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(英文版·原书第3版)

工程中的有限元方法

(美) T.R. 钱德拉佩特拉 著
A.D. 贝莱冈度



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出版说明

随着我国加入 WTO，国际间的竞争越来越激烈，而国际间的竞争实际上也就是人才的竞争、教育的竞争。为了加快培养具有国际竞争力的高水平技术人才，加快我国教育改革的步伐，国家教育部近来出台了一系列倡导高校开展双语教学、引进原版教材的政策。以此为契机，机械工业出版社陆续推出了一系列国外影印版教材，其内容涉及高等学校公共基础课，以及机、电、信息领域的专业基础课和专业课。

引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究，对引进原版教材提出了许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性严格把关。同时尽量考虑原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

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序

有限元方法(finite element method)是求解各种复杂数学物理问题的重要方法,利用该方法可以获取几乎任意复杂工程结构的各种信息,还可以直接就工程设计进行各种评判,可以就各种工程事故进行技术分析。据有关资料表明,一个新产品的问题有60%以上可以在设计阶段消除,国际上有90%的机械产品和装备都要采用数值计算进行分析。实际上,大规模计算在科学研究上,已成为探知复杂对象本质规律的定量分析手段;在工程设计和工艺设计上,可以成为替代大量实物试验的数字化“虚拟试验”,做到高效率 and 低成本。如美国 Boeing 公司设计的 B-777 飞机,就在计算机上完全实现了原型和制造工艺的“无纸设计”,其中大规模工程计算(模拟与仿真)起到核心技术的支撑作用。如对于新型轿车的设计和制造,如果采用全数字化的设计和高精度的模拟,可以减少近60%以上的实物试验,新型号的开发时间可以减少一半,开发费用也降低三分之一以上。可以看出,有限元方法已经作为一种成熟的分析手段,在科学研究、工程设计与评判中发挥着巨大作用。

本书英文原名为 Introduction to Finite Elements in Engineering,是近年来在国际上有限元分析教学方面的具有较大影响的大学教材之一,它的一个显著特点是:在介绍有限元方法基本原理的同时,提供相应的工程背景和建模技巧,书中所给出的实例和习题几乎都对应有或涉及到实际工程背景,使读者在学习过程中就体会和了解实际问题的有限元建模过程,可以说这正是有限元分析的重要目的之一。

本书的两个编著者之一 Dr. T. R. Chandrupatla 为美国 Rowan University 机械工程的教授和主任,曾在工业界从事机械设计工作,具有丰富的工程实际经验,也开展了有限元方法方面的学术研究;他长期从事有限元方面的教学工作,形成了在有限元教学中基本理论与实际工程相结合的显著特点。另一位作者 Dr. A. D. Belegundu 在 Pennsylvania State University 执教,也是从事机械系统及设计方面的研究,在结构有限元分析及优化方面发表了一大批学术论文,在学术和教学方面也有较大的影响。

Dr. T. R. Chandrupatla 与 Dr. A. D. Belegundu 合作,在1991年写出了本书的初版,由于特点鲜明在大学中广受欢迎和赞誉,取得很好的教学效果。后于1997年出版了该书的第2版。5年后,该书的第3版面市。本版完全保留了前两版的特点,在介绍有限元方法的同时,特别考虑了课程教学的特点,注重方法原理的论述与实用例题的展示,提供了200多个实例,还在所附光盘中提供了所有的计算机程序源代码。

本书语言流畅，推导严谨，实例丰富，特别注重实用性，可以作为机械、力学、土木、水利、航空航天等专业的学生开展有限元方法英文教学的教材。对于希望既学习有限元方法原理又希望学习相关英文专业术语表达的读者，该书更是一本不可多得的参考书。

曾攀

教授 所长

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前 言

本书的第1版在10多年以前问世，几年之后出版了第2版，我们收到了来自使用该书的教授、学生和从事实际工作的工程师的正面反馈意见，也了解到在过去20年中我们学校的学生使用该书的各方面情况。在这一新版中，已充分考虑了许多建议。本书的基本出发点是提供有限元方法的清晰理论、建模方法以及具体的计算机实现程序；在新版中依然保留前两版的教学特点。

在本书的许多章节中增加了一些新的材料，补充了实际算例和练习题，以帮助读者更好地学习和理解。而练习题更强调基本理论的理解和实际问题的考虑，所增加的理论和计算机程序涉及声学、轴对称四边形单元、共轭梯度算法以及特征值问题。在该版中还增加了三个附加程序，所提供的程序都在 Windows 平台上进行开发，并且都具有相同的编程结构以方便读者效仿使用。所采用的语言包括：Visual Basic、Microsoft Excel/Visual Basic、MATLAB，以及早期使用的 QBASIC、FORTRAN 和 C，相应的求解手册也作了更新。

第1章简要介绍有限元方法的历史背景和基本概念，对平衡方程、应力应变关系、应变位移关系和势能原理进行评述，引入 Galerkin 方法的概念。

第2章介绍矩阵和行列式的性质，引入 Gauss 消元法，讨论对称带状矩阵方程的求解和带状矩阵“特征顶线”（skyline）的处理方法，对 Cholesky 分解和共轭梯度法也作了讨论。

第3章通过对一维问题的分析来介绍有限元方法的基本概念和表达式，涉及有限元分析的主要步骤：形状函数的表达、单元刚度矩阵的推导、整体刚度矩阵的形成、边界条件的处理、方程的求解以及应力计算；同时给出了基于势能方法和 Galerkin 方法的表达形式，还考虑温度效应的处理。

第4章给出平面及三维桁架问题的有限元表达，对于整体刚度矩阵的组装，分别给出带状矩阵和具有“特征顶线”矩阵的形式，还提供基于这两种形式进行求解的计算机程序。

第5章介绍用于二维平面应力和平面应变问题求解的常应变三角形单元（CST），详细给出问题的建模过程及边界条件的处理方法，对于正交各向异性材料也给出相应的处理方法。

第6章介绍轴对称物体在承受轴对称外载时的建模过程，给出相应的三角形单元表达式，还提供几个实际问题的处理方法。

第7章介绍等参四边形单元和高阶单元的基本概念以及采用 Gauss 方法进行面积积分的数值方法，给出轴对称四边形单元的表达式以及基于共轭梯度法求解的过程。

第 8 章讨论梁单元及 Hermite 形状函数的应用, 涉及二维及三维框架结构。

第 9 章为三维应力分析, 包括四面体单元和六面体单元, 还介绍波前法的求解及实现过程。

第 10 章详细介绍标量场问题的处理。在其他各章中均将 Galerkin 方法和能量原理作为有限元方法推导的基本原理。在本章中, 仅采用 Galerkin 方法来进行推导。采用该方法可以直接对所给出的微分方程进行处理, 而不需要定义一个用来求最小值的等效泛函。该章分别就稳态热传导、扭转、一般流动、渗流、电磁场、管道中流动、声学等问题给出相应的 Galerkin 方法表达式。

第 11 章为动力学问题, 给出单元质量矩阵表达, 对一般特征值问题的特征值(自然率频)、特征向量(模态形状)的求解进行讨论, 给出求逆迭代法、Jacobi 法、三对角化法以及显式漂移法等求解方法。

第 12 章介绍前处理及后处理的概念, 给出二维问题网格自动划分的原理及实现方法, 对于三角形和四边形单元给出由单元值求取节点应力的最小二乘方法, 还介绍了后处理中的等直线技术。

对于大学本科生来说, 书中一些较深的内容可以忽略, 或根据某一新的完整内容体系, 按需要来采用本书的材料, 建议并鼓励在学习完第 5 章后就开始使用第 12 章中的程序, 这样可以帮助读者高效率地准备各种有限元分析的数据。

我们对 Nels Madsen(Auburn University), Arif Masud(University of Illinois(Chicago)), Robert L. Rankin(Arizona State University), John S. Strenkowsi(NC State University), 以及 Hormoz Zareh(Portland State University)表示感谢, 正是他们对本书的第 2 版进行了审阅并提出许多建设性的意见, 这对我们有很好的帮助作用。

本书的作者 Tirupathi Chandrupatla 对 J. Tinsley Oden 表示感谢, 正是他的教导和鼓励影响了 Chandrupatla 的一生; 还要感谢在 Rowan University 和 Kettering University 就读的学习该课程的学生; 感谢同事 Paris vonLockette 在本书第 2 版出版后的教学活动中所提出的富有价值的意见; 感谢技术编辑 Fran Daniele 在本书最后的出版中所进行的一丝不苟的工作。

本书的作者 Ashok Belegundu 感谢他在宾州州立大学(Penn State)的学生, 他们对教材内容和程序提出了很好的建议; 感谢机械与核工程系主任 Richard C. Benson 所给予的鼓励和支持; 感谢在声学系工作的 Victor W. Sparrow 教授和博士生 Dongjai Lee, 他们对本书的相关章节进行了讨论并提供了一些素材; 感谢已故的父亲, 正是他的鼓励一直激励我完成本书。

我们感谢 Prentice Hall 出版社的编辑 Laura Fischer, 她为我们完成了一项非常漂亮的工作。

T. R. 钱德拉佩特拉

A. D. 贝莱冈度

Preface

The first edition of this book appeared over 10 years ago and the second edition followed a few years later. We received positive feedback from professors who taught from the book and from students and practicing engineers who used the book. We also benefited from the feedback received from the students in our courses for the past 20 years. We have incorporated several suggestions in this edition. The underlying philosophy of the book is to provide a clear presentation of theory, modeling, and implementation into computer programs. The pedagogy of earlier editions has been retained in this edition.

New material has been introduced in several chapters. Worked examples and exercise problems have been added to supplement the learning process. Exercise problems stress both fundamental understanding and practical considerations. Theory and computer programs have been added to cover acoustics, axisymmetric quadrilateral elements, conjugate gradient approach, and eigenvalue evaluation. Three additional programs have now been introduced in this edition. All the programs have been developed to work in the Windows environment. The programs have a common structure that should enable the users to follow the development easily. The programs have been provided in Visual Basic, Microsoft *Excel/Visual Basic*, MATLAB, together with those provided earlier in QBASIC, FORTRAN and C. The Solutions Manual has also been updated.

Chapter 1 gives a brief historical background and develops the fundamental concepts. Equations of equilibrium, stress-strain relations, strain-displacement relations, and the principles of potential energy are reviewed. The concept of Galerkin's method is introduced.

Properties of matrices and determinants are reviewed in Chapter 2. The Gaussian elimination method is presented, and its relationship to the solution of symmetric banded matrix equations and the skyline solution is discussed. Cholesky decomposition and conjugate gradient method are discussed.

Chapter 3 develops the key concepts of finite element formulation by considering one-dimensional problems. The steps include development of shape functions, derivation of element stiffness, formation of global stiffness, treatment of boundary conditions, solution of equations, and stress calculations. Both the potential energy approach and Galerkin's formulations are presented. Consideration of temperature effects is included.

Finite element formulation for plane and three-dimensional trusses is developed in Chapter 4. The assembly of global stiffness in banded and skyline forms is explained. Computer programs for both banded and skyline solutions are given.

Chapter 5 introduces the finite element formulation for two-dimensional plane stress and plane strain problems using constant strain triangle (CST) elements. Problem modeling and treatment of boundary conditions are presented in detail. Formulation for orthotropic materials is provided. Chapter 6 treats the modeling aspects of axisymmetric solids subjected to axisymmetric loading. Formulation using triangular elements is presented. Several real-world problems are included in this chapter.

Chapter 7 introduces the concepts of isoparametric quadrilateral and higher order elements and numerical integration using Gaussian quadrature. Formulation for axisymmetric quadrilateral element and implementation of conjugate gradient method for quadrilateral element are given.

Beams and application of Hermite shape functions are presented in Chapter 8. The chapter covers two-dimensional and three-dimensional frames.

Chapter 9 presents three-dimensional stress analysis. Tetrahedral and hexahedral elements are presented. The frontal method and its implementation aspects are discussed.

Scalar field problems are treated in detail in Chapter 10. While Galerkin as well as energy approaches have been used in every chapter, with equal importance, only Galerkin's approach is used in this chapter. This approach directly applies to the given differential equation without the need of identifying an equivalent functional to minimize. Galerkin formulation for steady-state heat transfer, torsion, potential flow, seepage flow, electric and magnetic fields, fluid flow in ducts, and acoustics are presented.

Chapter 11 introduces dynamic considerations. Element mass matrices are given. Techniques for evaluation of eigenvalues (natural frequencies) and eigenvectors (mode shapes) of the generalized eigenvalue problem are discussed. Methods of inverse iteration, Jacobi, tridiagonalization and implicit shift approaches are presented.

Preprocessing and postprocessing concepts are developed in Chapter 12. Theory and implementation aspects of two-dimensional mesh generation, least-squares approach to obtain nodal stresses from element values for triangles and quadrilaterals, and contour plotting are presented.

At the undergraduate level some topics may be dropped or delivered in a different order without breaking the continuity of presentation. We encourage the introduction of the Chapter 12 programs at the end of Chapter 5. This helps the students to prepare the data in an efficient manner.

We thank Nels Madsen, Auburn University; Arif Masud, University of Illinois, Chicago; Robert L. Rankin, Arizona State University; John S. Strenkowsi, NC State University; and Hormoz Zareh, Portland State University, who reviewed our second edition and gave many constructive suggestions that helped us improve the book.

Tirupathi Chandrupatla expresses his gratitude to J. Tinsley Oden, whose teaching and encouragement have been a source of inspiration to him throughout his career. He also expresses his thanks to many students at Rowan University and Kettering University who took his courses. He expresses his thanks to his colleague Paris vonLockette, who gave valuable feedback after teaching a course from the second edition. We thank

our production editor Fran Daniele for her meticulous approach in the final production of the book.

Ashok Belegundu thanks his students at Penn State for their feedback on the course material and programs. He expresses his gratitude to Richard C. Benson, chairman of mechanical and nuclear engineering, for his encouragement and appreciation. He also expresses his thanks to Professor Victor W. Sparrow in the acoustics department and to Dongjai Lee, doctoral student, for discussions and help with some of the material in the book. His late father's encouragement with the first two editions of this book are an ever present inspiration.

We thank our acquisitions editor at Prentice Hall, Laura Fischer, who has made this a pleasant project for us.

TIRUPATHI R. CHANDRUPATLA
ASHOK D. BELEGUNDU

About the Authors

Tirupathi R. Chandrupatla is Professor and Chair of Mechanical Engineering at Rowan University, Glassboro, New Jersey. He received the B.S. degree from the Regional Engineering College, Warangal, which was affiliated with Osmania University, India. He received the M.S. degree in design and manufacturing from the Indian Institute of Technology, Bombay. He started his career as a design engineer with Hindustan Machine Tools, Bangalore. He then taught in the Department of Mechanical Engineering at I.I.T., Bombay. He pursued his graduate studies in the Department of Aerospace Engineering and Engineering Mechanics at the University of Texas at Austin and received his Ph.D. in 1977. He subsequently taught at the University of Kentucky. Prior to joining Rowan, he was a Professor of Mechanical Engineering and Manufacturing Systems Engineering at GMI Engineering & Management Institute (formerly General Motors Institute), where he taught for 16 years.

Dr. Chandrupatla has broad research interests, which include finite element analysis, design, optimization, and manufacturing engineering. He has published widely in these areas and serves as a consultant to industry. Dr. Chandrupatla is a registered Professional Engineer and also a Certified Manufacturing Engineer. He is a member of ASEE, ASME, NSPE, SAE, and SME.

Ashok D. Belegundu is a Professor of Mechanical Engineering at The Pennsylvania State University, University Park. He was on the faculty at GMI from 1982 through 1986. He received the Ph.D. degree in 1982 from the University of Iowa and the B.S. degree from the Indian Institute of Technology, Madras. He was awarded a fellowship to spend a summer in 1993 at the NASA Lewis Research Center. During 1994–1995, he obtained a grant from the UK Science and Engineering Research Council to spend his sabbatical leave at Cranfield University, Cranfield, UK.

Dr. Belegundu's teaching and research interests include linear, nonlinear, and dynamic finite elements and optimization. He has worked on several sponsored projects for government and industry. He is an associate editor of *Mechanics of Structures and Machines*. He is also a member of ASME and an Associate fellow of AIAA.

Contents

出版说明

序

前言

PREFACE

1 FUNDAMENTAL CONCEPTS

1

- 1.1 Introduction 1
- 1.2 Historical Background 1
- 1.3 Outline of Presentation 2
- 1.4 Stresses and Equilibrium 2
- 1.5 Boundary Conditions 4
- 1.6 Strain–Displacement Relations 4
- 1.7 Stress–Strain Relations 6
 - Special Cases, 7*
- 1.8 Temperature Effects 8
- 1.9 Potential Energy and Equilibrium;
The Rayleigh–Ritz Method 9
 - Potential Energy Π , 9*
 - Rayleigh–Ritz Method, 11*
- 1.10 Galerkin’s Method 13
- 1.11 Saint Venant’s Principle 16
- 1.12 Von Mises Stress 17
- 1.13 Computer Programs 17
- 1.14 Conclusion 18
 - Historical References 18
 - Problems 18

2 MATRIX ALGEBRA AND GAUSSIAN ELIMINATION

22

- 2.1 Matrix Algebra 22
 - Row and Column Vectors, 23*
 - Addition and Subtraction, 23*
 - Multiplication by a Scalar, 23*

	<i>Matrix Multiplication</i> , 23
	<i>Transposition</i> , 24
	<i>Differentiation and Integration</i> , 24
	<i>Square Matrix</i> , 25
	<i>Diagonal Matrix</i> , 25
	<i>Identity Matrix</i> , 25
	<i>Symmetric Matrix</i> , 25
	<i>Upper Triangular Matrix</i> , 26
	<i>Determinant of a Matrix</i> , 26
	<i>Matrix Inversion</i> , 26
	<i>Eigenvalues and Eigenvectors</i> , 27
	<i>Positive Definite Matrix</i> , 28
	<i>Cholesky Decomposition</i> , 29
2.2	Gaussian Elimination 29
	<i>General Algorithm for Gaussian Elimination</i> , 30
	<i>Symmetric Matrix</i> , 33
	<i>Symmetric Banded Matrices</i> , 33
	<i>Solution with Multiple Right Sides</i> , 35
	<i>Gaussian Elimination with Column Reduction</i> , 36
	<i>Skyline Solution</i> , 38
	<i>Frontal Solution</i> , 39
2.3	Conjugate Gradient Method for Equation Solving 39
	<i>Conjugate Gradient Algorithm</i> , 40
	Problems 41
	<i>Program Listings</i> , 43

3 ONE-DIMENSIONAL PROBLEMS

45

3.1	Introduction 45
3.2	Finite Element Modeling 46
	<i>Element Division</i> , 46
	<i>Numbering Scheme</i> , 47
3.3	Coordinates and Shape Functions 48
3.4	The Potential-Energy Approach 52
	<i>Element Stiffness Matrix</i> , 53
	<i>Force Terms</i> , 54
3.5	The Galerkin Approach 56
	<i>Element Stiffness</i> , 56
	<i>Force Terms</i> , 57
3.6	Assembly of the Global Stiffness Matrix and Load Vector 58
3.7	Properties of \mathbf{K} 61
3.8	The Finite Element Equations; Treatment of Boundary Conditions 62
	<i>Types of Boundary Conditions</i> , 62
	<i>Elimination Approach</i> , 63
	<i>Penalty Approach</i> , 69
	<i>Multipoint Constraints</i> , 74

- 3.9 Quadratic Shape Functions 78
- 3.10 Temperature Effects 84
 - Input Data File, 88*
 - Problems 88
 - Program Listing, 98*

4 TRUSSES

103

- 4.1 Introduction 103
- 4.2 Plane Trusses 104
 - Local and Global Coordinate Systems, 104*
 - Formulas for Calculating ℓ and m , 105*
 - Element Stiffness Matrix, 106*
 - Stress Calculations, 107*
 - Temperature Effects, 111*
- 4.3 Three-Dimensional Trusses 114
- 4.4 Assembly of Global Stiffness Matrix for the Banded and Skyline Solutions 116
 - Assembly for Banded Solution, 116*
 - Input Data File, 119*
 - Problems 120
 - Program Listing, 128*

5 TWO-DIMENSIONAL PROBLEMS USING CONSTANT STRAIN TRIANGLES

130

- 5.1 Introduction 130
- 5.2 Finite Element Modeling 131
- 5.3 Constant-Strain Triangle (CST) 133
 - Isoparametric Representation, 135*
 - Potential-Energy Approach, 139*
 - Element Stiffness, 140*
 - Force Terms, 141*
 - Galerkin Approach, 146*
 - Stress Calculations, 148*
 - Temperature Effects, 150*
- 5.4 Problem Modeling and Boundary Conditions 152
 - Some General Comments on Dividing into Elements, 154*
- 5.5 Orthotropic Materials 154
 - Temperature Effects, 157*
 - Input Data File, 160*
 - Problems 162
 - Program Listing, 174*

6 AXISYMMETRIC SOLIDS SUBJECTED TO AXISYMMETRIC LOADING

178

- 6.1 Introduction 178
- 6.2 Axisymmetric Formulation 179
- 6.3 Finite Element Modeling: Triangular Element 181
 - Potential-Energy Approach, 183*
 - Body Force Term, 184*
 - Rotating Flywheel, 185*
 - Surface Traction, 185*
 - Galerkin Approach, 187*
 - Stress Calculations, 190*
 - Temperature Effects, 191*
- 6.4 Problem Modeling and Boundary Conditions 191
 - Cylinder Subjected to Internal Pressure, 191*
 - Infinite Cylinder, 192*
 - Press Fit on a Rigid Shaft, 192*
 - Press Fit on an Elastic Shaft, 193*
 - Belleville Spring, 194*
 - Thermal Stress Problem, 195*
 - Input Data File, 197*
- Problems 198
 - Program Listing, 205*

**7 TWO-DIMENSIONAL ISOPARAMETRIC ELEMENTS
AND NUMERICAL INTEGRATION**

208

- 7.1 Introduction 208
- 7.2 The Four-Node Quadrilateral 208
 - Shape Functions, 208*
 - Element Stiffness Matrix, 211*
 - Element Force Vectors, 213*
- 7.3 Numerical Integration 214
 - Two-Dimensional Integrals, 217*
 - Stiffness Integration, 217*
 - Stress Calculations, 218*
- 7.4 Higher Order Elements 220
 - Nine-Node Quadrilateral, 220*
 - Eight-Node Quadrilateral, 222*
 - Six-Node Triangle, 223*
- 7.5 Four-Node Quadrilateral for Axisymmetric Problems 225
- 7.6 Conjugate Gradient Implementation
of the Quadrilateral Element 226
 - Concluding Note, 227*
 - Input Data File, 228*
- Problems 230
 - Program Listings, 233*