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Mathematical and Physical Theory of Turbulence

Edited by

John Cannon

Bhimsen Shivamoggi

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John Cannon
University of Central Florida
Orlando, U.S.A.

Bhimsen Shivamoggi
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Mathematical and Physical Theory of Turbulence

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Preface

Turbulence arises in practically all flow situations that occurs naturally or in modern technological systems. The turbulence problem poses many formidable intellectual challenges and occupies a central place in modern nonlinear mathematics and statistical physics. As the great physicist Richard Feynman mentioned, it is the last great unsolved problem of classical physics. The American Mathematical Society has listed the turbulence problem as one of the four top unsolved problems of mathematics. It is also one of the seven problems for which the Clay Institute has recently announced a $1 million prize.

The powerful notions of scaling and universality, which matured when renormalization group theory was applied to critical phenomena, had been manifested in turbulence previously. The recent dynamical system approach has provided several important insights into the turbulence problem. However, deep problems still remain to challenge conventional methodologies and concepts. There is considerable need and opportunity to advance and apply new physical concepts as well as new mathematical modeling and analysis techniques. There is also an ongoing need to bridge the gap between the grand theories of idealized turbulence and the harsh realities of practical applications. The turbulence problem continues to command the attention of physicists, applied mathematicians, and engineers.

Several turbulence research groups in Florida collaborated to hold an international turbulence workshop at the University of Central Florida, May 19–23, 2003. The sponsors of this workshop were: University of Central Florida (Department of Mathematics, College of Arts and Sciences, and Office of Research), Florida State University, Florida A&M University, University of Florida, Embry Riddle Aeronautical University, and area industrialist Dr. Nelson Ying. The idea was to bring together experts from physics, applied mathematics, and engineering working on this common problem and promote the influx of expertise into the subject from all these groups to the benefit of all in understanding complex issues of this problem.

This workshop aimed at a discussion of recent progress and some major unresolved basic issues in three- and two-dimensional turbulence and scalar compressible turbulence:

- Three-dimensional turbulence: theory, experiments, computational and mathematical aspects of Navier–Stokes turbulence
- Two-dimensional turbulence: geophysical flows and laboratory experiments
- Scalar turbulence: theory, modeling, and laboratory experiments
- Compressible/magnetohydrodynamics effects

There was enormous interest in this workshop worldwide. The following leading experts were invited speakers (*overview speakers):

Eberhard Bodenschatz (Cornell)
George Carnevale (Scripps Institution)*
Peter Constantin (University of Chicago)*
Gregory Falkovich (Weizmann Institute)*
Thomas Gatski (NASA Langley)
Sharath Girimaji (Texas A&M)
Marvin Goldstein (NASA Glenn)
Jerry Gollub (University of Pennsylvania)*
Jack Herring (NCAR)
Yukio Kaneda (Nagoya)
Shigeo Kida (Nagoya)
John Krommes (Princeton)
Jacques Lewalle (Syracuse)
Tasos Lyrintzis (Purdue)
David Montgomery (Dartmouth)
Sutanu Sarkar (University of California, San Diego)
Eran Sharon (University of Texas, Austin)
Siva Thangam (Stevens Institute)
Edriss Titi (University of California, Irvine)
Zelman Warhaft (Cornell)
Victor Yakhot (Boston University)*

Professor Peter Hilton, Distinguished Professor of Mathematics, University of Central Florida, was the banquet speaker.

The workshop program contained introductory overviews, specialist talks, and contributed talks, as well as panel discussions (one every other day). The workshop was a highly stimulating and enjoyable event and a great success. The purpose of these workshop proceedings is to distribute these overviews and contributed talks for the benefit of the turbulence research community at large.

The invited talks included in these proceedings are as follows: The invited talk by Gregory Falkovich provides an overview of the Lagrangian description of turbulence in general and the scalar diffusion problem in particular. David Montgomery provides an account of recent results in decaying two-dimensional turbulence via the entropy maximization approach. The discussion by Jack Herring describes the decay mechanism involving phase mixing of gravity waves in stratified turbulence. Jacques Lewalle describes application of wavelet scaling to investigate the regularity of Navier–Stokes equations. The invited talk by John Krommes provides an overview of some aspects of statistical theories of turbulence in strongly magnetized plasmas. Françoise Bataille discusses a general framework to develop eddy viscosity models for turbulent flows which do not exhibit the Kolmogorov energy spectrum. The invited talk by Siva Thangam provides an overview of the development of continuous turbulence models that are suitable for large-eddy simulation and Reynolds averaged Navier–Stokes formulation. The banquet talk by Peter Hilton provides fascinating personal reminiscences and anecdotes about three great mathematicians of the twentieth century — Alan Turing, Henry Whitehead, and Jean-Pierre Serre.

The contributed talks included in these proceedings are as follows: The contributed talk by Jacques Lewalle discusses the use of wavelets to describe several aspects of Navier–Stokes turbulence which are not well handled by traditional approaches. Mikhail Shvartsman discusses a connection between the governing equations, the constitutive theory, and the closure problem for the atmospheric boundary layer. The contributed talk by Peter Davidson discusses the significance of the law of conservation of angular momentum for freely evolving, homogeneous turbulence. Ekachai Juntasaro discusses a new concept of turbulence modeling in turbulent channel flow and turbulent boundary-layer flow.

John R. Cannon
Bhimsen K. Shivamoggi

The Editors

John Cannon completed his B.A. degree in mathematics at Lamar University in 1958. He attended graduate school at Rice University to study mathematics and completed his M.S. degree in 1960 and his Ph.D. degree in 1962. His research expertise is in partial differential equations and numerical analysis with applications to heat flow, fluid dynamics, chemical reactions, change of phase, flow in porous media, and inverse problems, as well as applications to biology and medicine. He has held faculty and research positions at Brookhaven National Laboratory, Università di Genova, Purdue University, University of Minnesota, The University of Texas at Austin, Università di Firenze, Colorado State University, the Hahn Meitner Institute in Berlin, Washington State University, The Office of Naval Research, Lamar University, and the University of Central Florida. He is the author of 2 books, 150+ refereed journal articles, 26 proceedings articles, and 5 technical reports.

Bhimsen Shivamoggi received his Ph.D. from the University of Colorado, Boulder. Following postdoctoral research appointments at Princeton University and the Australian National University, Canberra, he joined the mathematics faculty at the University of Central Florida, Orlando in 1984. He has worked on problems of stability and turbulence in fluids and plasmas. He has written several books on these topics and has held visiting appointments in Los Alamos National Laboratory, University of California at Santa Barbara, Technische Hochschule Darmstadt (Germany), Technische Universiteit Eindhoven (The Netherlands), Observatoire de Nice (France), Kyoto University (Japan), International Center for Theoretical Physics at Trieste (Italy), and University of Newcastle (Australia).

Contributors

F. Bataille
Centre de Thermique de Lyon
Villeurbanne Cedex, France

G. Brillant
Centre de Thermique de Lyon
Villeurbanne Cedex, France

George F. Carnevale
Scripps Institution of Oceanography
University of California San Diego
La Jolla, California

J. Clyne
National Center for Atmospheric Research
Boulder, Colorado

P. A. Davidson
Department of Engineering
University of Cambridge
Cambridge, United Kingdom

Douglas P. Dokken
Department of Mathematics
University of St. Thomas
St. Paul, Minnesota

Gregory Falkovich
Weizmann Institute of Science
Rehovot, Israel

J. R. Herring
National Center for Atmospheric Research
Boulder, Colorado

Peter Hilton
Department of Mathematical Sciences
State University of New York
Binghamton, New York
and
Department of Mathematics
University of Central Florida
Orlando, Florida

M. Yousuff Hussaini
Department of Mathematics and Computational Science and Engineering
Florida State University
Tallahassee, Florida

R. James
National Center for Atmospheric Research
Boulder, Colorado

Ekachai Juntasaro
School of Mechanical Engineering
Suranaree University of Technology
Nakhon Ratchasima, Thailand

Varangrat Juntasaro
Department of Mechanical Engineering
Kasetsart University
Bangkok, Thailand

Y. Kimura
Grade School of Mathematics
Nagoya University
Nagoya, Japan

J. A. Krommes
Plasma Physics Laboratory
Princeton University
Princeton, New Jersey

Jacques Lewalle
Department of Mechanical Engineering
Syracuse University
Syracuse, New York

David C. Montgomery
Department of Physics and Astronomy
Dartmouth College
Hanover, New Hampshire

Mikhail M. Shvartsman
Department of Mathematics
University of St. Thomas
St. Paul, Minnesota

Siva Thangum
Department of Mechanical Engineering
Stevens Institute of Technology
Hoboken, New Jersey

Stephen L. Woodruff
Center for Advanced Power Systems
Florida State University
Tallahassee, Florida

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1 A Mathematician Reflects: Banquet Remarks

Peter Hilton

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First, let me thank you very much for inviting me to participate in your conference and for giving me this opportunity to say a few informal words to you following an excellent dinner.

My area of research intersects yours precisely in our recognition of the importance of a good number system and a good notation, and our use of the same number system. Indeed, as I look back on my own career, my closest contact with turbulence — and this was, as in the case of this conference, a matter of international turbulence — was during World War II, when I was conscripted to work, from January 1942 until the end of the war in Europe in May 1945 at Bletchley Park, breaking the highest-grade German ciphers used for diplomatic and military traffic passing among the German government, the German High Command, and their naval, air force, and army commanders and U-boat captains. Those were indeed turbulent days, and I will say something about them in my remarks this evening. More generally, I will reflect on the wonderful mathematicians I have known during a career spanning more than 60 years, starting in British Military Intelligence, proceeding along a more conventional academic route, and continuing today, though at a gentler pace consistent with my growing maturity.

1.1 ALAN TURING

Our work at Bletchley Park, cracking the high-grade German codes, was wonderfully exciting, stimulating us to work tirelessly, over long stretches of time, buoyed by the intellectual challenge and our awareness of the importance of what we were achieving. However, for me, as a very young man — I started work at BP (as we called it) at the age of 18 — there was the thrill of working with some of the greatest British mathematicians of that period, getting to know them well and enjoying their friendship and collegiality. They were absolutely free of any outward awareness of their intellectual superiority and treated their younger colleagues as equals — which, as cryptanalysts but not as mathematicians, of course — we were! They seemed not to be aware that, however adept we might be at applying mathematical thinking and perhaps certain very specialized linguistic skills to the job in hand, we had no mathematical knowledge and experience to match theirs.

One lesson I learned from my experience at BP that I would like to share with you is this: to be able to apply mathematical reasoning to problems that are not intrinsically mathematical — in other words, to be able to apply mathematics — the essential prerequisites are a first-class mathematics education and strong interest in and incentive to solve the problem. Experience and familiarity with some scientific discipline, although desirable, are not essential. It is an interesting and, I believe, important fact that, in the group of mathematicians and young would-be mathematicians working on the most sophisticated ciphers used by the Germans, not one was an "applied" mathematician; all[1] were uncompromisingly "pure."

Within this group was one absolutely outstanding thinker: Alan Turing was unmistakably a genius. Alan's role was different from that of the other members of the team. He played no part in the day-to-day decryption of enemy signals; he was concerned with fundamental questions of cryptanalytical method, especially but not exclusively with the design of high-speed machines to expedite the deciphering process.

In 1948, it was my great good fortune to be appointed to a junior position in the mathematics department of the University of Manchester at the same time at which Alan was appointed[2] to a readership in mathematics, so I could continue to take advantage of the unique privilege of having Alan as colleague and friend. As I soon realized, Alan Turing had had in mind all along the possibility of building an *actual* machine to realize the concept of a "Turing machine"; this formed the basis of his seminal prewar paper, "On Computable Numbers." Of course, throughout the war and in the years immediately following, he was inventing the electronic computer. The realization of this dream in no sense interfered with his enormous success in facilitating the breaking of the German codes.

Unfortunately, many people today have the wrong impression of Alan Turing's personality and character. This misconception arises in part from a popular but quite inaccurate stereotype of genius, especially mathematical genius: such a person is thought to be a very narrow specialist, totally unable to deal with the usual demands of life.[3] A further source of confusion with regard to Alan Turing is the fine play by Hugh Whitemore, *Breaking the Code*, which was, in fact, a work of brilliant imaginative fiction inspired by Turing's life and the tragedy of his early death. Let me say here that Turing was an inspiring colleague and friend, a wonderful source of ideas, very approachable, and very versatile. In the very early 1950s, his picture of the future social impact of the computer was extraordinarily prophetic.

Let me also add, for the benefit of those who do not know the sad story, that Alan Turing committed suicide in 1954, just short of his 42nd birthday. Two years earlier, he had been bound over (i.e., convicted without penalty) on a charge of having a homosexual affair with a consenting adult — at that time a criminal offense — as a result of which he had to undertake not to repeat the "crime" and also lost his security clearance and became ineligible for a U.S. visa. What an appalling way to treat a man who not only was a genius but also had made a contribution of incalculable importance to the winning of World War II.

I benefited enormously from having been at BP. I had become friendly at a very early age with some of the greatest mathematicians of the century, and these friendships were to stand me in good stead for many years. Let me tell you about one such friend, Henry Whitehead, my friend and colleague at BP and for many more years afterwards and my research advisor at Oxford after the war.

1.2 HENRY WHITEHEAD

Henry, or — to give him his full name — John Henry Constantine Whitehead, was the son of the bishop of Madras and nephew of the famous philosopher Alfred North Whitehead. It was said that, after an academically undistinguished period as an undergraduate at Balliol College, Oxford, Henry took his bachelor's degree, went into the City (of London), made and lost a fortune in 2 years, and then realized he wanted to be a mathematician. He took a doctorate at Princeton University under the direction of the eminent geometer Oswald Veblen (who coined the term "analysis situs" for what has come to be called topology) and returned to Oxford as a fellow of Balliol College. There it soon

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[1] One, Jack Good, did become a statistician, but a theoretical statistician. I would rather describe him as a probabilist.

[2] It is relevant to mention here that the head of department at Manchester University was Professor Max Newman, who had headed the section at BP concerned with the machine decryption of German ciphers.

[3] The recent biography of Paul Erdös, *The Man Who Loved Only Numbers*, epitomizes this viewpoint.

became clear that he was one of the subtlest and most powerful of contemporary mathematicians, combining profound geometric intuition with a masterful capacity to employ and develop deep algebraic methods in the solution of geometric — or, rather, topological — problems.

I was fortunate to win an open scholarship in mathematics to Queen's College, Oxford, and started my undergraduate studies there in the fall of 1940. I met Henry soon after that, taking a course from him in projective geometry. The number of students taking this course rapidly decreased to four (all of whom subsequently became mathematicians); Henry's lectures were by no means easy to follow. However, we survivors recognized that he was not so much lecturing as thinking out loud, and that we were actually enjoying the privilege of watching how a great mathematician thought out his mathematical ideas. It was stimulating; it was awesome.

After four terms at Oxford I had to leave, in January 1942, to begin my service in British Military Intelligence at BP. To my delight, Henry arrived to join our team in the middle of 1943. Thus, the extraordinary situation came about in which I found myself, a mere mathematics student, teaching our cryptanalytic methods to perhaps the greatest British mathematician of his generation.

For me, the situation was even more remarkable because Henry Whitehead and I became firm friends! We shared many common interests outside mathematics: cricket, squash, racquets, politics, beer-drinking. Indeed, the irony is that the only significant interest we did *not* share was algebraic topology, Henry's areas of research, because I knew nothing about it. In 1945, Henry left BP to take up his new position as Waynflete Professor of Pure Mathematics in the University of Oxford. I, on the other hand, was far younger and had to spend another year in military service. In fact, I spent it very productively at the Post Office Engineering Research Station at Dollis Hill, London, where the Colossi, forerunners of the electronic computer and of crucial importance to us at BP, were built.

During that year, Henry invited me to spend a weekend with him and his family at their home in North Oxford. Over that weekend, he invited me to return to Oxford on my release from war service to work on my doctorate under his tutelage and to live in his home as a member of the family. It was a marvelous offer, but I did, briefly, hesitate. "But, Henry," I said, "I don't know what algebraic topology is." "Oh, don't worry about that," he responded, "you'll love it." Thus should the big decisions in one's life be made: on the basis of complete trust in the judgment of a very knowledgeable and discerning friend.

Henry was right, and I never looked back. I enjoyed the enormous privilege of living with him and his family almost as if I were a member of the family. In fact, they all called me "uncle," including Henry and his lovely wife, Barbara, and I remained a close family friend until Henry's sudden and untimely death while on leave of absence in Princeton in 1960.

1.3 JEAN-PIERRE SERRE

The third mathematician about whom I would like to speak is the great French mathematician J.-P. Serre, whom I got to know at the start of his distinguished career in 1952 when he was 25 years old. At that time I was a lecturer in mathematics at the University of Manchester, England, in the department headed by my wartime BP boss, Max Newman. Max Newman had built up a remarkably strong department, full of outstanding research mathematicians, but he always insisted on the great importance of good teaching and, to a slightly lesser extent, of fair examining.

Max encouraged us to invite outstanding mathematicians who were doing research of great importance to visit our campus so that we could learn from them and be stimulated in our research. Thus it was that I got to know of Serre's outstanding research in homotopy theory; his thesis, completed and accepted in 1951, was published (in French) in *Annals of Mathematics* at the end of that year. I am very proud to have been the first mathematician from outside France to have invited Serre to travel abroad to talk about his work.

I think it would require at least a semester-long course of lectures, rather than an after-dinner talk, to try to explain Serre's astonishing contribution in my field of research to a group of experts in a totally different field. Suffice it to say that he introduced into my field totally new methods that enabled him to answer fundamental questions that, prior to his work, it would have been pointless and frustrating, to ask. Our subject was never the same again.

Serre stayed with my wife and me during this visit and was a delightful guest, full of fun. We went for walks in the countryside and he proved to be very athletic, running and leaping over stiles and five-barred gates. Following his visit, we corresponded a lot, almost exclusively about mathematics. Then he invited me to visit Paris and stay with him and his wife. It was an astonishing experience. He was a tremendously hard worker and, after several hours spent discussing mathematics, he would propose we go to the Institut Henri Poincaré — to play ping-pong!

The visit exhausted and stimulated me. It led to what was probably the deepest and most important paper I have ever written.[4] I recall that I offered Serre joint authorship, but he declined, saying, very characteristically, that he was not yet old enough for such an honor.

Our correspondence continued for some time after my visit; at one point, I felt we had become so friendly that I asked him to call me "Peter." Such a step was a very significant one for a Frenchman in the 1950s, but Serre readily agreed, adding, "A titre de reciprocité, supprimez le Monsieur" ("By way of return, cut out the Mr"). Thus, he wrote "Cher Peter" and I wrote "Dear Serre." Of course, today, his close colleagues call him "Jean-Pierre."

It was a privilege to know this great mathematician and wonderful to get to understand his marvelous mathematical ideas. I am happy to be able to tell you that, some time after I decided to feature Jean-Pierre Serre as one of the most outstanding mathematicians I have known, he was selected by the Norwegian government as the first recipient of the recently instituted Abel Prize, designed to carry a very substantial financial award as well as a prestige entirely comparable with that of the Nobel prizes.[5]

1.4 EPILOGUE

I could go on to tell you of other great mathematicians I have known over a career spanning more than 60 years and of the fascinating visits I have made to mathematical centers all over the world.[6] However, I am well aware that an after-dinner talk should not be too long or too intellectually demanding. Thus, I will close with a few remarks about what I have learned over the years about doing and teaching mathematics.

First, I would like to echo Henry Whitehead in declaring my belief that mathematics is a first-class occupation. When trying to persuade a young colleague to remain a mathematician, Henry declared, "It is better to be second rate in a first-rate occupation than first rate in a second-rate occupation." This, I believe, is profoundly true, though it is also true that Henry was not faced with the agonizing choice confronting the young man seeking his advice.

Second, I wish to reject utterly the viewpoint that there is an unavoidable conflict in any university mathematics department between choosing good researchers and choosing good teachers. I argue that one should always try to choose the best mathematicians, for to do otherwise is to miss an opportunity and to risk a gradual descent into mediocrity. Of course, we must always insist on the importance of good teaching; however, the simple fact is that we know how to produce good researchers who love mathematics, but we do not know how to produce good teachers. Moreover, whereas the criteria of good mathematical research are generally agreed and judgments of the quality of an individual's research are fairly unanimous, the same is by no means the case when it comes to considerations of good teaching.

All we can say with virtual certainty is that, if someone is to be a good teacher of mathematics, that person must love mathematics; typically, that person will have demonstrated that love of mathematics

[4] A colleague was recently kind enough to describe it as a "landmark paper."
[5] There is no Nobel Prize in mathematics and there are several interesting theories as to why this is so!
[6] As I write this, I am looking forward to a visit to Zürich, Switzerland, next week to give a lecture to mark my 80th birthday.