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Preface

The Maxwell theory of electromagnetism was well established in the latter nineteenth century, when H. R. Hertz demonstrated the electromagnetic wave. The theory laid the foundation for physical optics, from which the quantum concept emerged for microscopic physics. Einstein realized that the speed of electromagnetic propagation is a universal constant, and thereby recognized the Maxwell equations to compose a fundamental law in all inertial systems of reference. On the other hand, the pressing demand for efficient radar systems during WWII accelerated studies on guided waves, resulting in today's advanced telecommunication technology, in addition to a new radio- and microwave spectroscopy. The studies were further extended to optical frequencies, and laser electronics and sophisticated semi-conducting devices are now familiar in daily life. Owing to these advances, our knowledge of electromagnetic radiation has been significantly upgraded beyond plane waves in free space. Nevertheless, in the learning process the basic theory remains founded upon early empirical rules, and the traditional teaching should therefore be modernized according to priorities in the modern era.

In spite of the fact that there are many books available on this well-established theme, I was motivated to write this book, reviewing the laws in terms of contemporary knowledge in order to deal with modern applications. Here I followed two basic guidelines. First, I considered electronic charge and spin as empirical in the description of electromagnetism. This is unlike the view of early physicists, who considered these ideas hypothetical. Today we know they are factual, although still unexplained from first principle. Second, the concept of "fields" should be in the forefront of discussion, as introduced by Faraday. In these regards I benefited from Professor Pohl's textbook, *Elektrizitätslehre*, where I found a very stimulating approach. Owing a great deal to him, I was able to write my introductory chapters in a rather untraditional way, an approach I have found very useful in my classes. In addition, in this book I discussed microwave and laser electronics in some depth, areas where coherent radiation plays a significant role for modern telecommunication.

I wrote this book primarily for students at upper undergraduate levels, hoping it would serve as a useful reference as well. I emphasized the physics of electromagnetism, leaving mathematical details to writers of books on "mathematical physics." Thus, I did not include sections for "mathematical exercise," but I hope that readers will go through the mathematical details in the text to enhance their understanding of the physical content.

In Chapter 21 quantum transitions are discussed to an extent that aims to make it understandable intuitively, although here I deviated from classical theories. Although this topic is necessary for a reader to deal with optical transitions, my intent was to discuss the limits of Maxwell's classical theory that arise from phase coherency in electromagnetic radiation.

It is a great pleasure to thank my students and colleagues, who assisted me by taking part in numerous discussions and criticisms. I have benefited especially by comments from S. Jerzak of York University, who took time to read the first draft. I am also grateful to J. Nauheimer who helped me find literature in the German language. My appreciation goes also to Springer-Verlag for permission to use some figures from R. W. Pohl's book *Elektrizitätslehre*.

Finally, I thank my wife Haruko for her encouragement during my writing, without which this book could not have been completed.

M. Fujimoto
September 2006

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Steady Electric Currents

1.1. Introduction

The macroscopic *electric charge* on a body is determined from the quantity of electricity carried by particles constituting the material. Although some electric phenomena were familiar before discoveries of these particles, such an origin of electricity came to our knowledge after numerous investigations of the structure of matter. Unlike the mass that represents mechanical properties, two kinds of electric charges different in sign were discovered in nature, signified by attractive and repulsive interactions between charged bodies. While electric charges can be combined as in algebraic addition, carrier particles tend to form neutral species in equilibrium states of matter, corresponding to *zero* of the charge in macroscopic scale.

Frictional electricity, for example, represents properties of rubbed bodies arising from a structural change on the surfaces, which is unrelated to their masses. Also, after Oersted's discovery it was known that the magnetic field is related to moving charges. It is well established that electricity and magnetism are not independent phenomena, although they were believed to be so in early physics. Today, such particles as electrons and atomic nuclei are known as *basic elements* composed of masses and charges of materials, as substantiated in modern chemistry. The electromagnetic nature of matter can therefore be attributed to these particles within accuracies of modern measurements. In this context we can express the law of electromagnetism more appropriately than following in the footsteps in early physics.

Today, we are familiar with various sources of electricity besides by friction. Batteries, for example, a modern version of Voltà's pile[1], are widely used in daily life as sources of *steady electric currents*. Also familiar is *alternating current* (AC) that can be produced when a mechanical work is converted to an *induction current* in a magnetic field. Supported by contemporary chemistry, in all processes where

[1] Voltà's pile was constructed in multi-layers of Zn and Cu plates sandwiched alternately with wet rags. Using such a battery, he was able to produce a relatively high emf for a weak current by today's standard.

electricity is generated, we note that positive and negative charges are separated from neutral matter at the expense of internal and external energies. Positive and negative charges $\pm Q$ are thus produced simultaneously in equal quantities, where the rate dQ/dt, called the *electric current*, is a significant quantity for electrical energies to be used for external work. Traditionally, the current source is characterized by an *electromotive force*, or "emf," to signify the driving force, which is now expressed by the *potential energy* of the source.

Historically, Voltà's invention of an electric pile (ca. 1799) played an important role in producing steady currents, which led Oersted and Ampère to discoveries of the fundamental relation between current and magnetic fields (1820). In such early experiments with primitive batteries, these pioneers found the law of the magnetic field by using small compass needles placed near the current. Today, the magnetic field can be measured with an "ammeter" that indicates an induced current. In addition to the *electron spin* discovered much later, it is now well established that charges in motion are responsible for magnetic fields, and all electrical quantities can be expressed in practical units of emf and current. As they are derived from precision measurements, these units are most suitable for formulating the laws of electromagnetism.

On the other hand, *energy* is a universal concept in physics, and the unit "joule," expressed by J in the MKS system[2], is conveniently related to practical electrical units. In contrast, traditional CGS units basically contradict the modern view of electricity, namely, that it is independent from mechanical properties of matter.

In classical physics, the electrical charge is a macroscopic quantity. Needless to say, it is essential that such basic quantities be measurable with practical instruments such as the aforementioned ammeter and the voltmeter; although the detailed construction of meters is not our primary concern in formulating the laws, these instruments allow us to set the standard for currents and emf's.

1.2. Standards for Electric Voltages and Current

Electric phenomena normally observed in the laboratory scale originate from the gross behavior of electrons and ions in metallic conductors and electrolytic solutions. It is important to realize that the *electronic charge* is the minimum quantity of electricity in nature under normal circumstances[3]. The charge on an electron has been measured to great accuracy: $e = -(1.6021892 \pm 0.000029) \times 10^{-19}$ C, where C is the practical MKS unit "coul."

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[2] MKS stands for meter-kilogram-second, representing basic units of length, mass, and time, respectively. Electric units can be defined in any system in terms of these units for mechanical quantities, however, in the MKSA system, the unit *ampere* for an electric current is added as independent of mass and space-time. CGS units are centimeter, gram, and second, representing mass and space-time similar to MKS system.

[3] According to high-energy physics, charged particles bearing a fraction of the electronic charge, such as $e/2$ and $e/3$, called "quarks" have been identified. However, these particles are short-lived, and hence considered as insignificant for classical physics.

In a battery, charges $+Q$ and $-Q$ are produced by internal chemical reactions and accumulate at positive and negative electrodes separately. Placed between and in contact with these electrodes, certain *materials* exhibit a variety of conducting behaviors. Many materials can be classified into two categories: *conductors* and *insulators*, although some exhibit a character between these two categories. Metals, e.g., copper and silver, are typical conductors, whereas mica and various ceramics are good insulators. Microscopically, these categories can be characterized by the presence or absence of mobile electrons in materials, where mobile particles are considered to be moved by charges $\pm Q$ on the electrodes. As mentioned, traditionally, such a driving force for mobile charges was called an electromotive force and described as a *force* F proportional to Q, although differentiated from a voltage difference defined for a battery. Mobile electrons in metals are by no means "free," but moved by F, drifting against an internal frictional force F_d. For *drift motion at a steady rate*, the condition $F + F_d = 0$ should be met, giving rise to what is called a *steady current*.

For ionic conduction in electrolytic solutions, Faraday discovered the law of electrolysis (1833), presenting his view of the ionic current. Figure 1.1 shows a steady electrolysis in a dilute $AgNO_3$ solution, where a mass M_{Ag} of deposited silver on the negative electrode is proportional to the amount of charge q transported during a time t, that is,

$$M_{Ag} \propto q. \tag{1.1}$$

The mass M_{Ag} can be measured in precision in terms of *molar number* N of Ag^+, hence the transported charge q can be expressed by the number N that is proportional to the time t for the electrolysis.

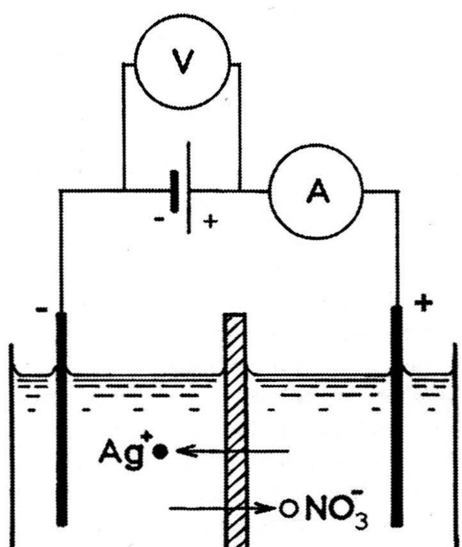

FIGURE 1.1. Electrolysis of $AgNO_3$ solution.

With accurately measured M_{Ag} and t, the current $I = q/t$ can be determined in great precision. Using an electrolytic cell, a current that deposits 1.1180 mg of silver per second is defined as one *ampere* (1A). Referring to the current of 1A, the unit for a charge is called 1 *coulomb*, i.e., 1C = 1A-sec.

To move a charge q through a connecting wire, the battery must supply an energy W that should be proportional to q, and therefore we define the quantity $V = W/q$, called the *potential* or *electric potential*. The MKS unit for W is J, so that the unit of V can be specified by J-C^{-1}, which is called a "volt" and abbreviated as V.

In the MKS unit system the ampere (A) for a current is a fundamental unit, whereas "volt" is a derived unit from ampere. Including A as an additional basic unit, the unit system is referred to as the "MKSA system." Nevertheless, a cadmium cell, for example, provides an excellent voltage standard: $V_{emf} = 1.9186 \times 0.0010$ V with excellent stability 1 μV/yr under ambient conditions.

A practical passage of a current is called a *circuit*, connecting a battery and another device with conducting wires. For a steady current that is time-independent, we consider that each point along a circuit can be uniquely specified by an electric potential, and a potential difference between two points is called *voltage*. The potential V is a function of a point x along the circuit, and the potential difference $V(+) - V(-)$ between terminals $+$ and $-$ of a battery is equal to the emf voltage, V_{emf}. If batteries are removed from a circuit, there is, naturally, no current; the potentials are equal at all points in the circuit—that is, $V(x) = \text{const}$ in the absence of currents.

1.3. Ohm Law's and Heat Energy

Electric conduction takes place through conducting materials, constituting a major subject for discussion in solid state physics. In the classical description we consider only idealized conductors, either metallic or electrolytic. In the former electrons are charge carriers, whereas in the latter both positive and negative ions are mobile, contributing to the electrolytic current. These carriers can drift in two opposite directions; however, the current is defined for expressing the amount of charges $|Q|$ transported per unit time, which is basically a *scalar* quantity, as will be explained in the following discussion. In this context how to specify the current direction is a matter of convenience. Normally, the current is considered to flow in the direction for decreasing voltage, namely from a higher to a lower voltage.

Because it is invisible the current is "seen" by three major effects, of which magnetic and chemical effects have already been discussed. The third effect is *heat* produced by currents in a conducting passage, for which Ohm (1826) discovered the basic law of *electrical resistance*. Joule (ca. 1845) showed later that heat produced by a current is nothing but dissipated energy in a conductor.

Consider a long conducting wire of a uniform cross-sectional area S. Figure 1.2 illustrates a steady flow of electrons through a cylindrical passage, where we consider a short cylindrical volume $S\Delta x$ between $x - \Delta x$ and x along the wire.

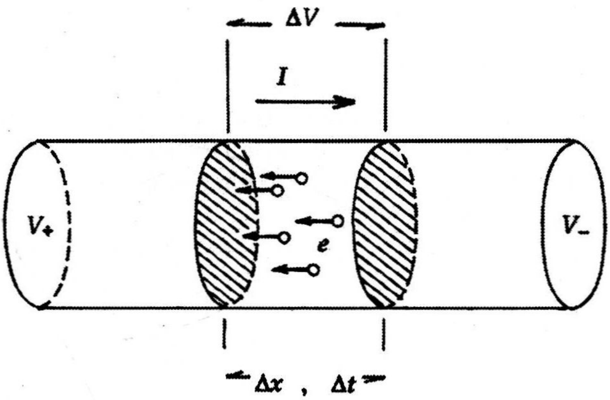

FIGURE 1.2. A simplified model for electronic conduction.

For a steady flow we can assume that the carrier density n is a constant of time between these points, at which the corresponding times are $t - \Delta t$ and t, respectively. If the potential is assumed to be higher to the left and lower to the right, the current should flow from the left to the right, as indicated in the figure. Assuming also that all electrons move in the direction parallel to the wire, the amount of charge $\Delta Q = neS\Delta x$ can be considered to move in and then out of the volume $S\Delta x$ during the time interval Δt, where $e < 0$ is the negative electronic charge. Therefore, the current is expressed as

$$I = \frac{\Delta Q}{\Delta t} = neS\frac{\Delta x}{\Delta t} = nev_\mathrm{d}S \tag{1.2}$$

where $v_\mathrm{d} = \Delta x/\Delta t$ is the drift velocity of electrons. We can therefore write

$$I = jS, \quad \text{where} \quad j = nev_\mathrm{d}, \tag{1.3}$$

called the *current density* in the area S. It is noted from (1.3) that the sign of j depends on the direction of v_d. For negative electrons, $e < 0$, the current is positive, $I > 0$, if $v_\mathrm{d} < 0$. For an ionic conduction of positive and negative carriers whose charges are $e_1 > 0$ and $e_2 < 0$, respectively, the equation (1.3) can be generalized as

$$j = n_1 e_1 v_1 + n_2 e_2 v_2, \tag{1.4}$$

where both terms on the right give positive contributions to j, despite the fact that ions 1 and 2 drift in opposite directions.

In the above simple model, the current density for a steady flow may not be significant, but for a distributed current it is an important measure, as will be discussed later for a general case. The MKSA unit of a current density is given by $[j] = \text{A-m}^{-2} = \text{C-m}^{-2}\text{-s}^{-1}$.

Next, we consider the potential difference between $x - \Delta x$ and x, which is responsible for the current through these points. For a small Δx, the potential difference ΔV can be calculated as

$$\Delta V = V(x - \Delta x) - V(x) = -F\Delta x,$$

where the quantity $F = -\Delta V/\Delta x$ represents the driving force for a *hypothetical unit charge* 1C. Nevertheless, there is inevitably a frictional force in the conducting material, as can be described by *Hooke's law* in mechanics, expressed by $F_d = -kv_d$, where k is an elastic constant.

For a steady current, we have $F = -F_d$, and therefore we can write

$$\Delta V = F_d\Delta x = kv_d\Delta x.$$

Eliminating v_d from this expression and (1.2), we obtain

$$\Delta V = \Delta RI, \quad \text{where} \quad \Delta R = \left(\frac{k}{neS}\right)\Delta x,$$

the *electrical resistance* between $x - \Delta x$ and x. For a uniform wire, this result can be integrated to obtain the total resistance from the relation, that is,

$$-\int_A^B dV = V(\text{A}) - V(\text{B}) = V_{\text{emf}} = \frac{Ik}{neS}\int_0^L dx = RI,$$

where

$$R = \frac{kL}{neS} = \rho\frac{L}{S} \tag{1.5}$$

is the resistance formula for a uniform conductor that can be calculated as the length L and cross-sectional area S are specified. The constant $\rho = \dfrac{k}{ne}$ is called the *resistivity*, and its reciprocal $1/\rho = \sigma$ is the *conductivity* of the material. Writing a potential difference and resistance between two arbitrary points as ΔV and R, Ohm's law can be expressed as

$$\Delta V = RI. \tag{1.6}$$

Obviously, the current occurs if there is a potential difference in a circuit. That is, if no current is present the conductor is static and characterized by a constant potential at all points. Values of ρ listed in Tables 1.1 and 1.2 are for representative industrial materials useful for calculating resistances.

The current loses its energy when flowing through a conducting material, as evidenced by the produced heat. Therefore, to maintain a steady current, the connected battery should keep producing charges at the expense of stored energy in the battery. Although obvious by what we know today, the energy relation for heat generation was first verified by Joule (ca. 1845), who demonstrated equivalence of heat and energy.

Figure 1.3 shows Joule's experimental setup. To flow a current I through the resistor R, a battery of V_{emf} performs work. The work to drive a charge ΔQ out of the battery is expressed by $\Delta W = V_{\text{emf}}\Delta Q$, and hence we can write $\Delta W = (RI)\Delta Q$, using Ohm's law.