Principles of Signals and Systems: Deterministic Signals

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Dedication

to Professor André Blanc-Lapierre
with thanks for all his years of collaboration

In memory of Nathalie

Preface

Signal processing and systems science are the cornerstones of a large number of scientific and technical activities. For example, it is impossible to have any understanding of communication techniques without a background in signals and systems. This is especially so in our "communication century".

The purpose of this work is to cover in three volumes a field stretching from the basic principles right up to research problems. In other words, our ambition is to help a novice in the subject to become a specialist in the field of signal processing. This journey obviously needs time and effort, and we will describe its various stages.

The first volume is devoted to deterministic signals, and provides the basic tools for describing continuous-time and discrete-time signals and systems. It corresponds largely to undergraduate level, and a knowledge of the topics it treats is absolutely indispensable for any student of electrical engineering. The second volume introduces the concept of stochastic signals and corresponds to the beginning of graduate studies. It is possible to work in electronics without a knowledge of stochastic processes, but this knowledge is necessary as soon as we wish to understand noise and fluctuation problems, which almost always appear at the limit of the performance of a system. The third volume introduces the reader to some research fields and, as this area is very broad, the topics chosen are those which correspond to the main interests of the author.

This work is neither the first nor the last in its field. As there are many other possible approaches to the presentation of this material, we will try to explain our philosophy. It results from long experience in the teaching of these subjects at various levels.

Let us begin by mentioning the spirit of concision. In its present form this first volume is a revised version of a French book entitled "Eléments de théorie du signal", the ambition of which was to cover the contents of the present volumes 1 and 2 in less than a hundred pages. It was of course a challenge and, although this version is much longer, the spirit remains the same. Our fundamental teaching philosophy is that details are confusing if the basic principles are not clearly presented. This explains the title of the book.

At this point we must consider the role of mathematics in signal theory. It is clear that a good understanding of the material covered needs *a priori* a sound knowledge of basic calculus such as integration derivation and the principle of functions of complex variables. However, the question is more complex. It is often thought that signal theory is just a particular field of mathematics, and I am sometimes asked by my students whether this is the case. The answer is clearly no. We do not present mathematical concepts for their own sake, but we use these concepts to help us make a clear description of signals and systems. The concept of frequency, for example, so fundamental to everyday life, is strictly in relation to Fourier transformation, which can be presented as a purely mathematical theory. The question appears particularly critical when using distributions. We have decided not to cover completely the subject of distribution theory which is the rigorous framework for the description of the Dirac impulse signal. This would have been too long to justify the few operations given in the text.

The understanding of this field requires the active cooperation of the reader. In order to help him, problems are presented at the end of each chapter. Some of these are elementary while others are deduced from recent research work and need more attention.

Finally, let us present some comments on the actual content. Paradoxically, the first chapter contains the subject matter of the whole book, and we usually spend many hours lecturing on this material. It introduces the basic idea of the representation of signals in connection with linear filtering. Everything that follows illustrates these ideas. We have also learnt after long practice in teaching that the transition from continuous-time to discrete-time signals is very difficult to grasp. For this reason an entire chapter is devoted to this question, and the origin, interpretation and consequences of the sampling theorem are analyzed in detail. Although the material covered may be classical, our objective was also to reflect recent advances in the field. An example of this is the theory of lattice filters, which illustrates very clearly the concept of dynamical filters and is introduced even at undergraduate level. Similarly many questions concerning dynamical filters are directly connected to properties of polynomials and rational functions. We have not hesitated to present the most fundamental properties both in the text and in the appendices. Lastly, we give very explicit and simple presentations of stability criteria, the origins of which are given in more detail in the appendices.

This volume is the result of many years of teaching and some points are the result of questions put by students, who thus become anonymous co-authors. I should like to extend to them a global acknowledgment for their help.

However, many colleagues at the Laboratoire des Signaux et Systèmes have made a direct contribution to the clarification of various questions concerning both the content of this book and its material realization. I am particularly indebted to C. Bendjaballah, D. Claude, B. Lumeau and O. Macchi for enlightening discussions and material help.

Lastly I would particularly like to thank Mrs. Eve Salinas for her contribution to the English version and to the material realization of the text. She played simultaneously the role of editor and typesetter in discussions concerning the structure of the text and in the typing of complex equations. It was a privilege for me to benefit from her cooperation.

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Contents

Chapter 1

Introduction to Signals and Systems

1.1 The concept of signals

In the general spirit of this book, we will not spend a lot of time defining the concepts of signals and systems precisely and axiomatically. Roughly speaking, a signal is a function of a number of parameters, one of which is usually time, and in our context a system is defined by its action on signals. This action is a particular example of *signal processing*, an expression describing a broad scientific field to which this book is only an elementary introduction. The term signal processing indicates that signals and systems are strongly interconnected, and it is in general difficult to discuss signals without introducing the systems for which they are used. For example, as described in more detail below, it is the practical importance of time-invariant linear filters which introduces the corresponding importance of the Fourier or Laplace representation of signals. Many other examples will be given throughout this text.

There are many types of signals, and from the outset it is worth introducing some properties which enable them to be classified. The most general example of the type of signal used in this book can be written in the form

$$y = s[x ; \omega] \tag{1.1}$$

In this expression the parameter x is a vector, generally complex, of dimension m, and y is a complex vector of dimension n. The symbol ω implies that the signal can also be random, or stochastic. This is discussed in more detail later. If ω is fixed, the signal becomes deterministic. Signals like (1.1) are important for describing vector fields, for example, but we can immediately simplify the structure by assuming that x is a real scalar. Then (1.1) becomes

$$y = s[t ; \omega] \tag{1.2}$$

where the parameter t is usually the time. Furthermore, it is frequently assumed that y is one dimensional, and so the signal becomes a scalar function of time.

Even with this simplification, four situations can be considered as a result of the dichotomy of the following properties.

(a) The parameter t can be continuous or discrete. This introduces the concept of continuous-time (CT) or discrete-time (DT) signals. This has no relation to a philosophical discussion on the concept of time. In signal problems, DT is a consequence of a very important operation called *sampling*. In the sampling procedure some points in time are selected and the DT signal is defined by the values of the signal at these times.

(b) The value of the signal can be continuous or discrete. The operation which transforms a continuous value into a discrete value is called *quantization*. Even if it is not completely sanctioned by use, the term *digital* signal refers to a DT signal with discrete values.

We now present some examples.

Example 1.1 Unit step signals The CT unit step signal $u(t)$ is defined by

$$u(t) = 0 \text{ for } t < 0 \text{ and } u(t) = 1 \text{ for } t > 0 \tag{1.3}$$

For $t = 0$ this signal has no definite value but, for reasons discussed later, it is convenient to take $u(0) = 1/2$.

In the DT case this signal is a sequence of numbers $u[n]$, where n is an integer, defined by

$$u[n] = 0 \text{ for } n < 0 \text{ and } u[n] = 1 \text{ for } n \geq 0 \tag{1.4}$$

Sometimes the value $u[0] = 1/2$ is taken instead of $u[0] = 1$.

Example 1.2 Rectangular signals Consider two times t_1 and t_2 ($t_1 < t_2$) and the signal

$$r(t; t_1, t_2) \stackrel{\Delta}{=} u(t - t_1) - u(t - t_2) \tag{1.5}$$

From definition (1.3) with $u(0) = 1/2$ we see that this signal is unity for $t_1 < t < t_2$, $1/2$ for $t = t_1$ or $t = t_2$ and zero for all other times. This signal is sometimes called the rectangular window and is widely used to isolate an interesting part of a given signal.

Example 1.3 Odd and even signals A signal $x(t)$ is odd or even if it satisfies $x(t) = x(-t)$ or $x(t) = -x(-t)$ respectively. From the relation

$$x(t) = (1/2)\{x(t) + x(-t)\} + (1/2)\{x(t) - x(-t)\} \tag{1.6}$$

we deduce that any arbitrary signal can be written as the sum of an odd component and an even component.

Example 1.4 Exponential signals An exponential signal has the form $\exp(st)$ in the CT case and z^n in the DT case where s and z are arbitrary complex numbers. If $s = \mathrm{j}2\pi\nu = \mathrm{j}\omega$, where $\mathrm{j}^2 = -1$, we obtain the signal $\exp(\mathrm{j}\omega t) = \cos(\omega t) + \mathrm{j}\sin(\omega t)$. In this expression ν is the *frequency* and ω the *angular frequency*. A similar expression is obtained in the DT case if $z = \exp(\mathrm{j}\omega)$.

It is important to note that the signal $u(t)\exp(st)$ is not an exponential signal. For reasons discussed later it is sometimes referred to as a causal exponential signal.

1.2 The concept of a linear system

For the moment we envisage a system as a kind of black box which transforms an input signal $x(t)$ into an output signal $y(t)$, where t can be continuous or discrete. There are many ways of describing a given system precisely, but for our purposes we assume that a system is known when the input–output relationship is well defined. This is sometimes referred to as an external or black box description of systems. We are interested in the action of the black box but not in its content which leads to the internal description. In the case of a linear system the external description is

$$y(t) = \int_{-\infty}^{+\infty} h(t, \theta)\, x(\theta)\, \mathrm{d}\theta \tag{1.7}$$

in the CT case and

$$y[k] = \sum_{l=-\infty}^{+\infty} h[k, l]\, x[l] \tag{1.8}$$

in the DT case. Note that k and l in (1.8) are integers, and we do not distinguish below between the notations $x[k]$, x_k or even $x(k)$ for a DT signal. We do not wish to discuss convergence problems here, so the integral and series given above are assumed to be convergent. In this case it is obvious that the input–output relationship is linear in the sense that the output corresponding to $\lambda_1 x_1 + \lambda_2 x_2$ is $\lambda_1 y_1 + \lambda_2 y_2$. If we are working with complex signals, λ_1 and λ_2 can of course also be complex.

Relationships (1.7) and (1.8) can be written symbolically in the form $y(t) = S\{x(t)\}$ or $y[k] = S\{x[k]\}$, where S refers to the word "system". However, it is important to avoid confusion with these expressions. Indeed, the relation $S\{.\}$ means a *functional* and not a function. More precisely, it is clear in (1.7) that the value of the output y at time t is calculated using the values of the input at all instants θ. This means that the system has a *memory* effect. However, if $y(t)$ is only a function of $x(t)$ at the same instant of time, the system is said to be instantaneous and the relation $y(t) = S\{x(t)\}$ is a simple function. Finally we could examine the following problem: can any linear system be written in the forms (1.7) and (1.8)? The answer is in the affirmative, but the proof is beyond the scope of this chapter. The function h appearing in the previous equations is called the *impulse response* of the linear system. In order to understand this expression, let us first consider the DT case. Suppose that the input $x[k]$ is $\delta[k-p]$ where $\delta[\ ]$ is the *Kronecker delta* signal defined by

$$\delta[k] = 1 \text{ if } k = 0 \text{ and } \delta[k] = 0 \text{ if } k \neq 0 \tag{1.9}$$

Then, from (1.8),

$$y[k] = h[k, p] \tag{1.10}$$

This means that $h[k, p]$ is the output at time k generated by an "impulse" input at time p. A similar procedure can be followed for the CT case, but now the input is the *Dirac function* or *Dirac distribution* $\delta(t)$ which is discussed in more detail below. In this case $h(t, s)$ is still the output at t generated by an impulse at s.

Among the set of all possible linear systems we will next consider the subset of *linear filters*, which is particularly important in the following discussion. For this purpose we will first define the concept of *time invariance* of a system. Consider a system S generating the output $y(t)$ when the input is $x(t)$. This system is said to be time invariant if, for any time translation τ (positive or negative), the output generated by $x(t-\tau)$ is $y(t-\tau)$. In other