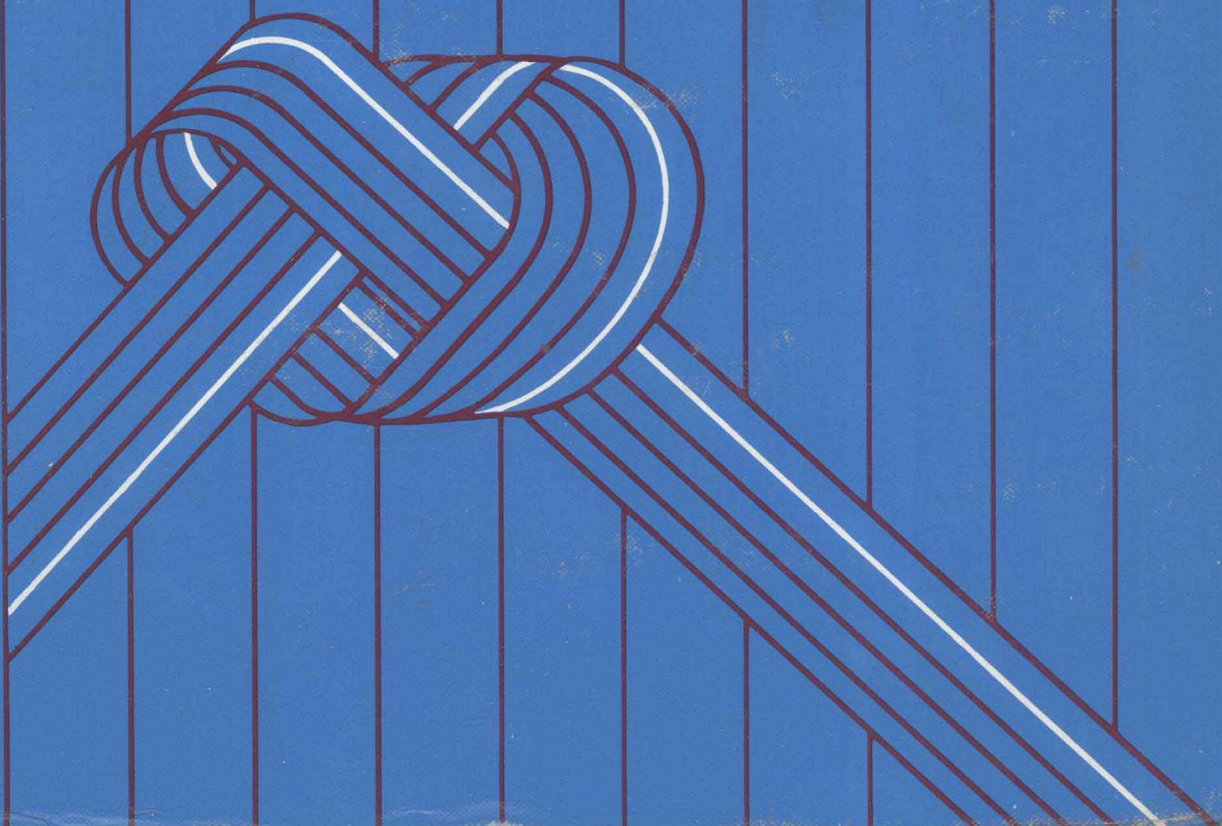


Robert H. Nicholson

Mathematics for Business and Economics



Mathematics for Business and Economics

Robert H. Nicholson
University of Richmond



McGraw-Hill Book Company

New York St. Louis San Francisco Auckland Bogotá Hamburg
Johannesburg London Madrid Mexico Montreal New Delhi
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

MATHEMATICS FOR BUSINESS AND ECONOMICS

Copyright © 1986 by McGraw-Hill, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

1234567890 HALHAL 898765

ISBN 0-07-046491-X

This book was set in Times Roman by General Graphic Services.
The editors were Sam Costanzo and Linda A. Mittiga;
the design was done by Caliber Design Planning;
the production supervisor was Leroy A. Young.
The drawings were done by J & R Services, Inc.
Halliday Lithograph Corporation was printer and binder.

Library of Congress Cataloging in Publication Data

Nicholson, Robert H.

Mathematics for business and economics.

Includes indes.

1. Business mathematics. 2. Economics, Mathematical.

I. Title.

HF5691.N45 1986 510'.2465 85-15190

ISBN 0-07-046491-X



Preface

The ever-increasing complexity of our modern economic structure and the impact on business of the computer as a computational tool have produced quantum leaps in quantitative training in business and economics and in other professional fields. This new focus has come primarily from the changing demands of employment within traditional industry, the emerging high-technology fields, the service sector, and government. As a result, many college and university courses in applied mathematics now stress business applications.

This text is designed specifically to meet the current demand for a compact and relevant text; that is, one that stresses brief theoretical explanations, easily understood applications, and the interpretation of quantitatively structured problems. The author's intent is to provide the methodology and techniques applicable to quantitative problems faced by a range of individuals with diverse interests. An attempt is made to keep applications as universal as possible so that each person can adapt them more easily to his or her specific field. Unnecessary mathematical jargon is eliminated without a loss in precision.

Several specific features of this book deserve elaboration. First, all concepts are presented by means of brief theoretical explanations relying on a minimum of proofs. Examples are introduced at the early stages of each topic. However, readers are offered explanations which are adequate for understanding a wide range of related applications. Applications are kept numerically simple and easy to comprehend so that principles are reinforced.

Second, the relation between problem formulation, mathematical computation, and the interpretation of results is emphasized throughout the book. The author believes that the ability to formulate quantitative problems from unstructured real-world situations and to translate the results of the analysis back to the real world is a skill equal in importance to computational ability. Thus, the issues of model formulation and construction are addressed in the text.

Finally, the book provides discussions of how aids such as the computer and hand calculator are used in specific analyses and of computational methods which have been modified because of computerized analyses. However, reliance on the computer has not been used as a justification for omitting computational methods which may be helpful in explaining particular analytical techniques. The relation between mathematical analysis and the computer is incorporated into the overall problem-solving process where appropriate.

The sequence to the chapters and the emphases given to topics within chapters are designed to make mathematics a relevant field of study. A basic theme of this book is that mathematics need not be, nor appear to be, abstract, but that it should provide a set of techniques which can be used to better understand the various business-related disciplines.

Little previous exposure to college-level mathematics is necessary to use this text. A familiarity with the basic terminology of management science and economics may be helpful in understanding some of the applications but is not a prerequisite for an overall comprehension of the material.

The textbook is adaptable to either one- or two-semester courses, and only the depth to which each topic is covered must be adjusted. Thus, if it is used for a one-semester course, instructors will not have to omit chapters and topics in a way which might reduce the book's usefulness. Alternatively, sufficient material and examples are included for a two-semester course which presents each topic in more detail. The book also may be used for courses in mathematical analysis either at the undergraduate level or for "refresher" courses at the introductory graduate level.

Thus, this book offers several distinct advantages to the student of mathematical analysis. The following chapter-by-chapter description further explains some of its techniques and learning aids.

The book contains 14 chapters. The first chapter is an overview of mathematical terms and problem-solving methodology which serves as an introduction to applied mathematics. In this chapter, readers will immediately encounter business use of the fundamentals of mathematics. Particular emphasis is on developing, understanding, and interpreting mathematical models. As this chapter makes clear, mathematical models provide a framework for applying mathematics to real-world problems.

Chapter 2 includes a review of basic mathematical concepts. Topics included are set theory, the real number system and its properties, exponents, and factoring. This chapter will be a helpful review for readers and will serve as a foundation for the study of applied mathematics.

Chapter 3 gives a summary of many of the mathematical functions used in business-oriented models. This material serves as the conceptual framework for many of the applications and models found in the text. Of special significance in this chapter are the definitions of a relation and a function.

Overall, the first three chapters provide an introduction to applied mathematics. That is, mathematical models provide the analytical structure within which basic mathematical concepts and functions are employed. These first three chapters will prepare and stimulate readers for the more specialized topics of later chapters.

Chapters 4 to 7 include topics in linear mathematics. Chapter 4 discusses linear functions and their characteristics. Here, the emphasis is on understanding the slope and intercept of a linear function. Chapter 5 extends this introduction to examine the solution of systems of linear equations and systems of linear inequalities. Matrix algebra, an additional set of techniques for working with groups of numbers and particularly for solving systems of linear equations, is the topic of Chapter 6.

The fundamentals of linear programming are covered in Chapter 7. This topic combines some of the earlier material on equations and inequalities with matrix algebra in describing a technique applied to many business problems. In general, linear programming is a method for addressing problems of optimization (e.g., minimizing cost or maximizing profit) given a set of constraints (e.g., maintaining product-quality standards or producing with a limited pool of resources). The placement of this chapter within the book and the particular focus of its presentation offer two advantages. First, following the chapter on matrix algebra, linear programming can be discussed using the concept of a matrix and the techniques of matrix algebra where applicable. Second, the chapter emphasizes computer-generated solutions to linear programming problems and the interpretation of such solutions. Thus, the “by-hand” computations of linear programming are discussed for completeness, but the focus is on the interpretation of results. These two aspects of Chapter 7 will enrich the reader’s understanding of this topic and increase his or her ability to apply linear programming to problems in related courses or in business.

Chapter 8 broadens the scope of the book by introducing four of the nonlinear functions used in business analysis: quadratic, cubic, exponential, and logarithmic functions. Particular emphasis is placed on the uses of the constant e ($e = 2.71828 \dots$) in analytical models.

The mathematics of finance is the topic of Chapter 9. This technique-oriented discussion uses many of the functional forms of Chapter 8 and applies them to problems involving various time and interest-rate considerations. The use of financial tables to solve problems is stressed in this chapter.

Differential and integral calculus are discussed in the next three chapters (Chapters 10, 11, and 12). These topics will include reference to both linear and nonlinear relations. Consequently, readers will find helpful at this point the material on functions found in Chapters 3 and 8.

Some of the specific topics included in these chapters are optimization techniques, evaluation of rates of change, area estimation, and the analysis of constrained functions. Chapter 11 is particularly important as it covers many of the business applications of differential calculus. The more theoretical discussions of Chapters 10 and 12 include very few proofs and are linked with applications as an essential part of the presentation. Thus, the entire presentation of calculus is at an applied level.

The principles of probability are included in Chapter 13. This material builds on the discussion of sets in Chapter 2 and that of integral calculus in Chapter 12. This chapter is not intended to be a course in business statistics, but rather is offered as an additional component of a complete overview of applied mathematics. For some, Chapter 13 also will provide introductory material for subsequent courses in sampling and statistical inference.

The final chapter (Chapter 14) is a restatement of the relation between problem formulation, mathematical models, and quantitative techniques. These three topics are the components of modern quantitative business analysis. In this summary chapter, the techniques of Chapters 3 to 13 are presented in a condensed way that reinforces their connection to the process of mathematical modeling. Many textbooks in applied mathematics do not stress this linkage adequately and in the proper perspective. The textbook theme of mathematical modeling is thus stressed in Chapter 14.

Problem sets including conceptual, computational, and application questions are presented within each chapter and at the end of each. The noncomputational review questions at the end of each chapter provide readers with a means of summarizing the definitions and concepts presented in the chapter. These questions may be particularly helpful in clarifying concepts before application problems are attempted. Answers to selected problems are included at the end of the book. The objectives of the problem sets are (1) to test an understanding of important concepts, (2) to extend this knowledge to business applications, and (3) to relate each topic to ideas presented earlier in the text.

In all, the book presents a unified approach to many different aspects of applied mathematics. Topics continually are related so that students will become more flexible in using a particular technique or combinations of techniques in solving problems.

It is anticipated that, after study of this text, readers will have acquired an appreciation of the business uses of applied mathematics. It is hoped that this book will be beneficial to both their education and their professional development.

I would like to express my thanks for the many useful comments and suggestions provided by colleagues who reviewed this text during the course of its development, especially to Allen Ashley, Adelphi University; Paul Baum, California State University, Northridge; Carl Cowen, Purdue University; Robert Limburg, St. Louis Community College; and Charles Margenthaler, Loyola College.

Contents

Preface xvii

1 An Introduction to Mathematical Analysis 1

- 1.1 Introduction 1
- 1.2 Mathematical Analysis in Organizations 2
- 1.3 Mathematical Modeling 2
 - Mathematical Models* 2
 - The Modeling Process* 3
- 1.4 Modeling Business Problems—Three Examples 6
- 1.5 Mathematical Modeling and This Textbook 9
- 1.6 Chapter Summary 9
- 1.7 Problem Set 10
 - Review Questions* 10

2 A Review of Basic Concepts 11

- 2.1 Introduction 11
- 2.2 Sets and Set Theory (optional) 12

	<i>The Algebra of Sets</i>	14
	<i>Exercises</i>	16
2.3	Real Numbers	18
	<i>Rational and Irrational Numbers</i>	18
	<i>Variables and Constants</i>	18
2.4	Equations and Inequalities	19
2.5	Properties of Real Numbers	20
	<i>Closure</i>	20
	<i>Commutativity</i>	20
	<i>Associativity</i>	20
	<i>Distributivity</i>	21
2.6	The Priority of Mathematical Operations	21
2.7	Exponents	22
	<i>Definition of an Exponent</i>	23
	<i>The Radical Sign</i>	23
	<i>Computational Rules</i>	24
	<i>Exercises</i>	25
2.8	Factoring: A Brief Summary	26
	<i>Exercises</i>	29
2.9	Chapter Summary	30
2.10	Problem Set	30
	<i>Review Questions</i>	30
	<i>Review Problems</i>	30
3	Mathematical Functions: An Introduction	35
3.1	Introduction	35
3.2	Relations and Functions	36
3.3	Dependent and Independent Variables	37
3.4	Drawing Functional Relationships	38
3.5	Functions with One Independent Variable	40
	<i>Polynomials with One Independent Variable</i>	40
	<i>Nonalgebraic Functions</i>	41
3.6	Functions with More than One Independent Variable	42
	<i>The Degree of a Polynomial with More than One Independent Variable</i>	42
3.7	Chapter Summary	43
3.8	Problem Set	44

<i>Review Questions</i>	44
<i>Review Problems</i>	45

4 Linear Functions 47

4.1	Introduction	47
4.2	Linear Models in Business	48
4.3	The Slope and the Intercept	49
	<i>Values for the Slope and Intercept</i>	50
	<i>Exercises</i>	54
4.4	The Standard Form of a Linear Equation	55
4.5	Determining the Equation of a Line	58
	<i>The Two-Point Method</i>	58
	<i>The Point-Slope Method</i>	61
	<i>Exercises</i>	62
4.6	Multivariate Functions: An Interpretation	63
4.7	Chapter Summary	64
4.8	Problem Set	65
	<i>Review Questions</i>	65
	<i>Review Problems</i>	66

5 Systems of Linear Equations and Inequalities 71

5.1	Introduction	71
5.2	Systems and Solutions	72
5.3	Linear Demand-and-Supply Analysis	74
5.4	Break-Even Analysis	76
	<i>Exercises</i>	78
5.5	The Elimination Method	80
	<i>Two Applications</i>	84
5.6	Inconsistent and Dependent Equations	87
	<i>Exercises</i>	89
5.7	Overconstrained and Underconstrained Systems of Equations	91
5.8	Inequalities: Properties and Operations	92
5.9	Solving Inequalities and Systems of Inequalities	94
	<i>Introduction</i>	94
	<i>Single Inequalities</i>	94

	<i>Systems of Inequalities</i>	97
	<i>Looking Ahead: Linear Programming</i>	101
	<i>Exercises</i>	101
5.10	Chapter Summary	103
5.11	Problem Set	104
	<i>Review Questions</i>	104
	<i>Review Problems</i>	105

6 Matrix Algebra 109

6.1	Introduction	109
6.2	Basic Concepts	110
6.3	Addition and Subtraction of Matrices	112
6.4	Matrix Multiplication	113
	<i>Exercises</i>	117
6.5	The Matrix Form of a System of Linear Equations	120
6.6	Solution of Systems of Linear Equations by Matrices	123
	<i>Exercises</i>	129
6.7	Solution of Systems of Linear Equations by the Inverse Matrix	129
	<i>The Gauss-Jordan Method of Inversion</i>	131
	<i>Exercises</i>	135
6.8	The Inverse Matrix: Additional Considerations	138
6.9	Input-Output Analysis	140
6.10	Chapter Summary	144
6.11	Problem Set	145
	<i>Review Questions</i>	145
	<i>Review Problems</i>	146

7 Linear Programming 151

7.1	Introduction	151
7.2	Three Linear Programming Problems	152
7.3	Linear Programming Problems in Algebraic Form	153
7.4	Linear Programming Problems in Matrix Form	154
7.5	Maximization and Minimization: A Geometrical Interpretation	156
	<i>Graphing the Constraints</i>	157

7.6	Other Solution Possibilities	165
	<i>Exercises</i>	166
7.7	The Simplex Method: Principles	168
7.8	The Simplex Method: Solution	170
7.9	Interpretation of the Optimal Tableau	174
	<i>Shadow Price Ranges</i>	175
	<i>Exercises</i>	178
7.10	The Dual in Linear Programming	180
7.11	Minimization By Maximizing the Dual	182
	<i>Formation of the Dual</i>	183
7.12	Interpretation of the Optimal Dual Tableau	188
	<i>Computing Primal-Shadow-Price Ranges from the Dual</i>	189
	<i>Concluding Remarks</i>	191
	<i>Exercises</i>	192
7.13	Additional Considerations	194
7.14	Computers and Linear Programming	195
7.15	Chapter Summary	195
7.16	Problem Set	197
	<i>Review Questions</i>	197
	<i>Review Problems</i>	198

8 Nonlinear Functions 203

8.1	Introduction	203
8.2	Quadratic Functions and the Quadratic Formula	204
	<i>The Quadratic Formula</i>	207
	<i>Exercises</i>	208
8.3	Cubic Functions	209
8.4	Exponential Functions	211
	<i>Properties of Exponential Functions</i>	211
	<i>Exponential Functions When b Is Greater than 1</i>	212
	<i>Exponential Functions When b Is between 0 and 1</i>	213
	<i>The Base e</i>	213
	<i>Exponential Growth and Exponential Decay</i>	214
	<i>A Modified Exponential Function</i>	216
	<i>Exercises</i>	218
8.5	Logarithmic Functions	220
	<i>Introduction</i>	220

	<i>Logarithmic Functions and Exponential Functions</i>	221
	<i>Logarithmic Functions in Business</i>	222
8.6	Chapter Summary	223
8.7	Problem Set	224
	<i>Review Questions</i>	224
	<i>Review Problems</i>	225
	Appendix: Using the Table of Natural Logarithms	227
	<i>Exercises</i>	230
9	The Mathematics of Finance	231
<hr/>		
9.1	Introduction	231
9.2	Sequences and Series	232
	<i>Finite and Infinite Series</i>	232
	<i>Geometric Series</i>	233
	<i>A Geometric Series: The Multiplier</i>	233
	<i>Exercises</i>	235
9.3	Simple Interest and Simple Discount	235
9.4	Compound Interest	237
9.5	Present Value with Compounding	239
	<i>Exercises</i>	240
9.6	The Future Value of an Annuity	241
	<i>Sinking Funds</i>	244
	<i>Exercises</i>	245
9.7	The Present Value of an Annuity	246
	<i>Loan Amortization</i>	249
	<i>Exercises</i>	251
9.8	A Perpetuity as an Infinite Geometric Series	252
	<i>Exercises</i>	254
9.9	Continuous Compounding	254
	<i>The Base e and Continuous Compounding</i>	254
	<i>Exercises</i>	257
9.10	Nominal Interest Rates and Effective Interest Rates	258
9.11	Chapter Summary	258
9.12	Problem Set	261

<i>Review Questions</i>	261
<i>Review Problems</i>	261

10 Differential Calculus: Principles 265

10.1	Introduction	265
10.2	Business Analysis and Calculus	266
10.3	Limits and Continuity	267
	<i>Limits</i>	267
	<i>Limit Theorems</i>	270
	<i>Continuity</i>	272
	<i>Exercises</i>	275
10.4	The Secant Slope and the Tangent Slope	275
10.5	The Derivative	278
	<i>Continuity and the Derivative</i>	278
	<i>Computing the Derivative by the Simple Power Rule</i>	280
	<i>The Derivative and the Slope</i>	282
10.6	Additional Rules of Differentiation	283
	<i>Exercises</i>	287
10.7	The Differentiation of Exponential Functions	288
10.8	The Differentiation of Logarithmic Functions	290
	<i>Exercises</i>	291
10.9	Higher-Order Derivatives	292
	<i>Exercises</i>	293
10.10	Implicit Differentiation	294
10.11	Partial Differentiation	298
	<i>Higher Partial Derivatives</i>	300
	<i>Exercises</i>	302
10.12	Chapter Summary	303
	<i>A Summary of the Rules of Differentiation</i>	304
10.13	Problem Set	305
	<i>Review Questions</i>	305
	<i>Review Problems</i>	306

11 Differential Calculus: Applications 309

11.1	Introduction	309
	<i>Marginal Measurements and Mathematical Models</i>	310

11.2	Identification of Stationary Points on Functions with One Independent Variable	310
	<i>Finding Stationary Points</i>	312
	<i>The Second-Derivative Test</i>	314
	<i>When the Second-Derivative Test Fails</i>	315
	<i>Exercises</i>	318
11.3	Differential Calculus and Profit Maximization	319
	<i>Optimizing a Profit Function</i>	319
	<i>Principles of Marginal Analysis</i>	320
	<i>Marginal Analysis and Profit Maximization</i>	324
	<i>Exercises</i>	327
11.4	Differential Calculus and Cost Analysis	329
	<i>Finding a Firm's Shut-Down Price</i>	329
	<i>Diminishing Marginal Returns of Inputs</i>	331
11.5	Marginal Productivity Analysis	333
	<i>Exercises</i>	336
11.6	Elasticity of Demand	337
	<i>Other Elasticity Measurements</i>	342
	<i>Exercises</i>	344
11.7	Identification of Stationary Points on Functions with Two Independent Variables	345
11.8	Constrained Optimization Models and Lagrangian Multipliers	348
	<i>The Method of Lagrangian Multipliers: A Restatement</i>	351
	<i>Exercises</i>	352
11.9	A Note on Functional Form	354
11.10	Chapter Summary	354
11.11	Problem Set	355
	<i>Review Questions</i>	355
	<i>Review Problems</i>	356
12	Integral Calculus with Applications	361
12.1	Introduction	361
12.2	Integral Calculus and Mathematical Models	362
12.3	Indefinite Integrals and the Antiderivative	363
12.4	The Definite Integral	365
12.5	The Rules of Integration	368
	<i>Integrals of Exponential and Logarithmic Functions</i>	370

12.6	Definite Integrals and the Rules of Integration	373
	<i>Exercises</i>	374
12.7	Integration By Parts	375
	<i>Exercises</i>	378
12.8	Using Tables of Integrals	378
	<i>Exercises</i>	380
12.9	Applications: Indefinite Integrals	380
	<i>Marginal Relationships and Total Relationships</i>	380
12.10	Applications: Definite Integrals	384
	<i>Exercises</i>	390
12.11	Improper Integrals	393
	<i>Exercises</i>	397
12.12	Chapter Summary	398
12.13	Problem Set	399
	<i>Review Questions</i>	399
	<i>Review Problems</i>	399

13 Probability: Principles and Applications 403

13.1	Introduction	403
13.2	Business Analysis and Probability	403
13.3	Counting Techniques	404
	<i>Tree Diagrams</i>	405
	<i>Subsets</i>	406
	<i>Permutations and Combinations</i>	407
	<i>Exercises</i>	410
13.4	Basic Definitions of Probability	411
	<i>Sample Spaces and Events</i>	411
	<i>Venn Diagrams</i>	411
	<i>Mutually Exclusive Events</i>	412
	<i>Joint Events and Conditional Events</i>	412
	<i>Dependent and Independent Events</i>	415
	<i>The Union of Events</i>	417
13.5	The General Rules of Multiplication and Addition	419
	<i>Exercises</i>	420
13.6	Bayes' Rule	421

	<i>Exercises</i>	425
13.7	Discrete Probability Distributions	427
	<i>The Binomial Probability Distribution</i>	427
13.8	Continuous Probability Distributions	429
	<i>The Mean and the Variance</i>	430
13.9	The Normal Distribution	432
	<i>Applications of the Normal Distribution</i>	434
	<i>Exercises</i>	438
13.10	Chapter Summary	439
13.11	Problem Set	440
	<i>Review Questions</i>	440
	<i>Review Problems</i>	441
14	Mathematical Models and Business Analysis: A Final Word	445
<hr/>		
14.1	Introduction	445
14.2	Linear Models	446
14.3	Nonlinear Models	447
14.4	Probabilistic (Stochastic) Models	448
14.5	Mathematical Analysis in Business: The Future	449
	Appendix: The Summation Operator	451
	Exercises	454
	Tables	456
	Glossary	477
	Answers to Even-Numbered Exercises	485
	Index	521