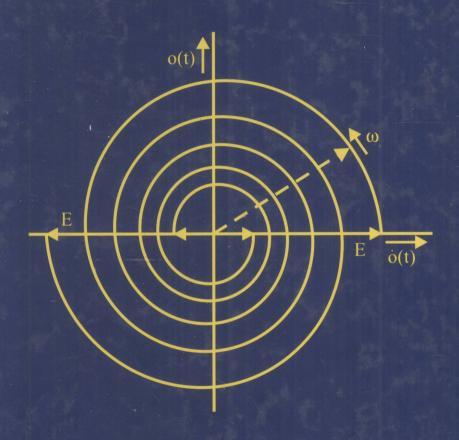
Oscillators and Oscillator Systems

Classification, Analysis and Synthesis



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OSCILLATORS AND OSCILLATOR SYSTEMS

Preface

In many of today's electronic systems, timing information plays an essential role in the information processing. To provide these systems with timing information, usually electronic oscillators are used, which generate periodic signals that can be used for timing purposes. Each electronic system poses different requirements on the oscillations produced by the oscillators, depending on the type and performance level of that specific system. It is the task of the designer to find the specifications for the desired oscillation and to implement an electronic circuit meeting these specifications. As the desired oscillations have to fulfill many requirements, the design process can become very complex. To find an optimal solution, the designer requires a design methodology that is preferably completely top-down oriented. To achieve such a methodology, it must be assured that each property of the system can be optimized independently of all other properties.

In this book, a systematic approach to the design of high-performance oscillators and oscillator systems is presented. The main problem in achieving a top-down design process is usually the complex relation between the *design parameters* and the *properties* of the design. However, as it is the designed system itself that determines this relation, it can be designed such that the relation becomes simple. In this book, a step is made towards an *orthogonal* design process for oscillators, in which each property of the oscillator can be optimized independently of all other properties. As in a practical design orthogonality of all properties is not necessary, nor desirable, this book provides insight in the design process of oscillators on every hierarchical level of the design, such that the designer can decide in which design step orthogonalization is useful to achieve an optimal design.

After an introduction in Chapter 1, and a view on design fundamentals in Chapter 2, in Chapter 3 a fundamental classification of oscillators is presented. To enable the designer to make strategic design decisions at the right hierarchical level of the design such a classification is of utmost importance. As in every oscillator a timing reference is present that is able to convert the timing infor-

mation into a measurable electrical quantity, the fundamental classification of Chapter 3 is based on the oscillator's internal timing reference. In this book, focus is completely on oscillators comprising linear timing references, that can be characterized by way of their pole patterns. This classification gives the designer a high-level tool to easily judge the properties of a specific class of oscillators with only a swift glance at the pole pattern. Therefore, from this classification many useful conclusions can be drawn about the strengths and weaknesses of every type of oscillator.

After in introduction into the noise behavior of oscillators in general in Chapter 4, Chapter 5 presents a new way of noise modeling specifically suited for the analysis of noise in relaxation (or first-order) oscillators. Although the noise behavior of first-order oscillators is principally difficult due to the non-linear character of these oscillators, it can be described very elegantly. For the description of the influence of the noise sources in first-order oscillators, it is shown in Chapter 5 that it is useful to develop effect-oriented, rather than cause-oriented models. The effect-oriented models developed in this chapter all focus on one parameter of the oscillation, as the designer is usually interested in one specific parameter. The developed models can be represented by simple filters followed by a sampling action, which makes them easily interpretable.

A second, important class of oscillators is the class of second-order oscillators. In these oscillators timing references are used that can be described with two poles. The noise behavior of second-order oscillators is the subject of Chapter 6. It is shown how the energy transport in oscillators effects the susceptibility to noise and how orthogonalization can be applied to the design of oscillators of this class. Special attention is being paid to orthogonal design of the timing reference and the circuits surrounding it, but also to the, very important, subject of interfacing between the separate circuit parts.

Another important aspect in oscillator design, tuning the oscillation frequency, is the subject of Chapter 7. In all oscillators, energy is transported during a cycle. The oscillation frequency is determined by two aspects of this energy transport: the amount of energy that is transported and the speed at which it is done. Furthermore, the energy flow in the timing reference can be subdivided into three flows. Energy can be dissipated, be supplied to and withdrawn from the timing reference each cycle, or be exchanged between circuit elements within the timing reference. In Chapter 7, it is shown which energy flows can best be used to tune an oscillator, and in which way this can best be done. A separate section of Chapter 7 covers the important subject of crystal tuning. Very low phase noise oscillators can be made using piezo-electric crystals, but generally these crystal oscillators can not easily be tuned. Therefore, several tuning meth-

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ods, among which several new methods, are compared. The tuning and noise behavior are studied as well as the power dissipation of these methods. Conclusions are drawn that tell the designer when to use which tuning method.

In Chapter 8, the subject of oscillator systems is covered. When an oscillator is to be designed, it is not always possible to meet the predefined requirements with an oscillator consisting of only one timing reference. In that case, using more timing references, or even more oscillators can be a good solution. In this chapter, many methods are described in which timing references or oscillators can be coupled, among which methods to improve the noise behavior of oscillators, to improve the tuning behavior, or to improve a quadrature phase relation. Moreover, coupled systems are described that can be used for resonance-mode selection, for example for use with resonators with many resonance modes, such as micromachined mechanical resonators, or overtone crystal oscillators.

Throughout this book, many examples have been incorporated to illustrate the underlying theory. Further, emphasis is on concepts and providing insight, rather than on mathematics. This makes this book not only useful to specialist in the field, but also to readers with little experience in the field of oscillator design.

This book is the result of the work done during my years as a Ph.D. student at the Electronics Research Laboratory of the Delft University of Technology, Faculty of Information Technology and Systems, Department of Electrical Engineering.

The work that has lead to the completion of this book could never have been done without the moral, intellectual and financial support of many. Personally, I would like to express my gratitude to Prof.dr.ir. Jan Davidse for convincing me to join the Electronics Research laboratory as a Ph.D. student. Secondly my gratitude goes to Prof.dr.ir. Arthur van Roermund for providing me with the opportunity to join the University, and for being a wonderful promoter for the full four years.

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Jan R. Westra Amstelveen, The Netherlands August 1999

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Chapter 1

Introduction

1.1 History of oscillators

Time most probably is the oldest element in nature. If only because the word 'old' does not have a meaning without the concept of time. With the creation of matter, nature was provided with the first timing references, as the elementary 'building blocks' of matter are atoms, consisting of electron clouds swarming around protons. In atoms, many periodical sequences take place, dividing time into more or less equal parts. On a much larger scale, the movement of the planets provides nature with a timing reference. This movement is the cause of the timing references that influence the life of every being: the day-night cycle and the cycle of the seasons. It is thus not surprising that the first timing references, that were more accurate than the movement of the planets, were meant to study this movement. The need for these accurate timing references emerged during the renaissance both for use in astronomy and, being a close relative of astronomy, in navigation. As Holland was the largest naval power in that time, it is also not surprising that the first to fulfill the desire for more accurate timing references was a Dutchman. Christiaan Huijgens invented the pendulum clock in 1656, thus paving the way to both safer and longer voyages. The first mathematical descriptions of oscillations were also made by Huijgens. In his Horologium Oscillatorium [6], he describes the exact isochronism of cycloidic oscillations, and improvements of the pendulum clock, making it useful for navigation at sea by the introduction of the balance; a mass-spring system that made the clock independent of the gravity field. Up to the 19th century, these were the most important technical advances in the field of timing references. A new impulse in the development of timing references was required after the invention of radio transmission, when electronic oscillators gained importance. The basis was laid by Barkhausen [2], formulating the criteria for oscillation, nowadays

known as The Barkhausen Criteria. In the early days of radio transmission, when the radio spectrum was almost empty and the most important active electronic components were the large and expensive electron tubes, emphasis was put largely on the simplicity of the implementation of the oscillator, rather than on its spectral purity. In these days, oscillators were thus mostly built using only one active component. The Clapp, Pierce, Colpitt and Hartley oscillators gained enormous popularity, just because they could be implemented using only one active component. As these systems had little design freedom, all design parameters tended to have complex interdependencies, which led to many papers and books analyzing the behavior of many specific oscillators, as well as giving general mathematical analyses [1,3–5,7–11]. In oscillator designs using only one active element, these interdepencies hampered the optimization of the design for each specific design criterion separately. When the design was optimized for one property, others could not be optimized. The resulting design was thus always a compromise chosen for the specific application.

Nowadays, electronic circuits are predominantly implemented in integrated circuits, the transistor being the most important active device. As thousands of transistors can be made in an extremely small area, neither the size, nor the cost of the complete circuitry is directly dependent on the number of active devices used. Instead, now the number of passive devices, coils and large capacitors, determines the size and cost of a circuit as, in present day technology, passive devices cannot be integrated easily in integrated circuits. As the number of active devices per circuit increased, transmitters could be made smaller, radio communication gained popularity and free space in the radio spectrum became scarce. The ease of integration and the ever growing popularity of radio communications became two mutually stimulating processes: On the one hand, the ever growing popularity led to the need for ever increasing accuracy in the transmitted signals so as not to disturb other transmissions in the spectrum. On the other hand, the ever increasing accuracy of the transmitted signals led to the development of circuits with ever growing complexity that could never have been designed without the aid of ever better design tools, both in CAD and in theoretical knowledge.

1.2 Aim and scope of this book

Every electronic design is a process of optimization procedures and trade-offs. A good design should always start with an optimization procedure at the highest possible hierarchical level. At this hierarchical level, the designer should be able to choose the right (type of) circuit for the job. Once he is able to specify his needs accurately; the requirements for an oscillator in a transmitting system will be completely different from an oscillator used in a measurement system. When the wrong design decisions are made, or when the design starts at a lower level

without being aware of the choices made at a higher level, precious design time can be lost trying to retrieve at the device level what has been lost at the system level. Once a specific system has been chosen, the designer should be aware that some quality parameters will be inherently good in that system, whereas others will not. Therefore, it is of utmost importance that the designer not only knows which quality parameters are important but also which quality parameters are not. Being aware of the *unimportant* parameters greatly facilitates the design and gives the designer the freedom to optimize for more important parameters.

The goal of this book is to present a systematic approach to the design of oscillators and oscillator systems. First of all, a classification of oscillators is presented. In this classification, properties are assigned to classes of oscillators at a high hierarchical level. It becomes clear which properties are associated with which level in the design hierarchy. Moreover, the designer becomes aware of which design trade-offs are to be made at which hierarchical level. The result is an oscillator design methodology. Furthermore, techniques that are derived from the systematic approach, are supplied to the designer to enable him or her to bring the performance as close as possible to the fundamental limits.

1.3 Survey of this book

After this introduction, we start in Chapter 2 with the fundamentals of design, and specifically the design of oscillators. We go through the various levels of the design and learn about the design fundamentals that form the basis for this book. For the oscillator design path outlined in Chapter 2, a basic classification of oscillators is required. This classification is presented in Chapter 3. Based on their type of internal timing reference, oscillators are subdivided into categories. The resulting classification is a design tool at a very high hierarchical level. Using this classification and the specifications of the oscillation, the designer is able to make the right strategic design decisions at the right hierarchical level.

Chapter 4 gives a general introduction to the subject of noise in oscillators. The influence of noise in oscillators is described at a mathematical level and several noise measures are introduced. At the end of this chapter, the *Bennet* noise model is introduced. This model can be used advantageously for the description of the influence of noise in oscillators. In chapters 5 and 6, we leave the highest hierarchical level to describe the noise behavior of two important classes of oscillators introduced in Chapter 3. At the end of these chapters, we are able to judge the noise performance of oscillators in a very early design stage, and design strategies are presented for each class. In Chapter 7, we take a closer look at different tuning strategies. Emphasis is laid on how oscillators can be tuned, while preserving the orthogonality of the design, completely in accordance with the design fundamentals presented in Chapter 2. At the end of Chapter 7, the

fundamental oscillator knowledge we have gained has paved the way to take a peek at a yet higher hierarchical level in Chapter 8. In this chapter it is shown that orthogonalization of design requirements is already possible at the system level. When it is impossible to combine design requirements in the design of one oscillator, orthogonalization at the system level can be the solution. In Chapter 9, the presented classification and design procedures are reviewed and conclusions are drawn.

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