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THE THEORY OF GROUP CHARACTERS AND MATRIX REPRESENTATIONS OF GROUPS

BY

DUDLEY E. LITTLEWOOD

PROFESSOR OF MATHEMATICS IN THE
UNIVERSITY COLLEGE OF NORTH WALES, BANGOR

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PREFACE

SINCE the discovery of group characters by Frobenius at the end of the last century, the development of the theory has been so spectacular, and the theory has shown such powerful contacts with other branches of mathematics, both pure and applied, that the inadequacy of its treatment by text-books is rather surprising. Indeed, until the publication last year of Murnaghan's treatise, *The Theory of Group Representations*, there was no book which devoted itself especially to the theory, and even Murnaghan's work was written specifically with a view to its applications to quantum theory and nuclear physics.

It has been my purpose in writing this book to give a simple and self-contained exposition of the theory in relation to both finite and continuous groups, and to develop some of its contacts with other branches of pure mathematics, such as invariant theory, group theory, and the theory of symmetric functions. There are three introductory chapters on matrices, algebras, and groups, so that no specialized knowledge is required of the reader beyond that obtained in an ordinary degree course in mathematics. Rather than to attempt any exhaustive treatment, it has been my aim to develop the nucleus of a theory which will bring to notice new problems to be solved.

The bibliography gives most of the original memoirs which have gone towards the development of the theory, together with text-books and authoritative references to relevant theories. I must express my debt to Murnaghan's book (no. 9 in the bibliography) and to two books by Weyl (nos. 13 and 14) in compiling this bibliography. Murnaghan's book also suggested certain additions to the last chapter.

I have to thank Prof. A. R. Richardson for his suggestions and comments concerning the writing of this book, Dr. A. J. Ward and Dr. A. C. Aitken for invaluable help in reading the proofs, and Mr. H. O. Foulkes for some corrections to the tables of characters.

D. E. L.

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p. 23. **The regular matrix representation**

This representation will not be *simply* isomorphic if there exists an element x of the algebra for which $ax = 0$ for all a of the algebra. The corresponding matrix X would be identically zero. A simply isomorphic representation, however, may be obtained in any case by adjoining a modulus to the algebra before obtaining the regular representation.

I

MATRICES

1.1. Linear transformations

CONSIDER the set of n linear equations

$$\begin{aligned} x_1' &= a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n, \\ x_2' &= a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n, \\ &\cdots\cdots\cdots\cdots\cdots \\ x_n' &= a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n, \end{aligned}$$

which may be written more concisely

$$x_p' = \sum_q a_{pq}x_q. \tag{1.1;1}$$

The coefficients a_{pq}, for the purposes of this book, are taken to be any complex numbers. More generally, they may be taken from any prescribed field, but the theory is simplified in the case of the complex numbers, owing to the fact that the nth degree equation in one variable has exactly n roots in this field; and this case is sufficient for the purposes of this book. For the more general theory the reader is referred to Turnbull and Aitken, *The Theory of Canonical Matrices* (London and Glasgow, 1932).

The equations (1.1; 1) are said to form a *linear transformation* in n variables. They may be regarded as effecting a mapping of an n-dimensional space into itself, i.e., relative to any assigned Cartesian coordinate system, the point $P' \equiv (x_1', x_2',\ldots, x_n')$ is made to correspond to the point $P \equiv (x_1, x_2,\ldots, x_n)$. The point P' is uniquely defined by P. This n-dimensional space is called the *carrier space*.

If the determinant of the coefficients is not zero, the equations (1.1; 1) may be solved for the x_r's in the form†

$$x_q = \sum a_{qp}' x_p' \tag{1.1;2}$$

and the point P is also uniquely defined by the point P'. Such a transformation is called *non-singular*; it effects a *bi-uniform mapping*. The transformation (1.1; 2) is called the *inverse transformation* of (1.1; 1).

† *Vide* any standard text-book on algebra, e.g. Chrystal.

1.2. Matrices

The transformation is completely defined by the n^2 quantities a_{pq}. We therefore associate with the transformation the array of numbers

$$\begin{bmatrix} a_{11}, & a_{12}, & \ldots, & a_{1n} \\ a_{21}, & a_{22}, & \ldots, & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1}, & a_{n2}, & \ldots, & a_{nn} \end{bmatrix}$$

This array is called a *matrix*, the *matrix of the transformation*. We write it shortly $[a_{st}]$, where s is the index of the row, and t the index of the column from which the given element is chosen. The letters s and t will in general be reserved throughout this book to indicate the row and the column of a matrix from which a typical element is chosen.

Two matrices are said to be equal if and only if they are identical, i.e.

$$[a_{st}] = [b_{st}]$$

if and only if $a_{pq} = b_{pq}$ for all p and q.

Now let $x''_p = \sum b_{pq} x'_q$ be a second transformation with matrix of coefficients $[b_{st}]$. The effect of taking the two transformations consecutively in order of definition is clearly to produce a third transformation,

$$x''_p = \sum c_{pq} x_q,$$

where

$$c_{pq} = \sum b_{pr} a_{rq}.$$

The matrix of this transformation is $[c_{st}]$.

We therefore define the *product* of two matrices by the rule

$$[b_{st}][a_{st}] = [\sum b_{sr} a_{rt}] = [c_{st}]. \tag{1.2;1}$$

Thus if two transformations are taken consecutively, the matrix of the combined transformation is the product of the matrices.

From the usual rule for the product of two determinants† we see that:

The determinant of the product of two matrices is equal to the product of the determinants.

It should be noted that the product $[a_{st}][b_{st}] = [\sum a_{sr} b_{rt}]$ is not in general equal to $[b_{st}][a_{st}]$. Multiplication is *not commutative*. Multiplication is, however, associative, and it is easily verified that

$$\{[a_{st}][b_{st}]\}[c_{st}] = [a_{st}]\{[b_{st}][c_{st}]\}.$$

The matrix with unity in each position in the leading diagonal, and zero elsewhere, i.e. the matrix $[\delta_{st}]$, where $\delta_{ij} = 0$ $(i \neq j)$ and $\delta_{ii} = 1$, which corresponds to the identical transformation, is called

† See Chrystal.

the unit matrix and will be denoted by I. If it is desired to convey the order, n^2, of the matrices, it will be written I_n. Clearly

$$[a_{st}]I = I[a_{st}] = [a_{st}].$$

If the determinant of $[a_{st}]$, which we shall denote by $|a_{st}|$, is not equal to zero, then, as we have stated, there exists an inverse transformation (1.1; 2), and hence a matrix $[a'_{st}]$, such that

$$[a_{st}][a'_{st}] = [a'_{st}][a_{st}] = I.$$

$[a'_{st}]$ is called the reciprocal of the matrix $[a_{st}]$, and is denoted by $[a_{st}]^{-1}$. In this case the matrix $[a_{st}]$ is said to be *non-singular.*

If, however, $|a_{st}| = 0$, then there is no reciprocal of $[a_{st}]$ and the matrix is said to be *singular.*

In addition to multiplication we define *addition* for matrices by the rule

$$[a_{st}]+[b_{st}] = [a_{st}+b_{st}].$$

Clearly addition is commutative and associative, and multiplication is distributive with respect to addition.

Permutation matrix

A matrix in which each row and each column has but one non-zero element, which is equal to unity, is called a *permutation matrix.* Transformation (see §1.3) by a permutation matrix has the effect of permuting the order of the rows of a matrix, and the columns also, in the same manner.

Diagonal matrices

If in a matrix $A = [a_{st}]$ we have

$$a_{ij} = 0 \quad (i \neq j),$$

the matrix is called a diagonal matrix. It is completely defined by the elements in the leading diagonal, namely $a_{11}, a_{22},..., a_{nn}$, and we shall use the concise notation

$$A = \text{diag}(a_{11}, a_{22},..., a_{nn}).$$

The notation may be made even more concise, in the case where a given element is repeated in consecutive positions, by the use of indices to indicate repetitions. Thus

$$\text{diag}([1]^3, [2]^2, [3]) = \begin{bmatrix} 1, & & & & & \\ & 1, & 0 & & & \\ & & 1, & & & \\ & & & 2, & & \\ & 0 & & & 2, & \\ & & & & & 3 \end{bmatrix}$$

The notation $\operatorname{diag}(A_1, A_2, \ldots, A_r)$ will also be used when the A_i represent square matrices, this signifying that the matrices A_i are placed in symmetric positions about the leading diagonal.

1.3. The transform of a matrix

Consider the effect of a change of the coordinate system in the carrier space on a transformation

$$x'_p = \sum a_{pq} x_q.$$

Let $y_1, y_2, \ldots, y_n$ be the coordinates of a point relative to the new system, with

$$x_p = \sum c_{pq} y_q,$$

the matrix $[c_{st}]$ being non-singular.

If $y'_1, \ldots, y'_n$ are the new coordinates of the transformed point, then

$$x'_p = \sum c_{pq} y'_q.$$

Now let the original point transformation referred to the new coordinate system take the form

$$y'_p = \sum b_{pq} y_q.$$

The coefficients b_{pq} may be found as follows:

$$x'_p = \sum a_{pq} x_q = \sum_{q,r} a_{pq} c_{qr} y_r,$$

$$x'_p = \sum c_{pq} y'_q = \sum_{q,r} c_{pq} b_{qr} y_r.$$

Thus

$$\sum_q a_{pq} c_{qr} = \sum_q c_{pq} b_{qr},$$

$$[a_{st}][c_{st}] = [c_{st}][b_{st}],$$

i.e.

$$[b_{st}] = [c_{st}]^{-1}[a_{st}][c_{st}]. \tag{1.3; 1}$$

The matrix $[b_{st}]$ is called the *transform of the matrix* $[a_{st}]$ *by the matrix* $[c_{st}]$.

Clearly $[a_{st}]$ is the transform of $[b_{st}]$ by the matrix $[c_{st}]^{-1}$. Matrices which are transforms of one another are called *equivalent* matrices. Since they may be regarded as corresponding to the same point transformation, but referred to different coordinate systems, equivalent matrices have many properties in common. In fact a very powerful method of finding the properties of a matrix is to find an equivalent matrix of simpler form, e.g. diagonal, and to find the properties of this second matrix. We shall pursue this method in the section on the 'classical canonical form'.

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1.4. Rectangular matrices and vectors

In a set of linear equations

$$x'_p = \sum a_{pq} x_q,$$

the number m, of new variables x'_p, may differ from the number n, of original variables x_q. Correspondingly we have a *rectangular matrix* $[a_{st}]$ with m rows and n columns. The product of two rectangular matrices $[a_{st}]$, $[b_{st}]$ may be obtained according to the same rule (1.2; 1) as for square matrices, but it is necessary that the number of rows in $[a_{st}]$ should be equal to the number of columns in $[b_{st}]$. It may be convenient to make up the number of rows or columns by adding zeros.

Of particular importance are matrices with one row, or one column. These are called *vectors*. The coordinates of a point in n-space may be formed into a vector $[x_s]$, and the transformation (1.1; 1) may then be expressed by matrix multiplication

$$[x'_s] = [a_{st}][x_s],$$

a form which lends itself to great neatness of expression.

A set of r vectors $X_1 = [x_{1s}]$, $X_2 = [x_{2s}]$,..., $X_r = [x_{rs}]$ are said to be *linearly dependent* if scalars, i.e. ordinary complex numbers, $\alpha_1, \alpha_2,..., \alpha_r$ can be found such that

$$\alpha_1 X_1 + \alpha_2 X_2 + ... + \alpha_r X_r = 0.$$

Otherwise they are *linearly independent*. It is clearly impossible to find a set of more than n linearly independent vectors of order n.

A square matrix $[a_{st}]$ of order n^2 may be regarded as composed of n column vectors

$$[a_{st}] = [A_t],$$

where

$$A_p = [a_{sp}].$$

If $|a_{st}| = 0$, the vectors A_p are linearly dependent, since a non-zero solution of

$$\sum a_{1p}\alpha_p = 0,$$
$$\sum a_{2p}\alpha_p = 0,$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$\sum a_{np}\alpha_p = 0$$

may be found, treated as equations in $\alpha_1, \alpha_2,..., \alpha_n$. This condition may be expressed: there is a vector $[\alpha_t]$ such that $[\alpha_t][a_{st}] = 0$.

If, further, the n vectors $[a_{sp}]$ are linearly dependent upon j of the vectors, then there will be $n-j$ linearly independent vectors $[\alpha_{qt}]$ $(1 \leqslant q \leqslant n-j)$ such that

$$[\alpha_{qt}][a_{st}] = 0,$$

and the matrix $[a_{st}]$ is said to be of *rank* j.

The condition for this is clearly that all $(j+1)$-rowed minors of the matrix $[a_{st}]$ have zero determinant, and hence is the same as the condition that the row-vectors $[a_{qt}]$ should be linearly dependent upon j of these. In this case we have also that $(n-j)$ linearly independent row-vectors $[\beta_{sp}]$ can be found such that

$$[a_{st}][\beta_{sp}] = 0 \quad (1 \leqslant p \leqslant n-j).$$

There is no difficulty in proving that the rank of a transform of a matrix is the same as the rank of the matrix. More generally, if X and Y are non-singular matrices, A and XAY have the same rank, for if $[\beta_{st}]$ is a rectangular matrix with $(n-j)$ columns and

$$Y^{-1}[\beta_{st}] = [\beta'_{st}],$$

then if

$$A[\beta_{st}] = 0,$$

also

$$XAY[\beta'_{st}] = 0.$$

1.5. The characteristic equation of a matrix

Let $A = [a_{st}]$ be any square matrix of order n^2, and I the unit matrix of the same order. If λ is a variable scalar, the matrix $[\lambda I - A]$ is singular for certain fixed values of λ called the *characteristic roots* of the matrix A. These values may be found by equating to zero the determinant

$$|\lambda I - A| = 0.$$

We obtain the *characteristic equation*

$$\lambda^n - a_1\lambda^{n-1} + a_2\lambda^{n-2} - \ldots + (-1)^n a_n = 0,$$

a_r being the sum of the determinants of the r-rowed principal (coaxial) minors, i.e. the minors in which the indices of the rows are the same as those of the columns, and a_n being the determinant of the matrix A itself.

Hence there are exactly n characteristic roots of a matrix of order n^2, some of which may be repeated.

The characteristic equation of a matrix has the important property that it is invariant for transformations of the matrix.

Equivalent matrices have the same characteristic equations.

The proof is quite simple, for since

$$[\lambda I - T^{-1}ST] = [T^{-1}][\lambda I - S][T],$$

we have also

$$|\lambda I - T^{-1}ST| = |T^{-1}||\lambda I - S||T|$$
$$= |\lambda I - S|.$$

1.6. The classical canonical form of a matrix

(a) All characteristic roots distinct

Of all the transforms of a given matrix, certain ones may be chosen as especially simple in form. These are the *canonical forms*. Of these the most important, and the only one we shall consider here, is the *classical canonical form*, which we now proceed to obtain. We treat first of the simpler case when all the characteristic roots of the matrix are distinct.

Let $A = [a_{st}]$ be a matrix of order n^2 with n distinct characteristic roots $\lambda_1, \lambda_2, \ldots, \lambda_n$.

Since $[\lambda_r I - A]$ is a singular matrix, we can find a vector $[b_{sr}]$ such that

$$[\lambda_r I - A][b_{sr}] = 0.$$

Hence

$$A[b_{sr}] = [b_{sr}]\lambda_r. \tag{1.6;1}$$

Further, the n vectors $[b_{sr}]$ are linearly independent, for since

$$(A - \lambda_r I)[b_{sr}] = 0,$$

if

$$\sum \alpha_r[b_{sr}] = 0,$$

multiplying on the left by $(A-\lambda_2 I)(A-\lambda_3 I)\ldots(A-\lambda_n I)$ we should obtain

$$\alpha_1[b_{s1}](\lambda_1-\lambda_2)(\lambda_1-\lambda_3)\ldots(\lambda_1-\lambda_n) = 0.$$

Hence $\alpha_1 = 0$, and similarly $\alpha_2 = \alpha_3 = \ldots = \alpha_n = 0$.

Thus, since the vectors $[b_{sr}]$ are linearly independent, the matrix $[b_{st}]$ is non-singular. Combining all the equations (1.6; 1) in matrix form we obtain

$$A[b_{st}] = [b_{st}][\lambda_s \delta_{st}],$$

where $\delta_{ij} = 0$ $(i \neq j)$, and $\delta_{ii} = 1$. $[\lambda_s \delta_{st}]$ is a diagonal matrix.

But since $[b_{st}]$ is non-singular we may put

$$[b_{st}]^{-1}A[b_{st}] = [\lambda_s \delta_{st}]$$
$$= \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n).$$

I. *Any matrix of which all the characteristic roots are distinct may be transformed into a diagonal matrix in which the diagonal elements are the characteristic roots of the matrix.*

This is the classical canonical form of the matrix for this case. It is clearly unique, save for the order in which the characteristic roots are placed in the leading diagonal.

From the existence of this form we deduce an important theorem namely

Every matrix satisfies its own characteristic equation.

Firstly, the product of two diagonal matrices is obtain by multiplying corresponding terms, e.g.

$$\text{diag}(\lambda_1, \lambda_2, ..., \lambda_n)\text{diag}(\mu_1, \mu_2, ..., \mu_n) = \text{diag}(\lambda_1 \mu_1, \lambda_2 \mu_2, ..., \lambda_n \mu_n).$$

It follows that if $A = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_n)$,

then $[A-\lambda_1 I][A-\lambda_2 I]...[A-\lambda_n I] = 0,$

i.e. $A^n - a_1 A^{n-1} + a_2 A^{n-2} - ... + (-1)^n a_n I = 0,$

and a diagonal matrix must satisfy its characteristic equation.

Secondly, since equivalent matrices have the same characteristic equation, and since also they must satisfy the same equation, for

$$(T^{-1}AT)^n = T^{-1}A^nT,$$

it follows that every matrix which can be transformed into a diagonal form, e.g. every matrix with distinct characteristic roots, must satisfy its characteristic equation.

Lastly, if A is any matrix whatsoever, we can find a matrix Z such that $A+\mu Z$ has all its characteristic roots distinct and satisfies its characteristic equation. We now take the limit as μ tends to zero, whence A satisfies its characteristic equation.

1.7. The classical canonical form of a matrix

(b) Multiple characteristic roots

Let $A = [a_{st}]$ be a matrix of order n^2 for which the characteristic root λ_1 is repeated r_1 times. Then a vector $[\alpha_{s1}]$ can be found such that

$$[a_{st}][\alpha_{s1}] = [\alpha_{s1}]\lambda_1.$$

If $[\alpha_{st}]$ is any non-singular matrix of which the first column is $[\alpha_{s1}]$, then the first column of $[a_{st}][\alpha_{st}]$ is $[\alpha_{s1}]\lambda_1$. Hence the first column of $[\alpha_{st}]^{-1}[a_{st}][\alpha_{st}]$ consists of λ_1 followed by $(n-1)$ zeros.

Let $$B = [\alpha_{st}]^{-1}[a_{st}][\alpha_{st}] = \left[\begin{array}{c|c} \lambda_1 & b_t \\ \hline 0 & b_{st} \end{array}\right],$$

in which s and t run from 2 to n.

Now B has the characteristic root λ_1 repeated r_1 times, since it is a transform of A, and hence $[b_{st}]$ has the same root repeated (r_1-1) times. Hence a matrix $[\beta_{st}]$ $(2 \leqslant s,t \leqslant n)$ can be found such that $[\beta_{st}]^{-1}[b_{st}][\beta_{st}]$ is of the form

$$\left[\begin{array}{c|c} \lambda_1 & c_t \\ \hline 0 & c_{st} \end{array}\right].$$

Thus the matrix $\left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & \beta_{st} \end{array}\right]$ transforms B into

$$\left[\begin{array}{c|c|c} \lambda_1 & \multicolumn{2}{c}{\alpha_t} \\ \hline 0 & \lambda_1 & c_t \\ \hline \multicolumn{2}{c|}{0} & c_{st} \end{array}\right].$$

Proceeding thus, a matrix K can be found such that

$$K^{-1}AK = \begin{bmatrix} \lambda_1, & \gamma_{12}, & \gamma_{13}, & \cdots, & \gamma_{1n} \\ & \lambda_1, & \gamma_{23}, & \cdots, & \gamma_{2n} \\ & & \ddots\lambda_1, & \cdots, & \gamma_{r_1 n} \\ & 0 & & & \\ & & & g_{st} & \end{bmatrix}.$$

Let K_1 be the rectangular matrix which consists of the first r_1 columns of K. Then

$$AK_1 = K_1 \begin{bmatrix} \lambda_1, & \gamma_{12}, & \gamma_{13}, & \cdots, & \gamma_{1r_1} \\ & \lambda_1, & \gamma_{23}, & \cdots, & \gamma_{2r_1} \\ & & \ddots & & \\ & 0 & & \lambda_1, & \gamma_{r_1-1,r_1} \\ & & & & \lambda_1 \end{bmatrix} = K_1 A_1.$$

Clearly $A_1-\lambda_1 I_{r_1}$ is a matrix with zeros on and below the leading diagonal, and

$$[A_1-\lambda_1 I_{r_1}]^{r_1} = 0.$$

Hence $$[A-\lambda_1 I_n]^{r_1}K_1 = K_1[A_1-\lambda_1 I_{r_1}]^{r_1} = 0.$$

Since K is a non-singular matrix the r_1 column vectors of K_1 are linearly independent.

Similarly, corresponding to each characteristic root λ_i of A repeated r_i times, we can find a rectangular matrix K_i with n rows and r_i columns such that

$$AK_i = K_i A_i,$$

where

$$A_i = \begin{bmatrix} \lambda_i & \alpha_{12}^{(i)}, & \alpha_{13}^{(i)}, & \ldots, & \alpha_{1r_i}^{(i)} \\ & \lambda_i, & \alpha_{23}^{(i)}, & \ldots & \\ & & \lambda_i, & \ldots & \\ & 0 & & \ddots & \\ & & & & \lambda_i \end{bmatrix} \qquad (1.7;1)$$

Let $T = [\tau_{st}]$ be the square matrix obtained by putting together all the rectangular matrices K_i. There will be n columns since $\sum r_i$ is the total number of characteristic roots, namely n. Also T is non-singular; for if

$$\sum_{r=1}^{n} \alpha_r[\tau_{sr}] = 0,$$

by multiplying on the left by $[A-\lambda_2 I_n]^{r_2}[A-\lambda_3 I_n]^{r_3}\ldots$, we obtain

$$\sum_{r=1}^{r_1} \alpha_r[\tau_{sr}] = 0,$$

and we should have all these α_r's zero, since these vectors are the linearly independent column vectors of K_1. Similarly, all the α_r's are zero, and T is non-singular.

We have

$$AT = T\begin{bmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & A_3 & \\ 0 & & & \ddots \end{bmatrix},$$

whence $T^{-1}AT$ is a matrix

$$T^{-1}AT = D = \begin{bmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & A_3 & \\ 0 & & & \ddots \end{bmatrix} \qquad (1\ 7;2)$$

$$= \operatorname{diag}(A_1, A_2, \ldots, A_r)$$

in which matrices of the type A_i (1.7;1) occur in positions about the leading diagonal, there being zeros above and below these

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