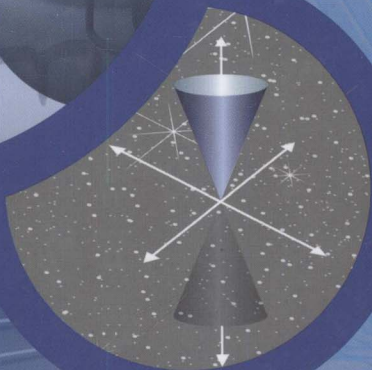
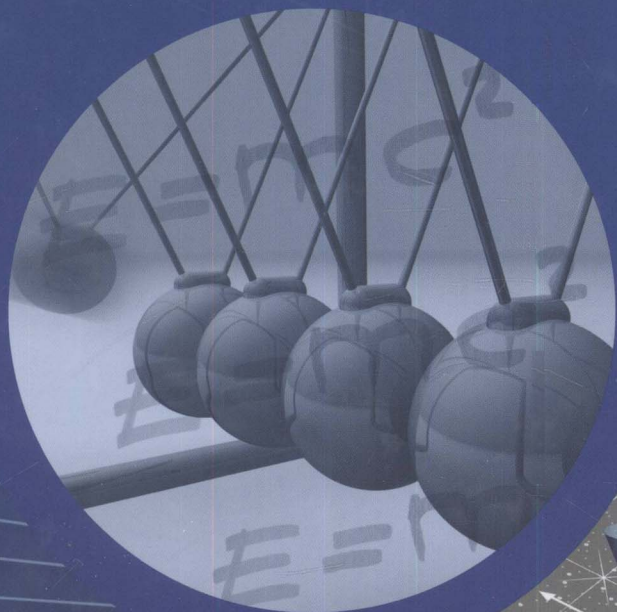


Dynamics and Relativity

J. R. Forshaw and A. G. Smith



DYNAMICS AND RELATIVITY

Jeffrey R. Forshaw and A. Gavin Smith

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Dedicated to the memory of Howard North and Edward Swallow.

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 Author's Preface vii

1. INTRODUCTORY OVERVIEW

1. SPACE, TIME AND MOTION

1.1 Describing Space and Time	2
1.1.1 Galilei and the classical particle	4
1.1.2 Unit vectors	6
1.1.3 Addition and subtraction of vectors	6
1.1.4 Multiplication of vectors	7
1.1.5 Time	7
1.1.6 Absolute space and time	7
1.2 Vectors and Coordinate Systems	12
1.2.1 Scalars and Vectors	12
1.2.2 Direction of motion	13
1.2.3 Kinematic motion	13
1.2.4 Newton's second law	13
1.2.5 Newton's third law	13
1.2.6 Newton's fourth law	13
1.2.7 Newton's fifth law	13
1.2.8 Newton's sixth law	13
1.2.9 Newton's seventh law	13
1.2.10 Newton's eighth law	13
1.2.11 Newton's ninth law	13
1.2.12 Newton's tenth law	13
1.2.13 Newton's eleventh law	13
1.2.14 Newton's twelfth law	13
1.2.15 Newton's thirteenth law	13
1.2.16 Newton's fourteenth law	13
1.2.17 Newton's fifteenth law	13
1.2.18 Newton's sixteenth law	13
1.2.19 Newton's seventeenth law	13
1.2.20 Newton's eighteenth law	13
1.2.21 Newton's nineteenth law	13
1.2.22 Newton's twentieth law	13
1.2.23 Newton's twenty-first law	13
1.2.24 Newton's twenty-second law	13
1.2.25 Newton's twenty-third law	13
1.2.26 Newton's twenty-fourth law	13
1.2.27 Newton's twenty-fifth law	13
1.2.28 Newton's twenty-sixth law	13
1.2.29 Newton's twenty-seventh law	13
1.2.30 Newton's twenty-eighth law	13
1.2.31 Newton's twenty-ninth law	13
1.2.32 Newton's thirtieth law	13
1.2.33 Newton's thirty-first law	13
1.2.34 Newton's thirty-second law	13
1.2.35 Newton's thirty-third law	13
1.2.36 Newton's thirty-fourth law	13
1.2.37 Newton's thirty-fifth law	13
1.2.38 Newton's thirty-sixth law	13
1.2.39 Newton's thirty-seventh law	13
1.2.40 Newton's thirty-eighth law	13
1.2.41 Newton's thirty-ninth law	13
1.2.42 Newton's fortieth law	13
1.2.43 Newton's forty-first law	13
1.2.44 Newton's forty-second law	13
1.2.45 Newton's forty-third law	13
1.2.46 Newton's forty-fourth law	13
1.2.47 Newton's forty-fifth law	13
1.2.48 Newton's forty-sixth law	13
1.2.49 Newton's forty-seventh law	13
1.2.50 Newton's forty-eighth law	13
1.2.51 Newton's forty-ninth law	13
1.2.52 Newton's fiftieth law	13
1.2.53 Newton's fifty-first law	13
1.2.54 Newton's fifty-second law	13
1.2.55 Newton's fifty-third law	13
1.2.56 Newton's fifty-fourth law	13
1.2.57 Newton's fifty-fifth law	13
1.2.58 Newton's fifty-sixth law	13
1.2.59 Newton's fifty-seventh law	13
1.2.60 Newton's fifty-eighth law	13
1.2.61 Newton's fifty-ninth law	13
1.2.62 Newton's sixtieth law	13
1.2.63 Newton's sixty-first law	13
1.2.64 Newton's sixty-second law	13
1.2.65 Newton's sixty-third law	13
1.2.66 Newton's sixty-fourth law	13
1.2.67 Newton's sixty-fifth law	13
1.2.68 Newton's sixty-sixth law	13
1.2.69 Newton's sixty-seventh law	13
1.2.70 Newton's sixty-eighth law	13
1.2.71 Newton's sixty-ninth law	13
1.2.72 Newton's seventieth law	13
1.2.73 Newton's seventy-first law	13
1.2.74 Newton's seventy-second law	13
1.2.75 Newton's seventy-third law	13
1.2.76 Newton's seventy-fourth law	13
1.2.77 Newton's seventy-fifth law	13
1.2.78 Newton's seventy-sixth law	13
1.2.79 Newton's seventy-seventh law	13
1.2.80 Newton's seventy-eighth law	13
1.2.81 Newton's seventy-ninth law	13
1.2.82 Newton's eightieth law	13
1.2.83 Newton's eighty-first law	13
1.2.84 Newton's eighty-second law	13
1.2.85 Newton's eighty-third law	13
1.2.86 Newton's eighty-fourth law	13
1.2.87 Newton's eighty-fifth law	13
1.2.88 Newton's eighty-sixth law	13
1.2.89 Newton's eighty-seventh law	13
1.2.90 Newton's eighty-eighth law	13
1.2.91 Newton's eighty-ninth law	13
1.2.92 Newton's ninetieth law	13
1.2.93 Newton's ninety-first law	13
1.2.94 Newton's ninety-second law	13
1.2.95 Newton's ninety-third law	13
1.2.96 Newton's ninety-fourth law	13
1.2.97 Newton's ninety-fifth law	13
1.2.98 Newton's ninety-sixth law	13
1.2.99 Newton's ninety-seventh law	13
1.2.100 Newton's ninety-eighth law	13
1.2.101 Newton's ninety-ninth law	13
1.2.102 Newton's hundredth law	13

2. ELECTRIC AND MAGNETIC FIELDS AND POTENTIALS

2.1 Electric and Magnetic Fields	14
2.1.1 Electric Fields	14
2.1.2 Magnetic Fields	14
2.1.3 Electric and Magnetic Fields	14
2.1.4 Electric and Magnetic Fields	14
2.1.5 Electric and Magnetic Fields	14
2.1.6 Electric and Magnetic Fields	14
2.1.7 Electric and Magnetic Fields	14
2.1.8 Electric and Magnetic Fields	14
2.1.9 Electric and Magnetic Fields	14
2.1.10 Electric and Magnetic Fields	14
2.1.11 Electric and Magnetic Fields	14
2.1.12 Electric and Magnetic Fields	14
2.1.13 Electric and Magnetic Fields	14
2.1.14 Electric and Magnetic Fields	14
2.1.15 Electric and Magnetic Fields	14
2.1.16 Electric and Magnetic Fields	14
2.1.17 Electric and Magnetic Fields	14
2.1.18 Electric and Magnetic Fields	14
2.1.19 Electric and Magnetic Fields	14
2.1.20 Electric and Magnetic Fields	14
2.1.21 Electric and Magnetic Fields	14
2.1.22 Electric and Magnetic Fields	14
2.1.23 Electric and Magnetic Fields	14
2.1.24 Electric and Magnetic Fields	14
2.1.25 Electric and Magnetic Fields	14
2.1.26 Electric and Magnetic Fields	14
2.1.27 Electric and Magnetic Fields	14
2.1.28 Electric and Magnetic Fields	14
2.1.29 Electric and Magnetic Fields	14
2.1.30 Electric and Magnetic Fields	14
2.1.31 Electric and Magnetic Fields	14
2.1.32 Electric and Magnetic Fields	14
2.1.33 Electric and Magnetic Fields	14
2.1.34 Electric and Magnetic Fields	14
2.1.35 Electric and Magnetic Fields	14
2.1.36 Electric and Magnetic Fields	14
2.1.37 Electric and Magnetic Fields	14
2.1.38 Electric and Magnetic Fields	14
2.1.39 Electric and Magnetic Fields	14
2.1.40 Electric and Magnetic Fields	14
2.1.41 Electric and Magnetic Fields	14
2.1.42 Electric and Magnetic Fields	14
2.1.43 Electric and Magnetic Fields	14
2.1.44 Electric and Magnetic Fields	14
2.1.45 Electric and Magnetic Fields	14
2.1.46 Electric and Magnetic Fields	14
2.1.47 Electric and Magnetic Fields	14
2.1.48 Electric and Magnetic Fields	14
2.1.49 Electric and Magnetic Fields	14
2.1.50 Electric and Magnetic Fields	14
2.1.51 Electric and Magnetic Fields	14
2.1.52 Electric and Magnetic Fields	14
2.1.53 Electric and Magnetic Fields	14
2.1.54 Electric and Magnetic Fields	14
2.1.55 Electric and Magnetic Fields	14
2.1.56 Electric and Magnetic Fields	14
2.1.57 Electric and Magnetic Fields	14
2.1.58 Electric and Magnetic Fields	14
2.1.59 Electric and Magnetic Fields	14
2.1.60 Electric and Magnetic Fields	14
2.1.61 Electric and Magnetic Fields	14
2.1.62 Electric and Magnetic Fields	14
2.1.63 Electric and Magnetic Fields	14
2.1.64 Electric and Magnetic Fields	14
2.1.65 Electric and Magnetic Fields	14
2.1.66 Electric and Magnetic Fields	14
2.1.67 Electric and Magnetic Fields	14
2.1.68 Electric and Magnetic Fields	14
2.1.69 Electric and Magnetic Fields	14
2.1.70 Electric and Magnetic Fields	14
2.1.71 Electric and Magnetic Fields	14
2.1.72 Electric and Magnetic Fields	14
2.1.73 Electric and Magnetic Fields	14
2.1.74 Electric and Magnetic Fields	14
2.1.75 Electric and Magnetic Fields	14
2.1.76 Electric and Magnetic Fields	14
2.1.77 Electric and Magnetic Fields	14
2.1.78 Electric and Magnetic Fields	14
2.1.79 Electric and Magnetic Fields	14
2.1.80 Electric and Magnetic Fields	14
2.1.81 Electric and Magnetic Fields	14
2.1.82 Electric and Magnetic Fields	14
2.1.83 Electric and Magnetic Fields	14
2.1.84 Electric and Magnetic Fields	14
2.1.85 Electric and Magnetic Fields	14
2.1.86 Electric and Magnetic Fields	14
2.1.87 Electric and Magnetic Fields	14
2.1.88 Electric and Magnetic Fields	14
2.1.89 Electric and Magnetic Fields	14
2.1.90 Electric and Magnetic Fields	14
2.1.91 Electric and Magnetic Fields	14
2.1.92 Electric and Magnetic Fields	14
2.1.93 Electric and Magnetic Fields	14
2.1.94 Electric and Magnetic Fields	14
2.1.95 Electric and Magnetic Fields	14
2.1.96 Electric and Magnetic Fields	14
2.1.97 Electric and Magnetic Fields	14
2.1.98 Electric and Magnetic Fields	14
2.1.99 Electric and Magnetic Fields	14
2.1.100 Electric and Magnetic Fields	14

Editors' Preface to the Manchester Physics Series

The Manchester Physics Series is a series of textbooks at first degree level. It grew out of our experience at the University of Manchester, widely shared elsewhere, that many textbooks contain much more material than can be accommodated in a typical undergraduate course; and that this material is only rarely so arranged as to allow the definition of a short self-contained course. In planning these books we have had two objectives. One was to produce short books so that lecturers would find them attractive for undergraduate courses, and so that students would not be frightened off by their encyclopaedic size or price. To achieve this, we have been very selective in the choice of topics, with the emphasis on the basic physics together with some instructive, stimulating and useful applications. Our second objective was to produce books which allow courses of different lengths and difficulty to be selected with emphasis on different applications. To achieve such flexibility we have encouraged authors to use flow diagrams showing the logical connections between different chapters and to put some topics in starred sections. These cover more advanced and alternative material which is not required for the understanding of latter parts of each volume.

Although these books were conceived as a series, each of them is self-contained and can be used independently of the others. Several of them are suitable for wider use in other sciences. Each Author's Preface gives details about the level, prerequisites, etc., of that volume.

The Manchester Physics Series has been very successful since its inception 40 years ago, with total sales of more than a quarter of a million copies. We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, for helpful criticisms and stimulating comments. Our particular thanks go to the authors for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and often accepting, the suggestions of the editors.

Finally we would like to thank our publishers, John Wiley & Sons, Ltd., for their enthusiastic and continued commitment to the Manchester Physics Series.

F. K. Loebinger

F. Mandl

D. J. Sandiford

August 2008

Authors' Preface

In writing this book, our goal is to help the student develop a good understanding of classical dynamics and special relativity. We have tried to start out gently: the first part of the book aims to provide the solid foundations upon which the second half builds. In the end, we are able, in the final chapter, to cover some quite advanced material for a book at this level (when we venture into the terrain of Einstein's General Theory of Relativity) and it is our hope that our pedagogical style will lead the keen student all the way to the denouement. That said, we do not assume too much prior knowledge. A little calculus, trigonometry and some exposure to vectors would help but not much more than that is needed in order to get going. We have in mind that the first half of the book covers material core to a typical first year of undergraduate studies in physics, whilst the second half covers material that might appear in more advanced first or second year courses (e.g. material such as the general rotation of rigid bodies and the role of four-vectors in special relativity).

The classical mechanics of Newton and the theory of relativity, developed by Einstein, both make assumptions as to the structure of space and time. For Newton time is an absolute, something to be agreed upon by everyone, whilst for Einstein time is more subjective and clocks tick at different rates depending upon where they are and how they are moving. Such different views lead to different physics and by presenting Newtonian mechanics alongside relativity, as we do in this book, it becomes possible to compare and contrast the two. Of course, we shall see how Newtonian physics provides a very good approximation to that of Einstein for most everyday phenomena, but that it fails totally when things whizz around at speeds approaching the speed of light.

In this era of electronic communications and online resources that can be researched at the push of a button, it might seem that the need for textbooks is diminished. Perhaps not surprisingly we don't think that is the case. Quiet time spent with a textbook, some paper and a pen, reading and solving problems, is probably still the best way to do physics. Just as one cannot claim to be a pianist without playing a piano, one cannot claim to be a physicist without solving physics problems. It is a point much laboured, but it is true nonetheless. The problems that really help develop understanding are usually those that take time to crack. The painful process of failing to solve a problem is familiar to every successful physicist, as is the excitement of figuring out the way forward. Our advice when solving the problems in this book is to persevere for as long as

possible before peeking at the solution, to try and enjoy the process and not to panic if you cannot see how to start a problem.

We have deliberately tried to keep the figures as simple as possible. A good drawing can often be an important step to solving a physics problem, and we encourage you to make them at every opportunity. For that reason, we have illustrated the book with the sorts of drawings that we would normally use in lectures or tutorials and have deliberately avoided the sort of embellishments that would undoubtedly make the book look prettier. Our aim is to present diagrams that are easy to reproduce.

A comment is in order on our usage of the word “classical”. For us “classical” refers to physics pre-Einstein but not everyone uses that terminology. Sometimes, classical is used to refer to the laws of physics in the absence of quantum mechanics and in that sense, special relativity could be said to be a classical theory. We have nothing to say about the quantum theory in this book, except that quantum theories that are also consistent with relativity lie at the very heart of modern physics. Hopefully this book will help whet the appetite for further studies in that direction.

We should like to express our gratitude to all those who have read the manuscript and provided helpful suggestions. In particular we thank Rob Appleby, Richard Battye, Mike Birse, Brian Cox, Joe Dare, Fred Loebinger, Nicola Lumley, Franz Mandl, Edward Reeves, David Sandiford and Martin Yates.

Finally, we would like to express particular gratitude to our parents, Thomas & Sylvia Forshaw and Roy & Marion Smith, for their constant support. For their love and understanding, our heartfelt thanks go to Naomi, Isabel, Jo, Ellie, Matt and Josh.

Jeffrey R. Forshaw

A. Gavin Smith

October 2008

Contents

Editors' Preface to the Manchester Physics Series	xi
Author's Preface	xiii
I INTRODUCTORY DYNAMICS	1
1 SPACE, TIME AND MOTION	3
1.1 Defining Space and Time	3
1.1.1 Space and the classical particle	4
1.1.2 Unit vectors	6
1.1.3 Addition and subtraction of vectors	6
1.1.4 Multiplication of vectors	7
1.1.5 Time	8
1.1.6 Absolute space and space-time	10
1.2 Vectors and Co-ordinate Systems	11
1.3 Velocity and Acceleration	14
1.3.1 Frames of reference	16
1.3.2 Relative motion	16
1.3.3 Uniform acceleration	18
1.3.4 Velocity and acceleration in plane-polar co-ordinates: uniform circular motion	20
1.4 Standards and Units	21
2 FORCE, MOMENTUM AND NEWTON'S LAWS	25
2.1 Force and Static Equilibrium	25
2.2 Force and Motion	31
2.2.1 Newton's Third Law	35
2.2.2 Newton's bucket and Mach's principle	39
2.3 Applications of Newton's Laws	41
2.3.1 Free body diagrams	41
2.3.2 Three worked examples	42
2.3.3 Normal forces and friction	46
2.3.4 Momentum conservation	49

2.3.5 Impulse	51
2.3.6 Motion in fluids	51
3 ENERGY	55
3.1 Work, Power and Kinetic Energy	56
3.2 Potential Energy	61
3.2.1 The stability of mechanical systems	64
3.2.2 The harmonic oscillator	65
3.2.3 Motion about a point of stable equilibrium	67
3.3 Collisions	68
3.3.1 Zero-momentum frames	68
3.3.2 Elastic and inelastic collisions	71
3.4 Energy Conservation in Complex Systems	75
4 ANGULAR MOMENTUM	81
4.1 Angular Momentum of a Particle	81
4.2 Conservation of Angular Momentum in Systems of Particles	83
4.3 Angular Momentum and Rotation About a Fixed Axis	86
4.3.1 The parallel-axis theorem	94
4.4 Sliding and Rolling	95
4.5 Angular Impulse and the Centre of Percussion	97
4.6 Kinetic Energy of Rotation	99
II INTRODUCTORY SPECIAL RELATIVITY	103
5 THE NEED FOR A NEW THEORY OF SPACE AND TIME	105
5.1 Space and Time Revisited	105
5.2 Experimental Evidence	108
5.2.1 The Michelson-Morley experiment	108
5.2.2 Stellar aberration	110
5.3 Einstein's Postulates	113
6 RELATIVISTIC KINEMATICS	115
6.1 Time Dilation, Length Contraction and Simultaneity	115
6.1.1 Time dilation and the Doppler effect	116
6.1.2 Length contraction	121
6.1.3 Simultaneity	123
6.2 Lorentz Transformations	124
6.3 Velocity Transformations	129
6.3.1 Addition of velocities	129
6.3.2 Stellar aberration revisited	130
7 RELATIVISTIC ENERGY AND MOMENTUM	135
7.1 Momentum and Energy	135

7.1.1 The equivalence of mass and energy	142
7.1.2 The hint of an underlying symmetry	144
7.2 Applications in Particle Physics	145
7.2.1 When is relativity important?	146
7.2.2 Two useful relations and massless particles	149
7.2.3 Compton scattering	152
III ADVANCED DYNAMICS	157
8 NON-INERTIAL FRAMES	159
8.1 Linearly Accelerating Frames	159
8.2 Rotating Frames	161
8.2.1 Motion on the earth	165
9 GRAVITATION	173
9.1 Newton's Law of Gravity	174
9.2 The Gravitational Potential	177
9.3 Reduced Mass	182
9.4 Motion in a Central Force	184
9.5 Orbits	186
10 RIGID BODY MOTION	197
10.1 The Angular Momentum of a Rigid Body	198
10.2 The Moment of Inertia Tensor	200
10.2.1 Calculating the moment of inertia tensor	203
10.3 Principal Axes	207
10.4 Fixed-axis Rotation in the Lab Frame	212
10.5 Euler's Equations	214
10.6 The Free Rotation of a Symmetric Top	216
10.6.1 The body-fixed frame	216
10.6.2 The lab frame	218
10.6.3 The wobbling earth	223
10.7 The Stability of Free Rotation	224
10.8 Gyroscopes	226
10.8.1 Gyroscopic precession	226
10.8.2 Nutation of a gyroscope	232
IV ADVANCED SPECIAL RELATIVITY	237
11 THE SYMMETRIES OF SPACE AND TIME	239
11.1 Symmetry in Physics	239
11.1.1 Rotations and translations	240
11.1.2 Translational symmetry	245

11.1.3 Galilean symmetry	246
11.2 Lorentz Symmetry	247
12 FOUR-VECTORS AND LORENTZ INVARIANTS	253
12.1 The Velocity Four-vector	254
12.2 The Wave Four-vector	255
12.3 The Energy-momentum Four-vector	258
12.3.1 Further examples in relativistic kinematics	259
12.4 Electric and Magnetic Fields	262
13 SPACE-TIME DIAGRAM AND CAUSALITY	267
13.1 Relativity Preserves Causality	270
13.2 An Alternative Approach	272
14 ACCELERATION AND GENERAL RELATIVITY	279
14.1 Acceleration in Special Relativity	279
14.1.1 Twins paradox	280
14.1.2 Accelerating frames of reference	282
14.2 A Glimpse of General Relativity	288
14.2.1 Gravitational fields	290
A DERIVING THE GEODESIC EQUATION	295
B SOLUTIONS TO PROBLEMS	297

Part I

Introductory Dynamics

1

Space, Time and Motion

1.1 DEFINING SPACE AND TIME

If there is one part of physics that underpins all others, it is the study of motion. The accurate description of the paths of celestial objects, of planets and moons, is historically the most celebrated success of a classical mechanics underpinned by Newton's laws¹. The range of applicability of these laws is vast, encompassing a scale that extends from the astronomical to the microscopic. We have come to understand that many phenomena not previously associated with motion are in fact linked to the movement of microscopic objects. The absorption and emission spectra of atoms and molecules arise as a result of transitions made by their constituent electrons, and the random motion of ensembles of atoms and molecules forms the basis for the modern statistical description of thermodynamics. Although atomic and subatomic objects are properly described using quantum mechanics, an understanding of the principles of classical mechanics is essential in making the conceptual leap from continuous classical systems with which we are most familiar, to the discretised quantum mechanical systems, which often behave in a manner at odds with our intuition. Indeed, the calculational techniques that are routinely used in quantum mechanics have their roots in the classical mechanics of particles and waves; a close familiarity with their use in classical systems is an asset when facing problems of an inherently quantum mechanical nature.

As we shall see in the second part of this book, when objects move at speeds approaching the speed of light classical notions about the nature of space and time fail us. As a result, the classical mechanics of Newton should be viewed as a low-velocity approximation to the more accurate relativistic theory of Einstein². To look carefully at the differences between relativistic and non-relativistic theories

¹ After Isaac Newton (1643–1727).

² Albert Einstein (1879–1955).