Fundamentals of METAL FORMING

Robert H. Wagoner
Jean-Loup Chenot

Fundamentals of METAL FORMING

Robert H. Wagoner
The Ohio State University

Jean-Loup Chenot
École Nationale Superieure des Mines de Paris

John Wiley & Sons, Inc.

New York Chichester Brisbane Toronto Singapore

ACQUISITIONS EDITOR	Cliff Robichaud
MARKETING MANAGER	Debra Riegert
PRODUCTION EDITOR	Ken Santor
MANUFACTURING COORDINATOR	Dorothy Sinclair
ILLUSTRATION COORDINATOR	Rosa Bryant

This book was set in 10/12 Palatino by John Wiley & Sons and printed and bound by Hamilton Printing Company. The cover was printed by New England Book Components, Inc.

Recognizing the importance of preserving what has been written, it is a policy of John Wiley & Sons, Inc. to have books of enduring value published in the United States printed on acid-free paper, and we exert our best efforts to that end.

The paper on this book was manufactured by a mill whose forest management programs include sustained yield harvesting of its timberlands. Sustained yield harvesting principles ensure that the number of trees cut each year does not exceed the amount of new growth.

Library of Congress Cataloging in Publication Data:

Wagoner, R. H. (Robert H.)
Fundamentals of metal forming/Robert H. Wagoner, Jean-Loup Chenot.
p. cm.
Includes index.
ISBN 0-471-57004-4 (cloth: alk. paper)
1. Metals—Mechanical properties. 2. Metal-work. I. Chenot, J. L. II. Title
TA460.W318 1997
671—dc20

95-47898
CIP

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Preface

We intend this book as preparation for senior undergraduate or first-year graduate students in the fundamental knowledge (mechanics, materials, basic numerical methods) needed to understand the analysis of metal-forming operations. It spans the considerable gap between traditional materials approaches (centering on material properties and structure) and purely mechanical ones (centering on force balances, and numerical methods). While seeking simplicity and comprehension, we do not avoid complex issues or mathematics (such as tensors) needed as preparation for further study leading to high-level modeling capability. In view of the progress in numerical methods and computation, these techniques are now so widespread as to be essential for many applications in the mechanics of materials.

The breadth of the intended audience (four distinct groups depending on educational discipline and rank) requires that various parts of the book be emphasized for a given course. For more than ten years, the first author has taught a senior undergraduate course in metal forming to metallurgical engineering students, in which certain parts of this book are used thoroughly while other parts are omitted. For example, such a course may consist of the following:

Senior-level course on Metal Forming

(For materials or mechanical engineering students)

Chapter 1	Complete
Chapter 2-4	Very briefly, little tensor manipulation
Chapter 7	von Mises and Hill's normal quadratic yield
Chapter 9	Complete
Chapter 10	Complete

The first author has also used the material in this book to teach a first-year graduate course to materials and mechanical engineering students on the subjects of elasticity and plasticity, where the emphasis has been different:

Graduate course on Elasticity and Plasticity

(For materials or mechanical engineering students)

Chapter 1	Second part, analysis of tensile test
Chapter 2-4	Covered moderately, very brief on large-strain measures
Chapter 5	Very brief, to introduce work methods (more for ME)
Chapter 6	Complete
Chapter 7	Complete
Chapter 8	Complete (less for ME)

Colleagues have used early versions of the material in this book to teach graduate courses on the subject of continuum mechanics to engineering mechanics students, with good results.

Graduate course in Continuum Mechanics
(For engineering mechanics or mechanical engineering students)

Chapter 2	Complete, brief review
Chapter 3	Complete
Chapter 4	Complete
Chapter 5	Complete
Chapter 6	Continuum aspects, omit crystal symmetry
Chapter 7	von Mises isotropic plasticity

The second author has taught the techniques and material in this book in a variety of settings, including: undergraduate courses in several French engineering schools, one-week training courses for engineers in industry, and graduate courses in the École des Mines and French universities.

The background of the students and the disciplines in which these courses were offered included continuum mechanics, plasticity and viscoplasticity, and general numerical methods and finite elements.

We are convinced that students learn by doing, not by reading, and thus have included numerous problems in each chapter, ranging in difficulty from using equations and derivations presented in the text (*Proficiency Problems*) to ones requiring creative and original thought (*Depth Problems*). Solutions for all of these problems are available in a Solutions Manual for this volume. We have added a third class of problems, *Numerical Problems*, requiring students to write and use their own computer programs. We believe that this approach, while frequently slow and painful, encourages greater understanding and mastery of the material.

This book is limited to a presentation of the fundamental mechanics, materials, and basic numerical concepts used in metal forming analysis. An advanced treatment that starts from this basis and develops the finite element method and illustrates its use in actual analysis, is nearing completion. Therefore, concepts particular to finite element analysis and complex forming application have been deferred to that volume.

Robert H.Wagoner, *Columbus, Ohio*
Jean-Loup Chenot, *Sophia-Antipolis, France*

February 1996

A note about notation

There are nearly as many kinds of notations for the various symbols representing mathematical quantities as there are authors in the field. We have attempted to follow a pattern that is intuitively consistent without striving for complete mathematical precision in each and every usage. In fact, no set of symbols, no matter how carefully defined, can replace understanding of the physical concepts and the underlying meaning of the equations and operations. We shall not attempt to cover every exception or eventuality in this note, but rather hope to provide a guide to our concept of the notation. In the first few chapters, we will provide examples of alternate notation while in later ones we will use the most convenient form.

We will not refer to curvilinear coordinates. Thus, all indices refer to physical axes or components and are placed in the subscript position. Also, vectors and tensors refer only to physically real quantities, rather than the generalized usage commonly found in the finite element community. Here are examples of the notation guidelines we follow.

SCALARS (a, A, α, t, T, a_1, a_{12}...)

Scalar quantities are physical quantities represented by a simple number, upper case or lower case and in any style (*italics*, Latin, Greek, etc.).

VECTORS (**a**, **b**, **c**...)

Physical vectors (as opposed to generalized 1-D arrays of numbers) will be represented by lower case bold letters*, with several subsidiary techniques for emphasizing the components rather than the entire vector quantity:

$$\mathbf{a} = a_1\hat{e}_1 + a_2\hat{e}_2 + a_3\hat{e}_3 = |\mathbf{a}|\hat{g} \leftrightarrow a_1, a_2, a_3 \leftrightarrow \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = [a_i] = [a] \leftrightarrow a_i$$

The vector **a** is shown as the sum of three scalar components multiplied by three base vectors ($\hat{\mathbf{e}}$). Since base vectors are the same as any other vector except for their intended use, any lower-case bold letter can represent them. We use $\hat{\mathbf{e}}$ specifically to emphasize the use of the vectors to represent other vectors (i.e. as base vectors). (In some cases, we will use $\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3$ to emphasize that the base vectors form a Cartesian set.) The superposed carat emphasizes that the vector is of unit length. The third representation emphasizes that the vector **a** can be decomposed into a magnitude $|\mathbf{a}|$ (or sometimes simply "a") and a direction represented by a unit vector $\hat{\mathbf{a}}$.

---

* We will on occasion use capital letters for vectors or their components to indicate some special quality or connection with another vector. An example is the expression of a given vector's components in an alternate coordinate system, or the same material vector at different times.

$$[A]^T \leftrightarrow \text{transpose of}[A]$$
$$[A]^{-1} \leftrightarrow \text{inverse of}[A]$$
$$|A| \leftrightarrow \text{determinant of}[A]$$
$$[A][B][C] \leftrightarrow \text{matrix multiplication}$$
$$[I] \leftrightarrow \text{identity matrix}$$

Each form can also include subscript indices to emphasize the components.

INDICIAL FORM (Aij, $B_{ij,}$ C_{ij}...)

The indicial forms of vectors, tensors, and matrices have been introduced above by the loose equivalence with other notations. Thus, for example, the multiplication of matrices [A] and [B] can be represented by $A_{ij}B_{jk} = C_{ik}$, which can be interpreted for any one component (the $i^{th}k^{th}$ one), or for all components (where i = 1,2,3; k = 1,2,3 for 3-D space, for example). However, the subscript j is a dummy index because we follow Einstein's summation convention where any repeated index is summed over to obtain the result. [i and k, conversely, are free indices because we are free to choose their values].

A few symbols are useful for representing matrix and tensor operations:

δ_{ij} = Kronecker delta, with the property that $\delta_{ij} = \begin{Bmatrix} 0 \text{ if } i \neq j \\ 1 \text{ if } i \neq j \end{Bmatrix}$

$$\varepsilon_{ijk} = \text{Permutation operator,}$$

with the property that $\varepsilon_{ijk} = \begin{Bmatrix} 0 & \text{if} & i & = j;\ i = k;\ j = k \\ 1 & \text{if} & ijk & = 1,2,3;\ 2,3,1;\ \text{or } 3,1,2 \\ -1 & \text{if} & ijk & = 3,2,1;\ 1,2,3;\ \text{or } 2,1,3 \end{Bmatrix}$

FUNCTIONS AND OPERATORS (L,F,K...)

We will denote functions and operators with script symbols without regard to the domain or range of the operation. These spaces will be clarified when the operator is first introduced. For example, $\mathbf{x} = \mathbf{L}(\mathbf{A})$ indicates that **L** is an operator which operates on a tensor and results in a vector. 2nd ranked tensors are often defined, for example, as linear vector operators [i.e. a linear operator that operates on a vector and has a vector result].

The symbol "$\leftrightarrow$" means that we will treat the other notations as equivalent even though they are not precisely equal. The right-hand side therefore emphasizes the components of **a** while ignoring the base vectors, which presumably have been defined elsewhere and are unchanged throughout the problem of interest. The brackets indicate an emphasis on all components, either in matrix or indicial form. (Note that the vector nature of **a** is not shown on the bracketed form, since the components need not apply to a physical vector.) The second "$\leftrightarrow$" indicates that a_i can be used to indicate a single component of **a** (the i^{th} one), or in the general sense, all of the components (i.e. for i=1,2,3 for 3-D space).

We will occasionally use a similar notation to denote numerical "vectors" such as those encountered in F.E.M. and other applications. In these cases, the range on the subscripts will be determined by the problem itself, rather than being limited to 3, as for usual 3-D space.

A few other vector notations are useful:

$\times$ indicates the vector cross product
. indicates the vector or tensor dot product

TENSORS OF RANK GREATER THAN 1 (**A**, **B**, **C**...)

Our tensor notation follows precisely the vector usage except we shall attempt to use upper case letters to represent tensors of rank greater than one. (Exceptions will apply to tensors which have almost ubiquitous usage, such as σ for the stress tensor or ε for the infinitesimal strain tensor.) Therefore, our tensor notation closely follows our vector notation:

$$\mathbf{A} \leftrightarrow \begin{bmatrix} A_{11}A_{12}A_{13} \\ A_{21}A_{22}A_{23} \\ A_{31}A_{32}A_{33} \end{bmatrix} = \left[A_{ij}\right] = [A] \leftrightarrow A_{ij}$$

MATRICES ([A], [a], [a],...)

Our matrix notation follows the bracketed forms introduced for physical vectors and tensors, without regard to whether the components represent physical quantities:

$$\begin{bmatrix} A_{11}A_{12}A_{13} \\ A_{21}A_{22}A_{23} \\ A_{31}A_{32}A_{33} \end{bmatrix} = \left[A_{ij}\right] = [A] \leftrightarrow A_{ij}$$

As before, the right-most form can be interpreted as a single component (the $i^{th}j^{th}$ one) or as all of the components (i = 1,2,3; j = 1,2,3 for general 3-D space). The bracketed forms emphasize all of the components, in the order shown by the first matrix.

We will tend to use lower case letters for column matrices, which correspond closely to vectors, but this may not always be possible while preserving clarity. Several other common usages for matrices:

Contents

4 Strain 118

5 Standard Mechanical Principles 157

6 Elasticity 189

7 Plasticity 219

CHAPTER 1

The Tensile Test and Basic Material Behavior

Fundamental and practical studies of metal mechanical behavior usually originate with the **uniaxial tension** test. Apparently simple and one-dimensional, a great deal of hidden information may be obtained by careful observation and measurements of the tensile test. On the other hand, the underlying physical complexity means that interpretation must be quite careful (along with the procedure followed to conduct the test) if meaningful results are to be realized.

In this chapter, we present a range of concepts at the simplest, most intuitive level. Many of these concepts will be expounded and presented in more mathematical form in subsequent chapters. This approach has the advantage of introducing basic mathematical and numerical ideas without the formalisms that are a source of confusion to many students when first encountered. However, the downside is that some of the introductions and treatments in this chapter are not precise and cannot be extended to more general situations without additional information.

Structurally, this chapter is divided into two parts: the first introduces the tensile test and the standard measures of material response. In the second section, we introduce all of the techniques necessary for analyzing a nonuniform tensile test. This analysis is not of great research interest because this 1-D form omits important aspects of the physical problem that can now be simulated using more powerful numerical methods. By omitting multi-axial complexity, however, we lose little of qualitative importance, and are able to illustrate the use of several numerical methods that will be needed in subsequent chapters, especially Chapter 10.

1.1 TENSILE-TEST GEOMETRY

Standard tensile test analysis is based on an ideal view of the physical problem. A long, thin rod is subjected to an extension (usually at a constant extension rate) and the corresponding load is measured.[1] The basic assumptions are that the loading is purely axial and the deformation takes place uniformly, both along the length of the specimen and throughout the cross-section. Under these conditions, it is sufficient to measure just two macroscopic quantities for much of the desired information: extension and load.

Two kinds of tensile specimens are used for standard tests: a round bar for bulk material (plates, beams, etc.), and a flat specimen for sheet products. Each is subject to ASTM specifications and has a nominal **gage length** of 2 inches. The gage length refers to the distance between ends of an extension gage put on the specimen to measure extension between these points. The **deforming length** is the length of the specimen that undergoes plastic deformation during the test. This length may change but should always be significantly longer than the gage length in order to ensure that the stress state is uniaxial and deformation is quasi-uniform over the gage length. Figure 1.1 shows the general geometry.

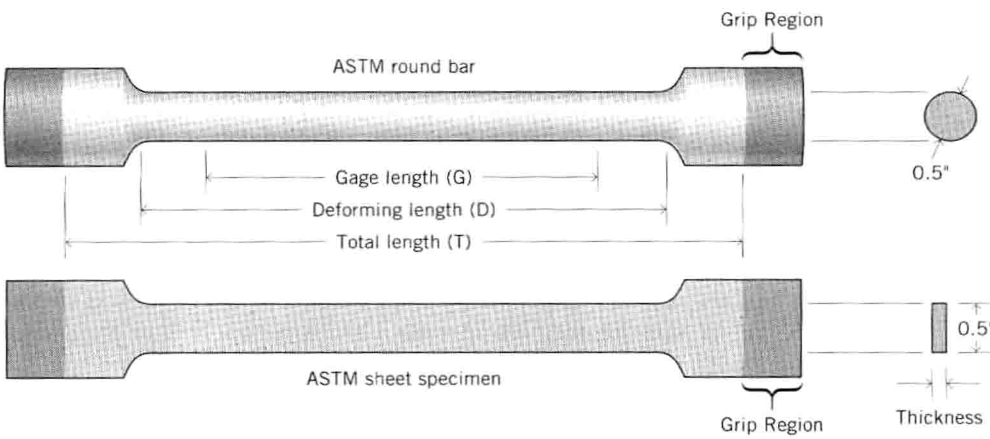

Figure 1.1 Standard tensile specimen shapes.

1.2 MEASURED VARIABLES

A standard tensile test is carried out by moving one end of the specimen (via a machine *crosshead*) at a constant speed, v, while holding the other end fixed. The primary variables recorded are **load** (P) and **extension** (Δl). Note that the extension could be obtained by multiplying v times t (time). This is done in some cases, but usually the "lash" (looseness) in the system necessitates use of an extension gage for accuracy. A typical load-extension curve appears in Figure 1.2.

[1] Variations on the basic method include imposing a certain load and measuring the extension or extension rate, or imposing jumps in rates or loads. These tests will not be considered in detail here, although they are very useful for materials research and for high-temperature deformation investigations.

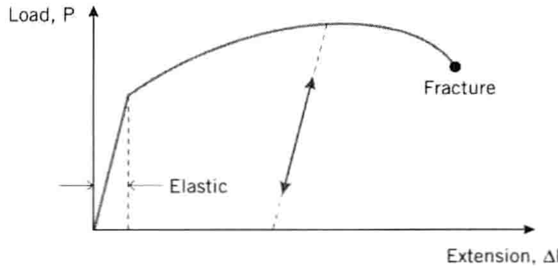

Figure 1.2 Typical response of a metal under uniaxial tension.

Note that the load-extension variables depend on specimen size. If, for example, the specimen were twice as large in each direction, the load would be four times as great, and the extension would be twice as great. Since we want to measure **material properties**, we normalize the measured variables to account for specimen size. The simplest way to do this is to normalize to the original specimen geometry.

1.3 ENGINEERING VARIABLES (NORMALIZED TO ORIGINAL SPECIMEN SIZE)

The variables may be defined as follows:

$\sigma_e =$ P/A_o = **engineering stress,**

$e =$ $\Delta l/l_o$ = **engineering strain** (sometimes called **elongation**)
(Note: l_o is usually equated to the gage length, L, Fig. 1.1.)

where the following notation is used:

$A_o =$ initial cross-sectional area

$l_o =$ G = initial gage length

$\Delta l =$ $l - l_o$ = change in gage length, extension

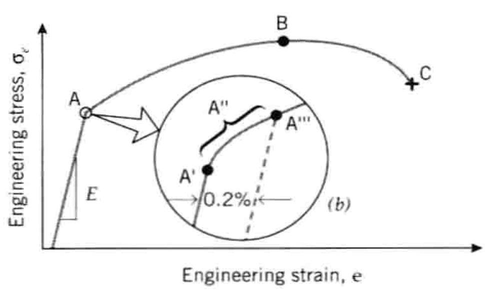

Figure 1.3 Points of special interest in a tensile test.

Engineering stress has units of force per area, and engineering strain is dimensionless (mm/mm, for example). Engineering strain is often presented as a percentage by multiplying by 100.

Other basic engineering quantities may be derived from the other ones as follows. The point at $A(e_y, \sigma_y)$ is of great interest and it may be used to define

E= **Young's modulus** = σ/e in the elastic region

σ_y = **yield point, yield stress, yield strength, elastic limit,** or **proportional limit.**

When finer resolution is available (Figure 1.3*b*), the apparent point *A* is seen as part of a smooth curve. This smoothness leads to ambiguity in defining *A*, so several other points are used:

$\sigma_{A'}$ = **proportional limit,** or **stress** at which the stress-strain curve ceases to be linear. (This is rather subjective and depends on the resolution and magnification of the curve.)

$\sigma_{A''}$ = **elastic limit** or **yield stress,** the minimum stress required to produce a permanent, plastic deformation. It can be found only by repeated loadings and unloadings. $\sigma_{A''}$ may be less than or greater than $\sigma_{A'}$ and $\sigma_{A'''}$.

$\sigma_{A'''}$ = **0.2% offset strength,** often called the **yield strength,** is obtained by drawing a line parallel to the elastic line but displaced by $\Delta e = 0.002$ (0.2%) and noting the intersection with the stress-strain curve.

The point $B(e_u, \sigma_{uts})$, at the maximum load sustainable by the specimen, defines

e_u = **uniform elongation** (elongation before necking begins)

σ_{uts} = **ultimate tensile strength**

The point C (e_t, σ_t) defines the limit of the tensile ductibilty or formability.

e_t = **total elongation**

e_{pu} = $(e_t - e_u)$ = **post-uniform elongation**

The **engineering-strain rate**, $\dot{e}$, is defined as de/dt, and is the rate at which strain increases. This quantity can be obtained simply by noting that all the strain takes place in the deforming length, D, so that the crosshead speed, v, is the same as the extension rate of D (See Fig. 1.1). That is,

$$\dot{e} = \frac{de}{dt} = \frac{dD/D_o}{dt} = \frac{dT/D_o}{dt} = \frac{v}{D_o} = \frac{\text{crosshead speed}}{\text{deforming length}} \tag{1.1}$$

The third equality is correct because the region outside of D is rigid; that is it does not deform, so that the velocity of all points outside of D is the same, and dD = dT (T = total length, Figure 1.1).

1.4 TRUE VARIABLES (NORMALIZED TO CURRENT CONFIGURATION)

Assume the original tensile test shown in Figure 1.4*a* is stopped at point X, and the specimen is unloaded to point Y. If the tensile test is then restarted, the dashed line will be followed approximately, and the specimen will behave as if no interruption occurred. If, instead, we remove the specimen and hand it to a new person to test, as in Figure 1.4*b*, the result will be quite different. The second person will measure the cross-sectional area and find a new number, A'_o, because the previous deformation reduced the width and thickness while increasing the length.

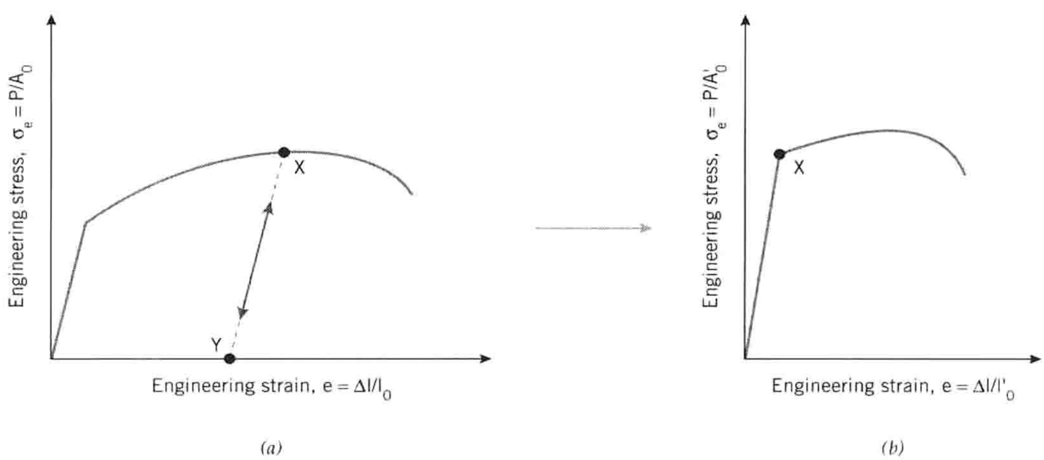

Figure 1.4 Interrupted tensile test

The same load will be required to deform the specimen, but the engineering stress will be different: $\sigma'_e = P/A'_o$. Obviously, if yield stress is to have a real material meaning, $\sigma_y \equiv \sigma'_y$, independent of who tests it. Similarly, a small extension at point X will produce different measured engineering strains for the same reason:

$$e_a = \frac{\Delta l}{l_o}, \quad e_b = \frac{\Delta l}{l'_o} \tag{1.2}$$

To take care of this problem, we introduce real or **"true" strain**, an increment of which refers to an infinitesimal extension per unit of *current* length. We limit ourselves to a small extension to insure that the current length is constant and well known. By assuming that the incremental strain over the current gage length is uniform, we can write mathematically that the true strain increment is $d\varepsilon = dl/l$ (not dl/l_o). We can express the total true strain as a simple integral:

$$\varepsilon = \int_{\varepsilon_t=o}^{\varepsilon_t} d\varepsilon = \int_{l_o}^{l} \frac{dl}{l} \Rightarrow \varepsilon = \ln\frac{l}{l_o} \tag{1.3}$$

Similarly, the real or true stress refers to the load divided by the *current* cross-sectional area:

$$\sigma_t = \frac{P}{A} \quad \left(\text{not } \frac{P}{A_o}\right) \tag{1.4}$$

Exactly analogous to the definition of engineering-strain rate, the **true-strain rate** is defined as $d\varepsilon/dt$. As in Section 1.3, this rate is simply related to the crosshead speed:

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{dD/D}{dt} = \frac{dT/D}{dt} = \frac{v}{D} = \frac{\text{crosshead speed}}{\text{current deforming length}} \tag{1.5}$$

1.5 RELATIONSHIP AMONG TRUE AND ENGINEERING VARIABLES

Definitions	Engineering	True	
Strain	$e = \dfrac{l_f - l_o}{l_o}$	$\varepsilon = \ln l_f/l_o$	(1.6)
Stress	$\sigma_e = P/A_o$	$\sigma_t = P/A$	(1.7)

Equation 1.6 yields the relationship between the two strain representations with only simple manipulation:

$$e = \exp(\varepsilon) - 1 \qquad \varepsilon = \ln(1 + e) \tag{1.8}$$

Equation 1.7 cannot be solved simultaneously until a relationship between the original and current cross-sectional area (A_o, A) is known. A