LECTURE NOTES IN PHYSICS

K. Hutter
A. A. F. van de Ven
A. Ursescu

Electromagnetic Field Matter Interactions in Thermoelastic Solids and Viscous Fluids

Springer

Kolumban Hutter
Alfons A. F. van de Ven
Ana Ursescu

Electromagnetic Field Matter Interactions in Thermoelastic Solids and Viscous Fluids

Springer

Authors

Kolumban Hutter
Bergstrasse 5
8044 Zürich
Switzerland
E-mail: hutter@vaw.baug.ethz.ch

Dr. Ana Ursescu
Hügelstrasse 21
64283 Darmstadt
Germany
E-mail: ana_ursescu@yahoo.com

Dr. Alfons A. F. van de Ven
Department of Mathematics
and Computer Sciences
Technical University Eindhoven
P.O. Box 513
5600 MB Eindhoven
The Netherlands
E-mail: a.a.f.v.d.ven@tue.nl

K. Hutter et al. *Electromagnetic Field Matter Interactions in Thermoelastic Solids and Viscous Fluids*, Lect. Notes Phys. 710 (Springer, Berlin Heidelberg 2006), DOI 10.1007/b 11803263

Library of Congress Control Number: 2006930673

ISSN 0075-8450
ISBN-10 3-540-37239-3 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-37239-4 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media
springer.com

Typesetting: by the authors and techbooks using a Springer LaTeX macro package
Cover design: WMXDesign GmbH, Heidelberg

Printed on acid-free paper SPIN: 11803263 54/techbooks 5 4 3 2 1 0

Lecture Notes in Physics

The Lecture Notes in Physics

The series Lecture Notes in Physics (LNP), founded in 1969, reports new developments in physics research and teaching – quickly and informally, but with a high quality and the explicit aim to summarize and communicate current knowledge in an accessible way. Books published in this series are conceived as bridging material between advanced graduate textbooks and the forefront of research to serve the following purposes:

- to be a compact and modern up-to-date source of reference on a well-defined topic;
- to serve as an accessible introduction to the field to postgraduate students and nonspecialist researchers from related areas;
- to be a source of advanced teaching material for specialized seminars, courses and schools.

Both monographs and multi-author volumes will be considered for publication. Edited volumes should, however, consist of a very limited number of contributions only. Proceedings will not be considered for LNP.

Volumes published in LNP are disseminated both in print and in electronic formats, the electronic archive is available at springerlink.com. The series content is indexed, abstracted and referenced by many abstracting and information services, bibliographic networks, subscription agencies, library networks, and consortia.

Proposals should be sent to a member of the Editorial Board, or directly to the managing editor at Springer:

Dr. Christian Caron
Springer Heidelberg
Physics Editorial Department I
Tiergartenstrasse 17
69121 Heidelberg/Germany
christian.caron@springer.com

Preface

This book is the second and substantially enlarged edition of the Springer Lecture Notes No. 88: *Field Matter Interactions in Thermoelastic Solids*, which appeared in 1978 and is out of print for about two decades. Since then, the basic issues addressed by the authors in that book have little changed: Because of the nonunique definition of the electromagnetic field quantities in ponderable bodies, constitutive postulates, e.g., for the stress tensor and other field quantities, must adequately be postulated, if two theories aiming to describe the same physical phenomenon yield for the same physical problem the same values of the observables. In the Lecture Notes, equivalence relations were established, which guarantee such equivalences for thermoelastic bodies, but no applications of the complex theory were given. Nevertheless, there was a continuous demand for the book, which was fulfilled by producing photocopies.

In the meantime, however, the authors continued to work on continuum problems of electromagneto-mechanical interactions, in which the theoretical models or simplifications were applied to practical problems. A.A.F. Van de Ven (AV) worked with students and postdoctoral fellows for more than two decades on problems of magnetoelastic instabilities, i.e., buckling of ferromagnetic and superconducting beams, plates and more complex structures, and on magnetoelastic vibrations of the same type of structures. In the latter problems, it is specifically the eigenfrequencies that need to be determined, inclusive of their dependence on the applied magnetic field or electric current. K. Hutter (KH) who was not involved with electrodynamics for 10 years, took up electromagnetic continua again about 15 years ago and concentrated on applications in fluids, as well as electrorheology. In this field, equivalence of formulations is equally a question of electromagneto-mechanical interactions. Here, the central theme is the postulation of adequate constitutive relations, which achieve the electrorheological effect, namely the transition from low viscous fluid behaviour to very high viscous response when the electric field is switched on. The application to plane Poiseuille flow of a theory was developed in a Ph.D. dissertation by W. Eckart, to 2D pipe flow with various arrangements of electrodes along the walls of the bottom and the lid of the 2D channel, which was made by Ana Ursescu (AU). Ursescu is joining us as the third author of this book.

This then outlines the content of the present book. In its first part it contains the material of the first edition and is due to AV and KH. Only small changes have been made to this text. Few adjustments were necessary because of the additions that were made. A few supplementing references are given to account for the recent literature. In the applications, the new chapter on *Magnetoelastic (In)stability and Vibrations* is due to AV and the chapter on *Electro-rheological Fluids* has been drafted by AU along with KH. The entire text has been screened for consistency and homogenization by AV and KH.

The increasing interest in electromagnetic problems in the last decade of the last century manifested itself in the appearance of a new journal in 1990, *International Journal of Applied Electromagnetics in Materials* (IJAEM), founded by K. Miya from Tokyo, and Richard K.T. Hsieh from Stockholm. The first plans for this new journal were made in 1986, during a IUTAM-Symposium in Tokyo alongwith Miya, Hsieh, Gerard Maugin, Francis Moon, Junji Tani, and one of the authors, AV. The birth of the new journal was accompanied by a series of ISEM symposia (International Symposia on Applied Electromagnetics in Materials, under the chairmanship of Miya) from 1988 until the present time. The 12th ISEM took place in 2005 in Salzburg, Austria.

One proviso to the style of the book should be mentioned. Reading the text is not easy. When developing results we are brief and we often outline the steps as to how a result is reached but do not present any details. Thus, the reader is expected to perform the in-between steps, or perhaps even consult the literature. Such an approach is almost unavoidable in electromagneto-mechanical interactions. The computations that are in principle not difficult, but generally involved and long, cannot be presented in detail as it would make the book twice as voluminous, and most likely rather boring over long stretches. We regard this as an acceptable compromise.

We wish to thank our sponsors and many of our friends in this field who have directly or indirectly contributed to this project. AV thanks the Technische Universiteit Eindhoven for their general support to the research on electromagnetoelastic interactions over many years (from 1975 onwards) and he especially thanks, many students for their contributions in particular M. Couwenberg, P. Rongen, P. Smits, and P. van Lieshout, who essentially supported the research on magnetoelastic instabilities. Moreover, he highly appreciates the cooperation and the many fruitful discussions in this area with A.O. J. Tani, Y. Shindo, K. Miya, Shu-Ang Zhou, B. Marusewski, and J.P. Nowacki. KH thanks the Darmstadt University of Technology and the Deutsche Forschungsgemeinschaft (German Research Foundation) for nearly 20 years of support. He also acknowledges the support of W. Eckart and AU in particular for their Ph.D. and postdoctoral fellowships in which the Electrorheological work presented in this book was created. He also thanks Professor K.R. Rajagopal and M. Růžička for their interest in our work on electrorheology.

We also thank E. Vasilieva and A. Maurer for compiling the text of the first edition and helping with the editing of the entire manuscript. Finally our thanks go to Dr. Ch. Caron from the Springer Verlag for his interest in this book and his willingness to publish it.

Darmstadt, Eindhoven
Spring 2006

K. Hutter
A. Ursescu
A.A.F. Van de Ven

Preface to the First Edition

The last two decades have witnessed a giant impetus in the formulation of electrodynamics of moving media, commencing with the development of the most simple static theory of dielectrics at large elastic deformations, proceeding further to more and more complex interaction models of polarizable and magnetizable bodies of such complexity as to include magnetic dissipation, spin-spin interaction and so on and, finally, reaching such magistral synthesis as to embrace a great variety of physical effects in a relativistically correct formulation. Unfortunately, the literature being so immense and the methods of approach being so diverse, the newcomer to the subject, who may initially be fascinated by the beauty, breadth and elegance of the formulation may soon be discouraged by his inability to identify two theories as the same, because they look entirely different in their formulation, but are suggested to be the same through the description of the physical situations they apply to. With this tractate we aim to provide the reader with the basic concepts of such a comparison. Our intention is a limited one, as we do not treat the most general theory possible, but restrict ourselves to non-relativistic formulations and to theories, which may be termed deformable, polarizable and magnetizable thermoelastic solids. Our question throughout this monograph is basically; what are the existing theories of field-matter interactions; are these theories equivalent, and if so; what are the conditions for this equivalence? We are not the first ones to be concerned with such fundamental ideas. Indeed, it was W.F. Brown, who raised the question of non-uniqueness of the formulation of quasistatic theories of magnetoelastic interactions, and within the complexity of his theory, he could also resolve it. Penfield and Haus, on the other hand, were fundamentally concerned with the question how electromagnetic body force had to be properly selected. This led them to collect their findings and to compare the various theories in an excellent monograph, in which they rightly state that equivalence of different formulations of electrodynamics of deformable continua cannot be established without resort to the constitutive theory, but at last, they dismissed the proper answer, as their treatment is incomplete in this regard. For this reason the entire matter was re-investigated in the doctoral dissertation of one of us (K. Hutter), but this work was soon found unsatisfactory and incomplete in certain points, although the basic structure of the equivalence proof as given in Chap. 3 of this tractate, was essentially already outlined there. Moreover, HUTTER was still not able to compare certain magnetoelastic interaction theories so that what he attempted remained a torso anyhow.

The difficulties were overcome by VAN DE VEN in a series of letters, commencing in fall of 1975, in which we discussed various subtleties of magnetoelastic interactions that had evolved from each of our own work. The correspondence was so fruitful that we soon decided to summarize our efforts in a joint publication. It became this monograph, although this was not our initial intention. Yet, after we realized that a proper treatment required a presentation at considerable length, we decided to be a little broader than is possible in a research report and to write a monograph, which would be suitable at least as a basis for an advanced course in continuum mechanics and electrodynamics (graduate level in the US). We believe that with this text this goal has been achieved. We must at the same time, however, warn the reader not to take this tractate as a basis to learn continuum mechanics and/or electrodynamics from the start. The fundamentals of these subjects are assumed to be known.

Our acknowledgements must start with mentioning Profs. J.B. ALBLAS (Technological University Eindhoven) and Y.H. PAO (Cornell University). They were the ones who initiated our interest in the subject of magnetoelastic interactions. While performing the research for this booklet and during our preparation of the various draughts we were supported by our institutions, the Federal Institute of Technology, Zürich and the Technological University, Eindhoven, and were, furthermore, encouraged by Prof. J.B. ALBLAS, Eindhoven, Prof. D. VISCHER, Zürich, Prof. I. MÜLLER, Paderborn, Prof. H. PARKUS, Vienna, Dr. Ph. BOULANGER, Brussels and Dr. A. PRECHTL, Vienna. The support and criticism provided by them, directly or indirectly, were extremely helpful. We are grateful to these people not only for their keen insight and willingness to discuss the issues with us, but also for their encouragement in general.

During the initial stage and again towards the end of the write-up of the final draught of this monograph K. HUTTER was financially supported in parts by the Technological University, Eindhoven, to spend a total of a two months period (September 1976 and April 1978) at its Mathematics Department. Without the hospitality and the keen friendship of the faculty and staff members of this department and especially of Prof. J.B. ALBLAS and his group, the work compiled in these notes would barely have been finished so timely. The burden of typing the manuscript was taken by Mrs. WOLFS-VAN DEN HURK. It was her duty to transform our hand-written draughts into miraculously looking typed sheets of over 200 pages. Her effort, of course, is gratefully acknowledged.

Eindhoven and Zürich
in the summer of 1978

K. Hutter
A.A.F. Van de Ven

List of Symbols

A_{ij}	Matrix of (anisotropic) heat conduction
$\boldsymbol{A}$	Matrix of thermal and electrical conduction
$\boldsymbol{A}, A_\alpha$	Vectorial electromagnetic potential
a	Large semi-axis of elliptical cross-section
$\boldsymbol{B}, B_i$	Magnetic induction field, magnetic flux density
$\boldsymbol{B}^M, B_i^M$; $\boldsymbol{B}^L, B_i^L$; $\boldsymbol{B}^S, B_i^S$	Magnetic induction in the MAXWELL-, LORENTZ- and statistical formulations
$\boldsymbol{B}^a, B_i^a$	Auxiliary magnetic induction field
$\overset{\circ}{\boldsymbol{B}}, \overset{\circ}{B}_i$	Magnetic induction in the rest frame
$\overset{\circ}{\boldsymbol{B}}{}^a, \overset{\circ}{B}{}_i^a$	Auxiliary magnetic induction in the rest frame
$\mathbb{B}, \mathbb{B}_\alpha$	LAGRANGEan magnetic induction
$\mathsf{B}, \mathsf{B}_\alpha$	LAGRANGEan magnetic induction, used in constitutive models

$\boldsymbol{B} = \overset{\circ}{\boldsymbol{B}} + \mathcal{O}(\dot{x}^2/c^2),\ B_i = \overset{\circ}{B}_i + \mathcal{O}(\dot{x}^2/c^2)$

$b^{(\mathrm{e})}_{\alpha\beta\gamma\delta}$	Electrostrictive constants
$b^{(\mathrm{m})}_{\alpha\beta\gamma\delta}$	Magnetostrictive constants
$\mathbf{b}, \mathrm{b}_\alpha$	Perturbed LAGRANGEan magnetic induction
$\boldsymbol{B}, B_{ij}$	Left CAUCHY–GREEN deformation tensor
BS	BIOT–SAVARD method
$\boldsymbol{B}_0$	Magnetic induction in the rigid-body state
b	Small semi-axis of elliptical cross-section, mean diameter of two concentric tori
C	Speed of light, thermal constant
C_w	Specific heat
$\boldsymbol{C}, C_{\alpha\beta}$	Right CAUCHY–GREEN deformation tensor
$\boldsymbol{C}^{-1}, C^{-1}_{\alpha\beta}$	Inverse of right CAUCHY–GREEN deformation tensor
$c_{\alpha\beta\gamma\delta}, c_{ijkl}$	Elastic constants, elasticities
CM	Combined method
c	Speed of light in vacuo
$\mathbb{C}$	Symbol for independent constitutive quantity
$d\boldsymbol{A}, dA_\alpha$	LAGRANGEan vectorial surface element

$d\boldsymbol{a}, da_i$	EULERian vectorial surface element
$d\boldsymbol{l}, dl_i$	EULERian vectorial length element
dv, dV	EULERian (LAGRANGEan) volume element
$\boldsymbol{D}, D_i$	Dielectric displacement
$\boldsymbol{D}^M, \boldsymbol{D}_i^M$	MAXWELLian dielectric displacement
$\boldsymbol{D}^a, D_i^a$	Auxiliary dielectric displacement
$\overset{\circ}{\boldsymbol{D}}, \overset{\circ}{D}_i$	Dielectric displacement in the rest frame
$\overset{\circ}{\boldsymbol{D}}{}^a, \overset{\circ}{D}{}_i^a$	Auxiliary dielectric displacement in the rest frame
$\mathcal{D}_i = \overset{\circ}{\mathcal{D}}_i + \mathcal{O}(\dot{x}^2/c^2)$	
$\mathbb{D}, \mathbb{D}_\alpha$	LAGRANGEan dielectric displacement
$\mathbb{D}^a, \mathbb{D}_\alpha^a$	Auxiliary LAGRANGEan dielectric displacement
$\mathbf{d}, \mathrm{d}_\alpha$	Perturbed LAGRANGEan dielectric displacement
$\mathbf{d}^a, \mathrm{d}_\alpha^a$	Perturbed LAGRANGEan auxiliary dielectric displacement
D	Plate rigidity
d	Distance of two concentric tori
d_{ij}	Stretching, rate of strain tensor
D_0	Regularization parameter for the constitutive relation of the extra stress tensor
$\boldsymbol{E}, E_{\alpha\beta}$	LAGRANGEan strain tensor, GREEN strain tensor
$\boldsymbol{e}, e_{ij}$	EULERian strain tensor, FINGER strain tensor
$\mathbf{e}, \mathrm{e}_{\alpha\beta}$	Perturbed GREEN strain tensor
$\boldsymbol{E}, E_i$	Electric field strength
$\boldsymbol{E}^C, E_i^C$; $\boldsymbol{E}^M, E_i^M$; $\boldsymbol{E}^L, E_i^L$; $\boldsymbol{E}^S, E_i^S$	Electric field strength in the CHU-, MAXWELL-, LORENTZ- and statistical formulations
$\boldsymbol{\mathcal{E}}, \mathcal{E}_i$	Electromotive intensity
$\overset{\circ}{\mathcal{E}}$	Electric field strength in the rest frame
$\mathbb{E}, \mathbb{E}_\alpha$	LAGRANGEan electric field strength,- electromotive intensity
e_{ijk}	Three-dimensional permutation tensor
e^{ABCD}	Four-dimensional perturbation tensor
$\mathbf{e}, \mathrm{e}_\alpha$	Perturbed LAGRANGEan electromotive intensity
E	YOUNG's modulus
$\boldsymbol{e}_i (i = 1, 2, 3)$	Unit vectors
e_{ij}	Infinitesimal strain tensor
E_0	Scale for electric field
$\boldsymbol{F}, F_i$	Specific body force, BIOT–SAVARD force
$\boldsymbol{F}^0$	BIOT–SAVARD force in the rigid-body state
$\mathbf{f}$	BIOT–SAVARD force in the perturbed state
$\boldsymbol{F}^e, F_i^e$	Electromagnetic body force
$\boldsymbol{F}^{\text{ext}}, F_i^{\text{ext}}$	External body force

$\boldsymbol{F}^{\text{Lorentz}}$	LORENTZ force
$\boldsymbol{\mathcal{F}}^{e}, \mathcal{F}^{e}_{\alpha}$	Total electric force due to surface tractions of the MAXWELL stress tensor just outside the body
$\boldsymbol{\mathcal{F}}^{\text{ext}}, \mathcal{F}^{\text{ext}}_{\alpha}$	Total force on a body due to external body force and surface traction distributions
$\boldsymbol{\mathcal{F}}$	Deformation gradient
$F_{\Delta p}(Q)$, F, $\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3$	Quantities measuring the electrorheological effect
$\boldsymbol{G}, G_{\alpha}$	LAGRANGEan electromagnetic momentum density
$\boldsymbol{g}, g_i$	EULERian electromagnetic momentum density
G	Elastic shear modulus
$\boldsymbol{G}, G_{\alpha\beta}$	LAGRANGEan strain tensor (GREEN strain tensor)
G	Shear modulus
G_I	Intermediate state
G_Y	Bending moment in a ring
$\boldsymbol{H}, H_i$	Magnetic field strength
$\boldsymbol{H}^{C}, H^{C}_{i}$, $\boldsymbol{H}^{M}, H^{M}_{i}$	Magnetic field strength in the CHU- and MAXWELL formulations
$\boldsymbol{H}^{a}, H^{a}_{i}$	Auxiliary magnetic field strength
$\boldsymbol{\mathcal{H}}^{a}, \boldsymbol{\mathcal{H}}^{a}_{i}$	Auxiliary effective magnetic field strength
$\overset{\circ}{\boldsymbol{\mathcal{H}}}, \overset{\circ}{\boldsymbol{\mathcal{H}}}_i$	Magnetic field strength in the rest frame
$\mathbb{H}, \mathbb{H}_{\alpha}$	LAGRANGEan effective magnetic field
$\mathbb{H}^{a}, \mathbb{H}^{a}_{\alpha}$	Auxiliary LAGRANGEan effective magnetic field strength
$\mathsf{H}, \mathsf{H}_{\alpha}$	LAGRANGEan magnetic field-strength, used in the constitutive relations
H^{AB}	Magnetic field-electric displacement four-tensor
$\mathbf{h}, \mathrm{h}_{\alpha}$	Perturbed LAGRANGEan effective magnetic field
$\mathbf{h}^{a}, \mathrm{h}^{a}_{\alpha}$	Perturbed LAGRANGEan auxiliary effective magnetic field
h	Gap width
I	Thermodynamic potential in determining the internal energy from electromagnetic integrability conditions
I_{AB}	Moment of inertia of the body relative to its center of mass in the rigid-body state
I_y	Moment of inertia of an area
J	JACOBIan determinant, determinant of the deformation gradient
J	Thermodynamic potential, determining the contribution of the deformation to the internal energy
$\mathbf{j}, \mathrm{j}_{\alpha}$	Perturbed LAGRANGEan electric current field
$\mathrm{j}^{1}_{\alpha\beta\gamma}$, $\mathrm{j}^{2}_{\alpha\beta}, \mathrm{j}^{3}_{\alpha\beta}, \mathrm{j}^{5}_{\alpha\beta}$, j^{4}_{α}	Coefficients arising in the constitutive relation for the LAGRANGEan conductive electric current

J	Homogeneous quadratic functional of the perturbation fields of which $\delta\mathsf{J}$ is the second variation of the action functional L
$\mathsf{J}_0, \mathsf{J}_1$	BESSEL functions of zeroth and first order
$\boldsymbol{J}, \boldsymbol{J}^{\mathrm{e}}, \boldsymbol{J}^{\mathrm{m}}$; $J_i, J_i^{\mathrm{e}}, J_i^{\mathrm{m}}$	Non-conductive current density,-electric, -magnetic current density
$\boldsymbol{\mathcal{J}}, \mathcal{J}_i$	Conductive current density
$\boldsymbol{\mathcal{J}}^{\mathrm{e}}, \mathcal{J}_i^{\mathrm{e}}$, $\boldsymbol{\mathcal{J}}^{\mathrm{m}}, \boldsymbol{\mathcal{J}}_i^{\mathrm{m}}$	Electric, magnetic current densities
$\mathring{\boldsymbol{\mathcal{J}}}, \mathring{\mathcal{J}}_i$	Conductive current in the rest frame
$\mathbb{J}, \mathbb{J}_\alpha$; $\mathbb{J}^{\mathrm{e}}, \mathbb{J}_\alpha^{\mathrm{e}}$; $\mathbb{J}^{\mathrm{m}}, \mathbb{J}_\alpha^{\mathrm{m}}$	LAGRANGEan conductive current density; LAGRANGEan electric (magnetic) conductive current density
$\mathcal{J}$	Common factor of J
$\mathcal{K}$	Magnetic part of J, $\mathcal{K} = \mathcal{B}_0^2 K, \mathcal{K} = I_0^2 K$
k	Longitudinal pressure gradient
L	Characteristic length
$L^{(e)}$	Thermoelectric constant
$L^{(m)}$	Thermomagnetic constant
$\boldsymbol{L}$	Action integral
$\mathcal{L}, \mathcal{L}_i$	LAGRANGE densities
$\boldsymbol{L}, L_i$	Specific body couple,
$\boldsymbol{L}, L_{ij}$	Dual tensor of the body couple vector
$\boldsymbol{L}^{\mathrm{ext}}, L_i^{\mathrm{ext}}$	Dual tensor of the external body couple vector
$\boldsymbol{\mathcal{L}}^e, \mathcal{L}_\alpha^e$	Total electromagnetic couple relative to the center of gravity in the rigid-body state due to the MAXWELL stress tensor just outside the body
$\boldsymbol{\mathcal{L}}^{\mathrm{ext}}, \mathcal{L}_\alpha^{\mathrm{ext}}$	Total external couple due to body couples, body forces and surface tractions
$\boldsymbol{M}^C, M_i^C$; $\boldsymbol{M}^M, M_i^M$; $\boldsymbol{M}^L, M_i^L$; $\boldsymbol{M}^S, M_i^S$	Magnetization density in the CHU-, MAXWELL- , LORENTZ- and statistical formulations
$\boldsymbol{\mathcal{M}}, \mathcal{M}_i$; $\mathcal{M}^M, \mathcal{M}_i^M$	Magnetization density, – in the MAXWELL formulation in the rest frame
$\mathbb{M}, \mathbb{M}_\alpha$	LAGRANGEan magnetization density
$\mathsf{\mathbf{M}}, \mathsf{M}_\alpha$	LAGRANGEan magnetization density per unit mass
m	Viscosity scale for the power law constitutive relation for the extra stress tensor

$\mathbf{m}, \mathrm{m}_\alpha$ $\mathbf{m}^C, \mathrm{m}_\alpha^C$ $\mathbf{m}^L, \mathrm{m}_\alpha^L$	Perturbed magnetization, – in the CHU- and LORENTZ formulations
$\mathrm{m}^1_{\alpha\beta\gamma}$ $\mathrm{m}^2_{\alpha\beta}, \mathrm{m}^3_{\alpha\beta}, \mathrm{m}^5_{\alpha\beta}$ $\mathrm{m}^4_{\alpha\beta}$	Coefficients arising in the constitutive relation for the perturbed magnetization
M_{rr}	Radial bending moment in a ring-like structure
$\mathbb{M}a$	MASON number
$\boldsymbol{n}, n_i$	EULERian unit normal vector
$\boldsymbol{N}, N_\alpha$	LAGRANGEan unit normal vector
n, n_0, n_∞	Exponents in the power law of the extra stress tensor
N_1, N_2	Normal stress differences
$n_{AB}(A, B = 1, 2, 3)$	Coefficients in the parameterization of η_{gen}
$\boldsymbol{O}, O_{ij}$	Orthogonal 3×3 matrix
$\boldsymbol{P}, P_i$	Polarization density
$\boldsymbol{P}^C, P_i^C$ $\boldsymbol{P}^M, P_i^M$ $\boldsymbol{P}^S, P_i^S$	Polarization density in the CHU-, MAXWELL- and statistical formulations
$\mathbb{P}, \mathbb{P}_\alpha$	LAGRANGEan polarization density
$\mathsf{P}, \mathsf{P}_\alpha$	LAGRANGEan polarization density per unit mass
$\mathbf{p}, \mathrm{p}_\alpha$	Perturbed LAGRANGEan polarization density
$\mathrm{p}^1_{\alpha\beta\gamma}$ $\mathrm{p}^2_{\alpha\beta}, \mathrm{p}^3_{\alpha\beta}$ p^4_α	Coefficients arising in the constitutive relation of the perturbed polarization
p	Pressure
q	Transverse load of a beam or plate
q	Perturbed LAGRANGEan charge density
$\mathbf{q}, \mathrm{q}_\alpha$	Perturbed LAGRANGEan energy flux
$\boldsymbol{q}, q_i$	Energy (heat) flux vector
$\boldsymbol{q}^S, q_i^S$	Energy (heat) flux vector in the statistical formulation
$\boldsymbol{Q}, Q_\alpha$	LAGRANGEan energy (heat) flux vector
$\mathcal{Q}$	Electric charge density, surface charge density
$Q, Q^{\mathrm{e}}, Q^{\mathrm{m}}$	Charge density, electric-, magnetic-
$\overset{\circ}{Q}$	Charge density in the rest frame
$\mathbb{Q}, \mathbb{Q}^{\mathrm{e}}, \mathbb{Q}^{\mathrm{m}}$	LAGRANGEan charge density, electric-, magnetic-.
$\mathrm{q}^1_{\alpha\beta\gamma}$ $\mathrm{q}^2_{\alpha\beta}, \mathrm{q}^3_{\alpha\beta}, \mathrm{q}^5_{\alpha\beta}$ q^4_α	Coefficients arising in the constitutive equation for the perturbed LAGRANGEan energy flux
Q	Quantity in the evaluation of I_0
$Q_j (j = 1, .., n)$	Contribution to the magnetic transverse load of beams and plates

Q	Discharge
r	Energy supply density
r^{e}	Electromagnetic energy supply density
r^{ext}	External energy supply density
$R_{\alpha\beta}$	Orthogonal matrix, describing the rotation of a rigid-body
R	Radius of a circular cross-section
$\boldsymbol{r}_1, r_2$	Position vectors on wires, –1,2
$\mathbb{R}e$	REYNOLDS number
s	Entropy supply density
s	Perturbation entropy density
$\left.\begin{matrix}\mathrm{s}^1_{\alpha\beta}\\ \mathrm{s}^2_\alpha, \mathrm{s}^3_\alpha\\ \mathrm{s}^4\end{matrix}\right\}$	Coefficients arising in the constitutive relation for the perturbed entropy density
$\mathscr{S}$	Set of equations characterizing the equilibrium state of a magnetoelastic system subject to an external field
$\mathscr{S}^1$	Analogous set as $\mathscr{S}$ describing the perturbed state
$\mathscr{S}^0$	Analogous set as $\mathscr{S}$ describing the rigid-body state
S_i	Element of $\mathscr{S}$
S_i^0	Element of $\mathscr{S}^0$
s_i	Element of $\mathscr{S}^1$
$\mathcal{T}, \mathcal{T}^0$	Stress tensor, - in rigid-body state
$\mathcal{T}^1$	Perturbed stress tensor
T_y	Traction in the y-direction along the boundary of the cross-section
T	Kinetic energy
$\boldsymbol{T}, T_{i\alpha}$	First PIOLA–KIRCHHOFF stress tensor
$T^M_{i\alpha}$	First PIOLA–KIRCHHOFF–MAXWELL stress tensor
$\boldsymbol{T}^p, T^p_{\alpha\beta}$	Second PIOLA–KIRCHHOFF stress tensor
$\boldsymbol{t}_i (i = 1, 2)$	Unit vector tangential to the i-th infinitely thin wire
t^e_{ij}	Extra CAUCHY stress tensor
$\boldsymbol{t}, t_{ij}$	CAUCHY stress tensor
$\boldsymbol{t}^M, t^M_{ij}$	CAUCHY–MAXWELL stress tensor
$\boldsymbol{t}^{(n)}, t^{(n)}_{ij}$	Stress vector, traction
$\mathbf{t}, \mathrm{t}_{i\alpha}$	Perturbed first PIOLA–KIRCHHOFF stress tensor
$\mathrm{t}_{\alpha\beta}\delta_{i\alpha}\mathrm{t}_{\alpha\beta}$	
$\left.\begin{matrix}\mathrm{t}^1_{\alpha\beta\gamma\delta}\\ \mathrm{t}^2_{\alpha\beta\gamma}, \mathrm{t}^3_{\alpha\beta\gamma}\\ \mathrm{t}^4_{\alpha\beta}\end{matrix}\right\}$	Coefficients arising in the constitutive relation of the perturbed first PIOLA–KIRCHHOFF stress tensor
U	Internal energy density
$\bar{\boldsymbol{U}}$	Particle displacement from the initial to the intermediate configuration
$\boldsymbol{u}$	Particle displacement from the intermediate to the final state

U	HELMHOLTZ free energy, internal energy
u	Elliptical coordinate
$\boldsymbol{V}, V_i$	Velocity, arising in the LORENTZ transformation
V	Transverse displacement in beam and plate bending
v	Elliptical coordinate
VM	Variational method
V_0	Scale for material velocity
V	Electric potential
v_1	Longitudinal velocity
v_2	Transverse velocity
W_N	Speed of propagation
w_n	Speed of displacement
W^M	MAXWELL modified internal energy density
W	Bending energy of beams or plates, part of the elastic energy independent of $\boldsymbol{B}$
$w^{(m)}(z) = u^{(m)}(z) + iv^{(m)}(z)$	Complex displacement
$\boldsymbol{x}$, x_i	Position vector
$\boldsymbol{X}, X_\alpha$	Particle position in reference configuration
$Z = (D_0^2 + \frac{\dot{\gamma}^2}{2})$	Regularized second stretching invariant in simple shear flow

Greek Symbols

$\alpha_1, \ldots, \alpha_6$	Scalar, parameters in the isotropic representation of the extra stress tensor
$\alpha_{AB}(A = 1, \ldots, 6; B = 1, 0)$	Scalar, parameters in the isotropic representation of the extra stress tensor
$\hat{\beta}_i(i = 1, 2)$	Parameters in the constitutive equation for the shear stress
β	Thickness to width ratio
β_1, β_2	Parameters arising in the CASSON-like constitutive function
$\beta_{\alpha\beta}$, β_{ij}	Coupling matrix of thermal and electrical conduction
γ	Entropy production density
$\dot{\gamma}$	Shear rate in simple shear flow
$\gamma = 0.5772$	EULER number
δ	Regularization parameter in the constitutive relation for stress
δJ	Second variation of the action integral L
δL	First variation of the action integral
$\delta_{\alpha\beta}, \delta_{ij}$	KRONECKER delta
ε	Effective permittivity
$\varepsilon = \pi a/(4l)$	Slenderness parameter