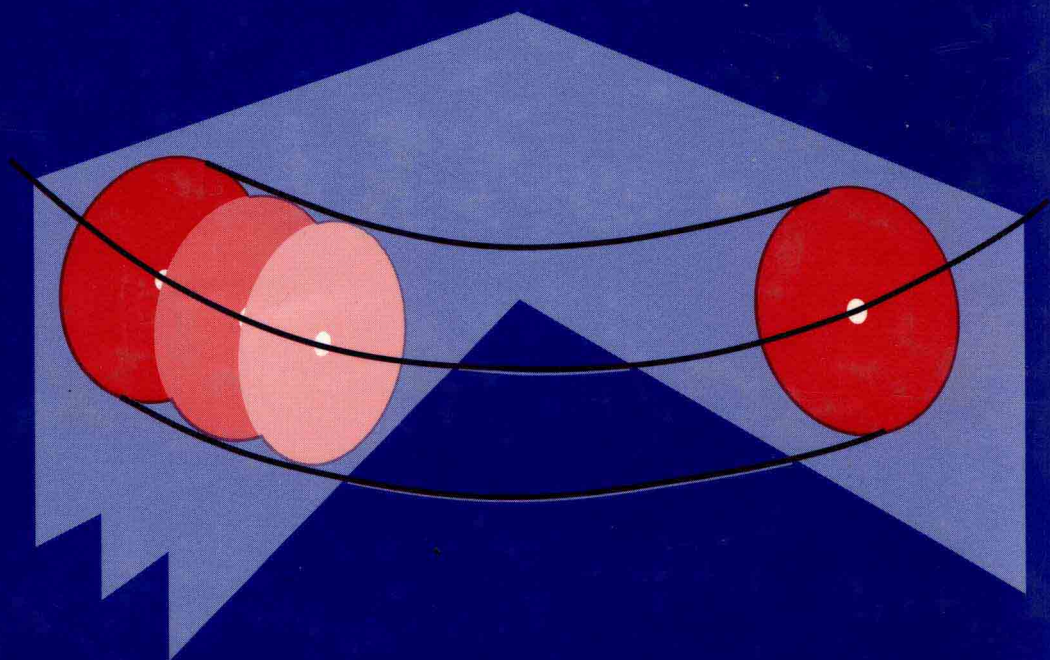


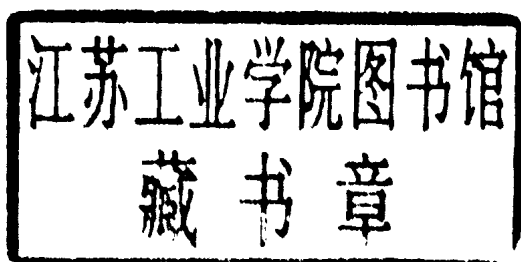
# CAD/CAE Descriptive Geometry

Daniel L. Ryan



# **CAD/CAE Descriptive Geometry**

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# Preface

Modern descriptive geometry is a mixture of plane geometry (mathematics), orthographic projection (technical drawing), and high-speed communication methods (digital computing). For readers who are unfamiliar with parts of this mixture, this book will introduce the fundamentals of each and the application of these processes. For those readers with some background, this book will appear slow paced and very elementary; the topics covered in this book are for those who want a sound foundation in fundamentals.

Constructive geometry, as we use it today, evolved from the transition between the industrial and the electronic revolutions. The process began before the industrial revolution and followed progress into the present electronic age. Therefore, constructive geometry is part of a design language and is used for one or more of the following purposes: (1) study, (2) construction, (3) maintenance, or (4) creation of an industrial product or project. When constructive geometry is used inside a digital computer, it becomes part of computer-aided design (*CAD*). We use *CAD* to produce industrial drawings of all types and descriptions. In order to produce these drawings, we must understand where modern descriptive geometry came from, what it is now, and where it might lead us in the future.

Constructive geometry within *CAD* is commonplace in American industry today. Because *CAD* is taken for granted in the industrial marketplace and used whenever necessary, it is often overlooked in the college curriculum. The most visual part of *CAD* is computer graphics, which is covered in most curricula. No attempt has been made here to present computer graphics as a subject; computer graphics are used throughout the book as the tool for presentation of the constructive geometry used in *CAD*.

The selection of material in this textbook was based on the premise that the reader has access to a personal computer or PS/2 compatible *CAD* workstation running Autodesk, Inc., software. Therefore, each chapter presents a basic geometry topic and covers it in detail using the *CAD* workstation. It is the author's belief that this combination is necessary in an engineering or technology course because *CAD* requires ingenuity, inventiveness, imagination, and patience. An ability to use constructive geometry in design solutions can be practiced after exposure to the chapter materials.

# **CAD/CAE Descriptive Geometry**

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# 1 Descriptive Geometry Conventions

Modern descriptive geometry is a mixture of plane geometry (mathematics), third quadrant orthographic projection (engineering drawing), and high-speed communication methods (digital computing). For readers who are unfamiliar with parts of this mixture, this book will introduce the fundamentals of each and the application of these constructive geometry processes. To those readers with some background, this book will appear slow paced. The topics covered in this book are for those who want a sound foundation in fundamentals.

Constructive geometry, as we use it today, evolved from the transition into the electronic revolution. This chapter begins before the electronic revolution and follows the progress into the present electronic age. Therefore, constructive geometry is part of a design language and is used for one or more of the several purposes of (1) study, (2) construction, (3) maintenance, or (4) creation of an industrial product or project (computer-aided engineering [CAE]). When descriptive geometry is used inside a digital computer, it becomes part of computer-aided design (CAD). We use CAD to produce industrial drawings of all types and descriptions. In order to produce these drawings we must understand where modern descriptive geometry came from, what it is now, and where it might lead us in the future.

## 1.1 DESCRIPTIVE GEOMETRY HISTORY

One of the earliest evidences of descriptive geometry is found in the book on architecture by Marcus Vitruvius, who lived at the time of Augustus and was appointed superintendent of military designs for the Roman army. In his book, Marcus Vitruvius dealt with the concept of horizontal and vertical planes of construction of public buildings. This was the first recorded orthographic projection concept.

The first known mathematical principle of descriptive geometry appears in Albrecht Dürer's book *Geometry and Perspective*, published in Nuremberg in 1525. The first recorded three-view, first quadrant drawing appears in this work. It would be 200 years (1738) before another army officer Francois Frezier would publish a treatise on stereotomy for stonecutting. But this almost forgotten treatise was read by a brilliant young French mathematician Gaspard Monge. Monge was employed at the military school at Mezieres in 1768. He designed the fortifications used by the French army. While working on the many computations required, he discovered a geometrical solution for defilement. The ordinary mathematical procedures for the same solutions were very laborious. When Monge first presented his procedures in which he associated precise and mathematical elements to several common-scale

elevation and plan drawings, the French army was skeptical of these simple and direct methods. After testing, his methods became a French military secret, and Monge was prohibited from revealing them until he lectured at the Ecole Polytechnique in 1795. His book on descriptive geometry was published in 1801.

One of Monge’s students at the Polytechnique, Claude Crozet, was employed in 1816 to be an instructor of descriptive geometry at West Point. In 1821 the first United States-based information on descriptive geometry was published, and in 1826 Charles Davies, who succeeded Crozet, published the first text book on descriptive geometry. Modernizations of this first text were published in 1860 and 1865 by Professors Warren and Church and were used until 1910 when Henry Miller of the University of Michigan introduced a series of editions lasting until 1945. Miller’s work was the last textbook effort to preserve the Monge abstract principles. The texts appearing after 1945 introduced more direct methods for solving abstract geometrical relationships. Noteworthy among the authors of such texts were Street, Grant, Pare, Loving, Hill, Hood, Palerlee, Springer, Hoelscher, plus a score of present-day authors.

1.2 DEFINITIONS

The process of learning modern descriptive geometry can be mastered in three steps:

- 1. *Comprehension*: The subject matter is in the form of printed words, diagrams, observations of demonstrated skills, and CAD models.
- 2. *Clarification*: The learner carefully reads and considers step by step what is perceived through the comprehension/presentation (step 1).
- 3. *Retention*: Further study, the solution of practice problems, and participation in evaluation methods determine the amount of clarification (step 2) that has taken place.

In the above three steps there will be a need for a system of written and graphical symbols. These symbols appear throughout this textbook and will appear at the CAD workstation used for the practice problems. It is important, therefore, that each reader make this chapter a reference dictionary to be used throughout the remainder of the text. Do not attempt to memorize the material presented in this chapter; instead you should refer back to it as needed.

1.3 ABBREVIATIONS

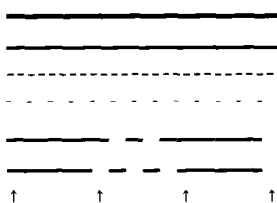
In a short time all abbreviations will have been memorized from constant use; for now refer back to them as often as needed.

Written Symbol	Graphical Symbol
AP = apex	$\Lambda$
AU = auxiliary	$\alpha$
AX = axis (major: AX, minor: ax)	A
BA = base	B
BL = brilliant line	$\beta$
BP = brilliant point	$\gamma$
CN = cone	C

Written Symbol	Graphical Symbol
CV = convolute	$\psi$
CP = coordinate plane	$\rho$
CO = coplanar	l
CY = cylinder	o
DG = diagonals	/
DL = direct light	L
EL = element	E
EV = edge view	$\sigma$
FP = frontal plane (reference frontal: F)	F
GX = generatrix	G
GL = ground line	-
HP = horizontal plane (reference horizontal: H)	P
IL = indirect light	I
IN = intersection	X
LS = line of shadow	S
NL = normal	N
OB = oblique	$\diamond$
OP = orthographic projection	=
PA = parallel (parallelism)	//
PD = perpendicular (perpendicularity)	$\perp$
PL = plane	■
PP = profile plane (reference profile: P)	□
PR = projection	●
PT = point	↔
RY = rays	R
RT = right	r
RV = revolution	$\theta$
SF = surface	$\Sigma$
SH = shade	$\delta$
SP = spheroid	T
SW = shadow	Θ
TA = tangent	$\tau$
TR = trace	$\Omega$
VP = vanishing point	x
VC = visual cone	$\Psi$
VR = visual ray	$\zeta$

## 1.4 NOTATIONS

You will notice that most of the figures, diagrams, and monitor displays include the listed two-letter abbreviations, graphical symbol, and a conventional system of line segments, which are



Object  
Construction lines  
Projection lines  
Hidden lines  
Center lines, traces, rotation lines  
Axes of revolution, reference lines  
Direction of sight

## 1.5 SYSTEM OF MEASUREMENT

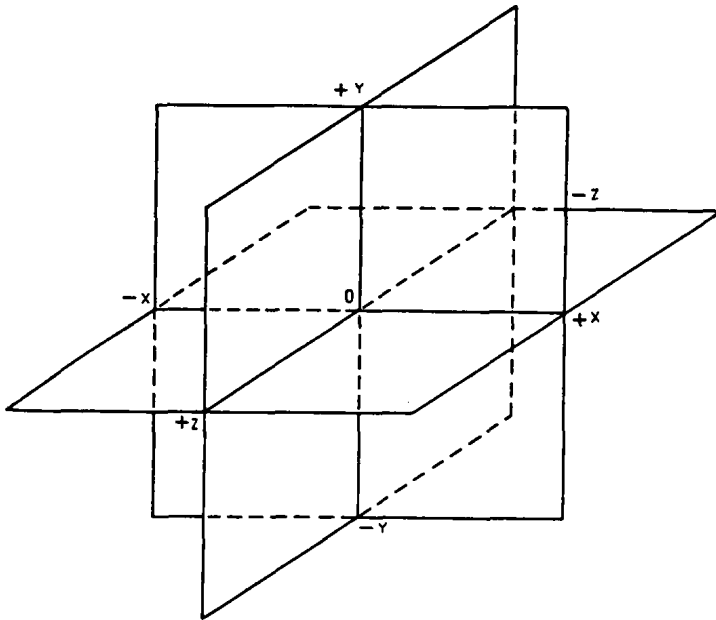
Three mutually perpendicular planes, one horizontal (*HP*) and two vertical (*FP* and *PP*) are shown in Figure 1.1. These reference planes serve as the system of measurement or coordinate planes for the description and storage of points, lines, planes, and solids. These reference planes (*HP*, *FP*, and *PP*) intersect in the three axes labeled *OX*, *OY*, and *OZ* in Figure 1.1. The origin (*O*) is the intersection of the three axes. Four quadrants are formed by the intersecting axes, namely

- 1—where *X* and *Y* measurements are both positive (+),
- 2—where *X* is negative (−) and *Y* is positive,
- 3—where *X* is negative and *Y* is negative and,
- 4—where *X* is positive and *Y* is negative.

Measurements are made to points in one of these four quadrants where

- X* = distance to point right or left of *O*, as  $+X$  or  $-X$ ,  
*Y* = distance to point above or below *O* as  $+Y$  or  $-Y$  and,  
*Z* = distance to point before or behind *O*, as  $+Z$  or  $-Z$ .

The point measurements (coordinates) are always given in *X*, *Y*, and *Z* order, and the distances are displayed in graphic display units (GDU). A GDU is set at the CAD workstation and can be anything from SI (metric) units to English feet and inches for architectural applications. For this book, the units of measurement are all in millimeters, unless otherwise noted. So, a point located (25,50,75) is 1 inch (25 millimeters) to the right, 2 inches above, and 3 inches in front of the origin.



**Figure 1.1** Coordinate system.

## 1.6 SPACE COORDINATION

In this section we shall see that the system of measurement described in section 1.5 can be used to define space in much the same way a mathematician describes points in a plane system of measurement. We shall see later in this text further similarities: (1) parametric descriptions of a line in space are quite similar to an equation of a line in a plane, (2) distance descriptions (true lengths) are a simple extension of the equation for the plane, (3) planes in space can be described by the location of three or more lines, and (4) solids can be described simply by means of equations.

For this section we will need to review the euclidean postulates for space coordination.

**EP1** There is a set (group) of points (real numbers, mapped as ordered triplets). Certain subsets (members of the group) are called lines; at least two points are needed to define a line. Certain other subsets are called planes; at least three points are needed to define a plane, and solids contain at least four points not in any one plane.

**EP2** There is exactly one plane containing any three noncolinear points.

**EP3** Each plane satisfies all other postulates.

**EP4** Each plane separates space into half-spaces with the following properties: (1) if two points are in the same half-space subset, a line between them does not intersect the plane, and (2) if two points are in different half-spaces, a line between them will intersect the plane.

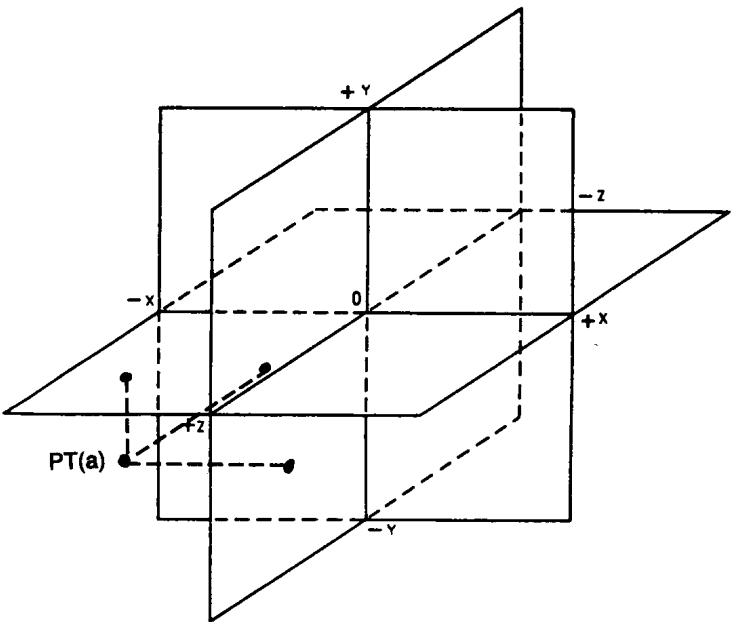
From these few postulates all the theorems of modern descriptive geometry can be derived. However, we will not derive any theorems in this text. All that is required is an understanding of the elementary aspects of space geometry and the ability to visualize and display some space figures. In this regard, you should observe that the CAD display problem is one of representing three-dimensional relations in a plane, namely, the monitor face. Planes will be displayed by plotting a parallelogram, suggesting a portion of a plane. Lines that are behind planes as viewed in Figure 1.1 are dashed (hidden).

## 1.7 PROJECTIONS

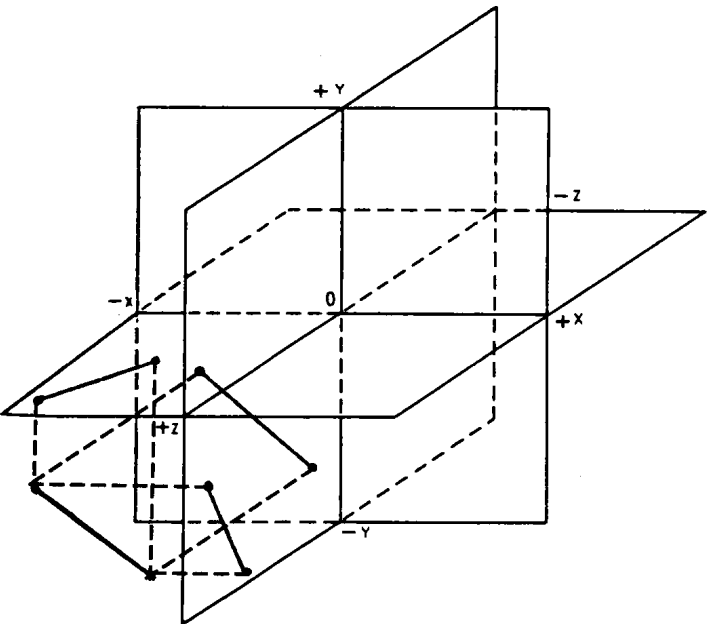
Just as in section 1.6 we defined a one-to-one mapping of ordered pairs of real numbers onto the points of the plane, so for space projection we analogously define a one-to-one mapping of ordered triplets or real numbers onto reference planes. These projections depend on certain choices:

1. The projection of a point is the base of the perpendicular from the point to the reference plane as shown in Figure 1.2.
2. The projection of a line is the locus of the projections of all points of the line upon the reference plane.
3. The projection of a nonperpendicular line is a straight line as shown in Figure 1.3, while the projection of a perpendicular line is a point.





**Figure 1.2** Projection of a point upon a reference plane.



**Figure 1.3** Projection of a line upon a reference plane.