
Douglas A. Haith

***ENVIRONMENTAL
SYSTEMS
OPTIMIZATION***

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PREFACE

The problems associated with managing land, water, air, and energy resources have never been simple. It often seems that the complexities of environmental management, which includes the control of pollution and the allocation of resources, are so intimidating that rational analysis is futile. This is partially because of the political nature of environmental decisions. However, a substantial source of complexity is the intricate physical interactions within environmental systems. Wastes are transported by water or air from one location to another with attendant chemical transformations. Land used for one purpose may limit other uses, and energy planning must consider a bewildering array of competing sources and consumers. It can be difficult to trace the impacts of pollution control and resource management decisions, and it often seems impossible to determine which of several alternatives best meet management objectives.

Fortunately, there is an analytical process that promises to reduce the complexities of environmental problems to manageable levels. During and after World War II a problem-solving approach known as systems analysis evolved that has proved very useful in the resolution of complex management problems. Systems analysis has been applied recently with considerable success to environmental management. The applications that have been most fruitful are based on the mathematical modeling of environmental systems and the use of optimization techniques to identify promising management decisions.

This book provides students and practicing professionals with an introduction to the application of systems analysis and, most particularly, mathematical modeling and optimization techniques to environmental management. It is intended for engineers, biologists, economists, and planners who are interested in solving pollution control and resource allocation problems.

Two principal topics are emphasized. The first is the use of mathematical models that reduce environmental problems to mathematical relationships that can be manipulated to evaluate the effects of management alternatives. The second is the application of optimization methods such as search techniques, linear programming, dynamic programming, and integer programming to determine which management alternatives are better than others. Many examples are offered in the text; even though the standard applications of water and air pollution control and solid waste management are discussed fully, the examples include more recent applications such as land disposal of wastes, nonpoint source pollution, land use management, energy planning, and multi-objective planning. Artificial problems have been avoided; two chapters are devoted to extensive analyses of environmental problems based on "real-world" data.

The basic concepts of systems analysis are relatively simple and require only a year of college calculus to interpret. Mathematical proofs are based on intuition, graphical methods, and simple algebra. Computer programming capability is not a

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prerequisite for understanding the material in the book, but it is required for many of the exercises. Previous experience or coursework in environmental management is not necessary, but would be useful in understanding some of the examples.

The book is based on material presented in a one-semester, upper-class course at Cornell University. Although the course is required or recommended for environmental majors in several fields, it has attracted many other students who desire an introductory systems analysis course. All or portions of the book will serve similar purposes elsewhere. To the extent possible, the chapters are self-contained, thereby facilitating its use as a reference source.

Many individuals have contributed to this book. Early versions were reviewed by J. Robert Cooke, Ronald B. Furry, Charles D. Gates, Daniel P. Loucks, Jerry R. Stedinger, and Michael F. Walter. They corrected my logic and mathematics, and made many substantial suggestions for improvement. My students suffered through early drafts; their reactions forced me to revise and clarify continually, my ideas and writing. Carol L. Beasley, Merrill G. Floyd, Vivian Kahane, Linda Indig, Kevin J. Murphy, and their colleagues at Wiley provided the steady level of encouragement and competence that transform manuscripts into published books. Karen E. Rizzo typed the manuscript and its several revisions with great efficiency, and she also prepared preliminary drawings for many of the figures. My deepest gratitude is reserved for Ellen, Robert, and Benjamin, who have given me more encouragement and love than any husband and father deserves.

Douglas A. Haith

To Charles R. Scherer, Ph.D. (May 13, 1943–January 1, 1979)

*Near the snow, near the sun, in the highest fields,
See how these names are fêted by the waving grass
And by the streamers of white cloud
And whispers of wind in the listening sky.
The names of those who in their lives fought for life,
Who wore at their hearts the fire's centre.
Born of the sun, they travelled a short while toward the sun,
And left the vivid air signed with their honour.*

From "I think continually of those . . ." by Stephen Spender.
Reprinted by permission from Stephen Spender, *Collected Poems*
1928–1953, Random House, New York, 1955.

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CHAPTER 1

ENVIRONMENTAL SYSTEMS ANALYSIS

The management of environmental problems is a challenging venture. These problems involve land, water, air, and energy resources that significantly affect human activities and attitudes. A major difficulty is that individual parts of environmental problems function together to produce unwanted results. For example, the water pollution associated with a wastewater discharge to a stream is related to many factors: waste sources and properties, waste collection, treatment processes, method and location of discharge, transport of the wastes in the stream, and the effects of the wastes on biota and human use. Each component can be and often is analyzed separately, but a water pollution problem results from the interactions and collective effects of a water pollution *system*.

There are obvious advantages in treating environmental problems as systems. Problems can be considered in their totality, and the most effective points of control can be sought. In a wastewater discharge example this might produce combinations of source reductions, treatment methods, and discharge locations that are more effective and possibly less costly than improved treatment alone. A consequence of a systems perspective on environmental quality is the broadening of possible control options and subsequent opportunities for efficient, integrated management strategies.

Systems are collections of things that function together; the study of these collections is called *systems analysis*. This is a general definition that includes many professional disciplines and applications, from computer science to sociology. Common to many applications involving problem solving is a *systems approach* involving three steps:

1. Definition of the relevant system and objectives.
2. Generation and evaluation of alternatives for meeting the objectives.
3. Selection of an alternative.

These steps are an obvious (if somewhat optimistic) prescription of logic. However, several refinements can lend a unique and powerful character to the approach. The first is an emphasis on quantitative analysis. Although the systems approach can be carried out in a qualitative, descriptive fashion, its most impressive results are produced with numbers. When system components, objectives, and management alternatives are described numerically, it is easier for analysts to communicate their results; the ultimate users of the analyses can interpret problem solutions more readily. Quantification does not eliminate subjectivity, but it discourages vagueness.

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A second refinement follows from the decision to quantify. In any but the very simplest system there are usually many combinations of management options for the various components. The numerical accounting of all resulting system interactions is a tedious bookkeeping process that can be accomplished efficiently by using mathematical models. Models are approximations or abstractions of the actual system and include mathematical descriptions of objectives, component interactions, and management methods. Mathematical models are essentially experimental tools. The analyst can vary parameters of the models and use model outputs as predictions of the performance of the real system. Thus mathematical models help to generate and evaluate rapidly alternative solutions to a problem.

A final refinement to the systems approach is to impose conditions on the types of solutions we wish to obtain from models. These conditions are dictated by efficiency needs. Most mathematical models are capable of evaluating an infinite number of alternatives, and it is clearly desirable to discover alternatives that most closely meet the objectives of the problem being analyzed. Such alternatives can be found through the use of optimization techniques. When these techniques are combined with a mathematical model of a system, an *optimization model* results.

This book is about environmental systems analysis, or the application of the systems approach to environmental management. However, emphasis is on the quantitative aspects of systems analysis, especially the use of optimization models to aid in the development of solutions to environmental problems. This does not imply that other forms of systems analysis are not useful in environmental management nor does it mean that optimization models will always produce ideal solutions to environmental problems. Nevertheless, successful applications of optimization have indicated that the procedures promise to improve the analytical capability of environmental engineers, planners, and scientists.

ELEMENTS OF BENEFIT/COST ANALYSIS

Management of an environmental problem requires objectives or criteria against which alternative solutions can be measured. One general objective is the maximization of net social benefits. The quantification of these benefits in monetary terms is a major interest of benefit/cost analysis. The analysis has two key components, one of which is the idea of resource allocation. The environment (water, air, land, energy) can be viewed as a resource that should be used to improve social welfare. The second component is the concept of social accounting, which requires that the benefits and costs to *all* users of a resource affected by an environmental problem should be determined. Benefit/cost analysis can be illustrated by the water pollution situation in Figure 1-1.¹

A factory is discharging into a river a toxic waste that affects the catch by a downstream commercial fishery. It is assumed that the factory and fishery are the river's

¹This example has its genesis in a comparable problem presented in A. V. Kneese, *The Economics of Regional Water Quality Management*, Johns Hopkins Press, Baltimore, 1964.

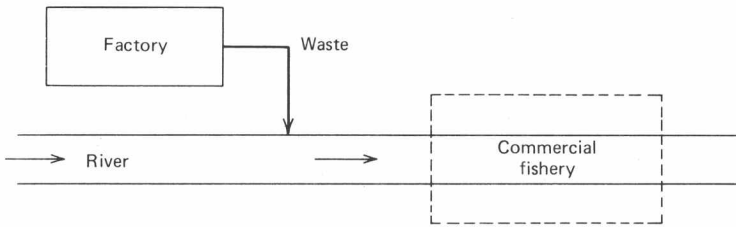


Figure 1-1 Water pollution problem used to illustrate benefit/cost analysis.

only users. The discharge, with its resulting effect, is an example of an economic *externality*, which is defined as a cost or benefit produced by one economic unit that is incurred by other economic units. Thus the factory's waste disposal has an associated cost, but that cost is borne by the fishery, not the factory. The cost is "external" to the factory's accounting of income and costs. Implicit in the concept of externality is the idea of unfairness. It does not seem just that the fishery should pay for the factory's waste disposal. However, justice and fairness are often ambiguous. Consider the example of an estate maintained by a wealthy family near a large metropolitan area. Parts of the estate might consist of forests and rolling fields that lie along public roads. The result for travelers (by car, by bicycle, or on foot) is a scenic rural vista within an otherwise highly developed area. Many, if not most, travelers derive pleasure from this scenery. The fields are an externality in that they produce uncompensated benefits incurred by the traveling public. Fairness would dictate that the estate owners be compensated for the benefits they are providing. This externality may not seem as obvious as the waste discharge, yet it is clearly comparable. In the waste discharge case the environmental resource is being degraded by one party without subsequent compensation to other users. Conversely, the scenic externality involves the uncompensated improvement of an environmental resource by one party that provides benefits for many others.

Returning to the example in Figure 1-1, what are the possible solutions to this problem? The two extreme solutions are to do nothing or to ban the discharge entirely. Although both solutions are defective, they are not without merit. The "do-nothing" approach is based partly on the realization that waste products are an unavoidable consequence of human activity. Something must be done with them, and river discharge is a logical disposal means. Moreover, it has long been known that the environment has an impressive ability to assimilate wastes. For example, given a sufficient supply of oxygen, much of the organic matter in wastes can be fairly rapidly oxidized to carbon dioxide and water by microbes in streams and rivers. Waste assimilation can thus be considered a valuable use of the environmental resource. The difficulty with this approach is that the waste *assimilation capacity* of an environmental resource (in this case, a river) is finite. Excessive waste discharges may not be assimilated, or at least not in a desirable fashion. When the river oxygen is depleted, the river will become a septic, vile-smelling sewer. In this case the organic wastes may still be assimilated (degraded), but most people would not be pleased with the results. Similarly, toxic substances may be assimilated by cycling through aquatic food chains and ultimately destroying fish and other wildlife or endangering human food sources.

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Juxtaposed to the do-nothing solution is the banning of waste discharge, presumably on the grounds that this would return the river to a natural (unpolluted) condition. The merits of this solution are less obvious, but it certainly appeals to real social preferences for cleanliness, pure water, protection of wildlife, and the like. However, banning the waste discharge in this case carries the implicit assumption that commercial fishing is a better or more valuable use of the river than waste assimilation. Of course, fishing is a disturbance of the river's natural condition, and it might also be consistent to ban the fishery. In this case, there would be no users of the environmental resource. Other users, such as people fishing for sport, boaters, and swimmers, may subsequently appear. The new users are similar to the fishery in that they interfere (if only in a minor way) with the river's natural condition. The approach of prohibiting waste discharges leads either to the prevention of all uses of the river or the favoring of certain uses over others. The latter implies a judgment concerning the relative value of uses and leads to a third approach to resolving the water pollution problem, the application of benefit/cost analysis.

The benefit/cost approach is based on the concept that the river is a resource that should be used in the most beneficial way. This requires an examination of the monetary consequences of the waste discharge to both users of the river. Let us assume that the factory is presently incurring no waste disposal cost and that its cheapest alternative to river waste disposal would cost \$50,000. The fishery's current profits are \$10,000, but it is estimated that elimination of the waste discharge would increase catches sufficiently to raise profits to \$30,000. Thus the present (do-nothing) alternative provides the factory with \$50,000 in benefits at a cost of \$20,000 to the fishery, for a net benefit of \$30,000. Conversely, prohibiting the waste discharge eliminates both factory benefits and fishery costs, producing no net benefits. The conclusion of the analysis is that the present use of the river for both waste discharge and fishing is more valuable than its use for fishing alone. Of course, better alternatives may exist. For example, diversion of a portion of the waste from the river for \$10,000 might increase the fishery profits to \$25,000, for a net benefit of $(\$50,000 - 10,000) - (\$30,000 - 25,000) = \$35,000$. Each possible alternative could be evaluated similarly and the alternative yielding the greatest net benefit (most valuable use of the resource) selected.

Benefit/cost analysis is the aggregation of the monetary cost and benefits to all economic units affected by the various solutions to an environmental problem. This is an indication of the value of the environmental resource to society. The indicator is imperfect for many reasons, the most important of which is that it ignores social preferences for the *distribution* of costs and benefits. For example, if the fishery is owned by a native or minority group whose economic development is being encouraged by government action, the loss of \$20,000 profits may be more important than a \$50,000 waste disposal cost incurred by the factory. Clearly, there are social goals that are inadequately accounted for in benefit/cost analysis, and the net benefit criterion is seldom used as an exclusive method for selecting solutions to environmental problems. Nevertheless, it is an essential tool for environmental management, primarily because it forces the explicit accounting, in quantitative (monetary) terms, of the beneficial and adverse effects of environmental pollution. In addition, it requires a determination of the economic impact of pollution control alternatives.

Benefit/cost analysis provides the rationale for *effluent charges*, which are imposed on waste discharges. Effluent charges are in essence a "price" or "user fee" that a waste discharger pays for using the environment for waste assimilation. The charge should be equal to an opportunity cost, or the value of the environmental resource in its most productive alternative use. In the factory/fishery example, this opportunity cost is \$20,000, or the lost profits of the fishery. Note that effluent charges do not necessarily reduce pollution. In the example, the factory would discharge as long as the effluent charge was less than \$50,000. The objective of an effluent charge is to internalize the externalities associated with environmental pollution; that is, the waste discharger is forced to include the off-site costs of resulting pollution in cost and revenue accounting.

There are several other difficulties in applying benefit/cost analysis (with or without effluent charges) to real-world problems. Suppose, for example, that instead of a fishery, the second use of the river is as part of a public park that provides boating, swimming, sport fishing, scenic beauty, and the like. Furthermore, assume that the waste discharge interferes with these uses. In order to apply benefit/cost analysis, the monetary costs (or lost benefits) incurred by the users as a result of the waste discharge must be determined. Since the individual users generally would not maintain a revenue and cost accounting of their recreational activities, the estimation of the river's value to the park users could be exceedingly difficult. Without such an estimate, the net benefits of waste management alternatives could not be computed.

This type of situation can be dealt with by the imposition of an *environmental quality standard*, which is a law or regulation specifying a minimum allowable quality level, in measureable parameters, for an environmental resource. In this case the standard might be a maximum concentration of a toxic chemical in the river at the park. The standard would imply that the factory must reduce toxic waste discharges to the point where the resulting river levels meet the standard. Environmental standards are set by political processes that implicitly consider trade-offs between competing uses of the environmental resource.

Quality standards can also be a mechanism for handling social preferences for benefit and cost distributions. For example, in the factory/fishery situation, if commercial fishing is considered by government to be a favored activity, a water quality standard might be set in terms of parameters affecting fish mortality. Once the standard has been set, the net benefits of alternatives that meet the standard can be determined and the alternative selected that maximizes net benefits. Thus environmental standards can be a means of dealing with the imperfections of benefit/cost analysis.

Environmental quality standards can be implemented when the cause-and-effect relationships between a waste discharge and environmental quality parameters can be determined. This is not always easy, since most potential pollutants undergo physical, chemical, and biological changes when introduced into the environment. If there were many waste discharges to the river upstream of the park, it could be difficult to determine how much each of these discharges should be controlled to meet the quality standard. In this case it might be advisable to impose *effluent standards*, which would limit the amount and characteristics of each waste discharge more or less equally. If these standards are met, water quality presumably will improve, although the improvement is not necessarily obtained in an economically efficient manner. For exam-

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ple, the control of just the major discharges closest to the park might be less expensive than control of all upstream discharges, and water quality improvements may be comparable.

Effluent standards have two major advantages, administrative convenience and equity. The administration of water pollution control programs can be costly and difficult. Effluent standard compliance can be readily monitored by sampling and analysis of waste discharges. Violations are eliminated by additional removal of the offending substances. Although environmental quality standards can also be monitored, violations are not readily corrected, since the relative contribution of each discharge to the violation must be determined. Effluent standards also have an element of fairness to them, particularly when they are set on a uniform basis for each category of waste discharge.

Environmental quality and effluent standards are common environmental management tools. Standards tend to disaggregate benefit/cost analysis. In the case of effluent standards, the decomposition is complete, with each waste discharger able to maximize his or her own net benefits (minimize net costs) of meeting the standard. Application of benefit/cost analysis results in three different types of environmental management problems.

1. Maximization of the total net monetary benefits (minimization of the total monetary costs) to all users of an environmental resource.
2. Maximization of the total net monetary benefits (minimization of the total net monetary costs) to all waste dischargers of meeting an environmental quality standard.
3. Maximization of the total net monetary benefits (minimization of the total net monetary costs) to each discharger of meeting an effluent standard.

Most environmental problems are variations of, or a combination of, types two and three. The basic concepts of benefit/cost analysis are preserved, but constraints (in the form of standards) are imposed on the analysis. Such constraints imply that the values of certain resource uses either cannot be evaluated monetarily or that the monetary values that can be obtained do not reflect social preferences.

As a final note *cost-effectiveness* refers to a search for the least expensive way to meet an objective. The determination of an environmental management alternative that satisfies standards at minimum cost is a cost-effectiveness problem.

AN EXAMPLE OF THE SYSTEMS APPROACH

Although the mathematical modeling of environmental problems is emphasized in this book, we have observed that systems analysis can be applied without resorting to models. The basic ideas of systems analysis are illustrated in the following water

quality problem.² This same problem is analyzed in Chapter 2, using a mathematical model.

EXAMPLE 1-1 Wastewater Management

A metal refining factory has a waste disposal problem. For 1 kg of metal produced, 3 kg of waste are created. The waste is contained in wastewater at a concentration of 2 kg/m³. Wastewater has been discharged to a nearby river with partial treatment. The government has imposed an effluent standard of 100,000 kg/wk on the factory's waste discharge. The factory has a production capacity of 55,000 kg of metal per week. Metal is sold at a price of \$1.30/kg and production costs are \$0.90/kg. The factory's wastewater treatment facility has a capacity of 70,000 m³/wk. However, the facility's efficiency (fraction of waste removed) varies with waste loading. If W is the wastewater inflow to treatment in 10⁴ m³/wk then, for W between 0 and 70,000 m³/wk, treatment efficiency is given by $1 - 0.06W$. Thus the more heavily the plant is loaded, the less efficient it is at removing the waste from the wastewater. Wastewater treatment costs are \$0.20/m³. □

This wastewater management problem, which is illustrated in Figure 1-2, involves the determination of an efficient way for the factory to meet its effluent standard. Although the example is deceptively simple, it has similarities to other environmental pollution problems.

1. Waste output is related to production level. It is a direct function of factory's output of a useful product (refined metal).
2. The waste material or pollutant is flushed from the factory in water. This is known as *wastewater*, the strength of which is measured by its concentration of pollutant (2 kg/m³).

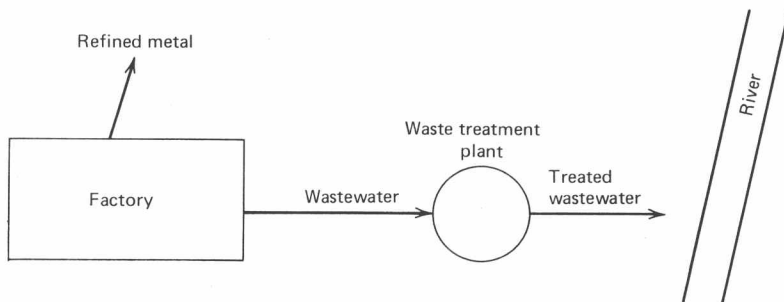


Figure 1-2 Wastewater management example.

²This example was suggested by a comparable problem in M. B., Fiering, J. J. Harrington, and R. J. deLucia, "Water Resources Systems Analysis," Information Canada, Ottawa, 1971.