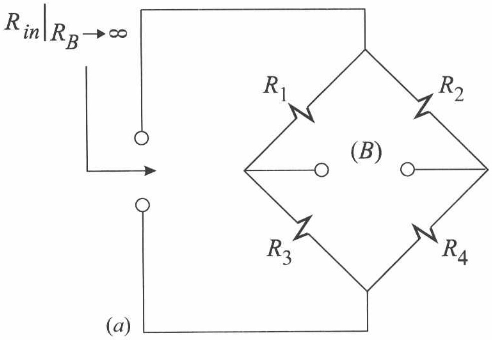

Figure 1.2

In Fig. 1.2*b*, we determine the resistance looking into the network from port (*B*) with the *input port short* and obtain by inspection:

$$\mathscr{R}^{(B)} = R_1 \parallel R_3 + R_2 \parallel R_4 \tag{1.2}$$

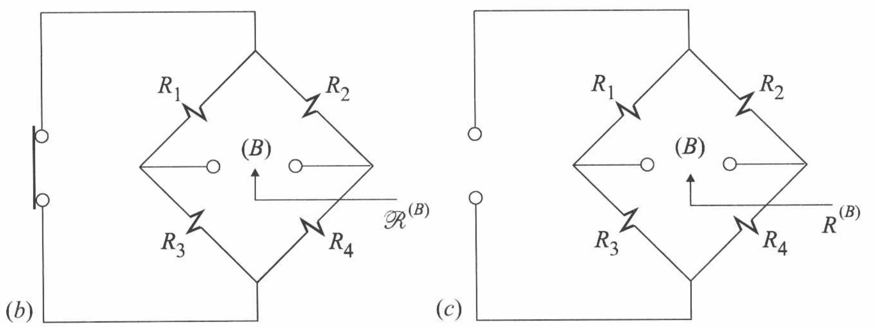

Figure 1.2 (*cont.*)

In Fig. 1.2*c*, we determine the resistance looking into the network from port (*B*) with the *input port open* and obtain by inspection:

$$R^{(B)} = (R_1 + R_2) \parallel (R_3 + R_4) \tag{1.3}$$

We now assemble these three separate and independent calculations to obtain the input resistance R_{in} in Fig. 1.1 using the following formula given by the EET:

$$R_{in} = R_{in}|_{R_B \to \infty} \frac{1 + \dfrac{\mathscr{R}^{(B)}}{R_B}}{1 + \dfrac{R^{(B)}}{R_B}} \tag{1.4}$$

Upon substituting Eqs. (1.1), (1.2) and (1.3) in (1.4):

$$R_{in} = (R_1 + R_3) \| (R_2 + R_4) \frac{1 + \dfrac{R_1 \| R_3 + R_2 \| R_4}{R_B}}{1 + \dfrac{(R_1 + R_2) \| (R_3 + R_4)}{R_B}} \tag{1.5}$$

Equation (1.5) is a low-entropy result because in it R_{in} is expressed in terms of series and parallel combinations of resistances and ratios of such resistances added to unity. Such an expression, for a given set of typical element values, can be easily approximated using rules of series and parallel combinations wherever applicable. In this expression, we can also see the contribution of the bridge resistance, R_B, to the input resistance, R_{in}, directly.

We can also appreciate two important advantages of the method of EET used in deriving R_{in} above. First, since the method of EET requires far less algebra than nodal analysis, it is considerably faster and simpler. Second, since the EET requires three separate and *independent* calculations, any *error in the analysis does not spread* and remains confined to a portion of the final answer. In a sense, this kind of analysis yields modular answers – if there is anything wrong with a particular module, it can be replaced without affecting the entire answer. This not only makes the analysis faster, but also the debugging of the analysis faster as well.

1.3 Input impedance of a bridge circuit with a dependent source

In this section we consider the effect of a dependent current source,[2,5] $g_m v_1$, in Fig. 1.3, on the input resistance R_{in}. This circuit is borrowed from a well-known

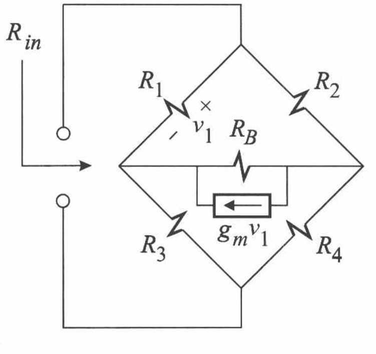

Figure 1.3

textbook by L. O. Chua and Pen-Min Lin[5] in which the authors determine the contribution of the transconductance, g_m, to the input resistance, R_{in}, using the parameter-extraction method. Because of the considerable amount of matrix algebra required by the parameter-extraction method, which would become prohibitively complex if all elements were in symbolic form, Chua and Lin have assigned numerical values ($R_1 = 1\,\Omega$, $R_2 = 0.2\,\Omega$, $R_3 = 0.5\,\Omega$, $R_4 = 10\,\Omega$ and $R_B = 0.1\,\Omega$) to all the resistors and determined:

$$R_{in} = \frac{96.3 + 5.1g_m}{137.7 + 10.5g_m}\,\Omega \tag{1.6}$$

We will now show how to determine R_{in} in three simple steps by applying the EET to the dependent current source $g_m v_1$. To demonstrate the superior power of this method of analysis, we will keep all circuit elements in symbolic form.

In Fig. 1.3, we designate the dependent current source as the extra element and set it to zero by letting $g_m = 0$. This reduces the circuit to the bridge circuit in Section 1.2, as shown in Fig. 1.4*a*. Hence, we have from Eq. (1.5):

$$R_{in}|_{g_m \to 0} = (R_1 + R_3) \| (R_2 + R_4) \frac{1 + \dfrac{R_1 \| R_3 + R_2 \| R_4}{R_B}}{1 + \dfrac{(R_1 + R_2) \| (R_3 + R_4)}{R_B}} \tag{1.7}$$

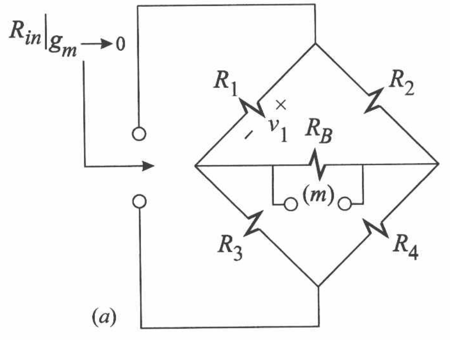

Figure 1.4

The EET now requires us to perform two additional calculations as shown in Figs. 1.4*b* and *c* in which the dependent current source is replaced with an independent one, i_m, pointing in the opposite direction. In Fig. 1.4*b* we determine the transresistance, v_1/i_m, which is the *inverse* of the transconductance gain g_m of the dependent source, with the input port short. Inspecting Fig. 1.4*b*, we see that $R_1 \| R_3$ and $R_2 \| R_4$ form a voltage divider connected across an equivalent Thevinin voltage source, $i_m R_B$, in series with a Thevinin resistance, R_B, so that we have:

$$\frac{v_1}{i_m R_B} = \frac{R_1 \parallel R_3}{R_B + R_2 \parallel R_4 + R_1 \parallel R_3} \tag{1.8}$$

It follows that the inverse gain, with the input port short, is given by:

$$\mathscr{G}^{(m)} = \left.\frac{v_1}{i_m}\right|_{(in)\to short} = \frac{R_1 \parallel R_3}{R_B + R_2 \parallel R_4 + R_1 \parallel R_3} R_B \tag{1.9}$$

Similarly, we can determine in Fig. 1.4c that the inverse gain, with the input port open, is given by:

$$\overline{G}^{(m)} = \left.\frac{v_1}{i_m}\right|_{(in)\to open} = \frac{R_B \parallel (R_3 + R_4)}{R_1 + R_2 + R_B \parallel (R_3 + R_4)} R_1 \tag{1.10}$$

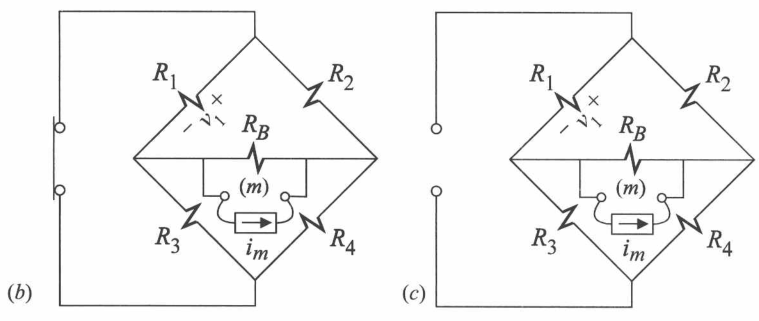

Figure 1.4 (*cont.*)

We can now assemble the final answer using the three separate calculations in Eqs. (1.7), (1.9) and (1.10) according to the following formula given by the EET:

$$R_{in} = R_{in}|_{g_m \to 0} \frac{1 + g_m \mathscr{G}^{(m)}}{1 + g_m \overline{G}^{(m)}} \tag{1.11}$$

Upon substituting, we get:

$$R_{in} = (R_1 + R_3) \parallel (R_2 + R_4) \frac{1 + \dfrac{R_1 \parallel R_3 + R_2 \parallel R_4}{R_B}}{1 + \dfrac{(R_1 + R_2) \parallel (R_3 + R_4)}{R_B}} \tag{1.12}$$

$$\times \frac{1 + \dfrac{g_m R_B}{1 + (R_B + R_2 \parallel R_4)/R_1 \parallel R_3}}{1 + \dfrac{g_m R_1}{1 + (R_1 + R_2)/R_B \parallel (R_3 + R_4)}}$$

Hence, by doing far less algebra than that required by the parameter-extraction method, we have obtained a low-entropy symbolic expression which is far superior to the one given in Eq. (1.6)

The EET, quite naturally, also allows for the value of a dependent source to become infinite so that a particular transfer becomes simplified in the same manner as that of an ideal operational amplifier circuit. In the case of R_{in} in Fig. 1.3, the EET allows us to write:

$$R_{in} = R_{in}|_{g_m \to \infty} \frac{1 + \dfrac{1}{g_m \overline{\mathscr{G}}^{(m)}}}{1 + \dfrac{1}{g_m \overline{G}^{(m)}}} \tag{1.13}$$

in which $\overline{G}^{(m)}$ and $\overline{\mathscr{G}}^{(m)}$ are the same as before and $R_{in}|_{g_m \to \infty}$ is determined in Fig. 1.5. The gain from v_1 to $g_m v_1$ reminds us of an opamp connected in some kind of

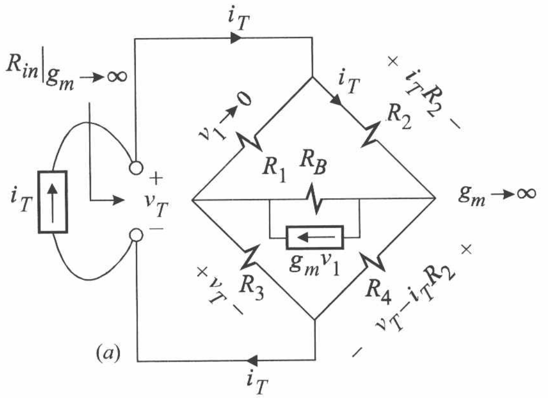

Figure 1.5

feedback fashion whose details we do not need to know at all. Now, if we let g_m become infinite, then $v_1 \to 0$ very much in the same manner as the differential input voltage of an opamp tends to zero when the gain becomes infinite and the output voltage stays finite. We can see in Fig. 1.5 that, with $g_m \to \infty$ and $v_1 \to 0$, the current through R_1 becomes zero and i_T flows entirely through R_2 creating a voltage drop $i_T R_2$ across it. At the same time, v_T appears across R_3 causing a current v_T/R_3 to flow through it. We can also see that the voltage drop across R_4, when $v_1 = 0$, is equal to $v_T - i_T R_2$ so that the current through it is simply $(v_T - i_T R_2)/R_4$. Summing the currents at the lower node of the bridge, we obtain:

$$i_T = \frac{v_T}{R_3} + \frac{v_T - i_T R_2}{R_4} \tag{1.14}$$

It follows from Eq. (1.14) that:

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