

# STATISTICAL METHODS

for

## ENGINEERS and SCIENTISTS

*SECOND EDITION, REVISED AND EXPANDED*

ROBERT M. BETHEA  
BENJAMIN S. DURAN  
THOMAS L. BOULLION

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# **Statistical Methods for Engineers and Scientists**

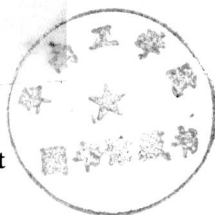
**Second Edition, Revised and Expanded**



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In gratitude and affection for their patience,  
understanding, and assistance during the preparation  
of our text, we dedicate this book to Nancy Jo, Do-  
lores, and Nancy.

## PREFACE TO THE SECOND EDITION

We have used the first edition of this book as a text many times. Based on this experience and the comments made by our students, colleagues, and others, we have revised and updated (in terms of computer usage) the text to make it a better instrument for classroom instruction or self-study. Our suggestions for course arrangements remain as stated in the Preface to the First Edition. This text is also useful for professional engineers and scientists especially because of its emphasis on regression, experimental design, analysis of variance, and computerized data reduction and interpretation.

The major changes include an approximately forty percent increase in the number of homework problems, incorporation of the Statistical Analysis System (SAS\*) programs throughout, addition of several topics, and a reorganization of several chapters.

Chapters 1-7 remain essentially the same as in the first edition except for the inclusion of more homework problems and blending in the use of the SAS System in several worked examples. The chi-square goodness-of-fit test has been added in Chapter 7.

The major reorganization has taken place in Chapters 8, 9, and 11. Some material, including the three-way analysis of variance, has been moved from Chapter 8 to Chapter 11. A section dealing with multiple comparisons has been added in Chapter 8. Several changes have been made in Chapter 9, including an extension of the nonlinear regression section. Several topics have been

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\* SAS is the registered trademark of SAS Institute Inc., Cary, North Carolina 27511-8000.

added in Chapter 11, including analysis of covariance and nested designs. Numerous examples, worked by the use of the SAS System programs, which are included, have been woven into these chapters. In addition to the major revisions, slight changes have been made throughout the text to improve readability and presentation.

We wish to thank those persons, societies, publishers, and organizations listed in the Preface to the First Edition who have graciously extended their permissions to reproduce and use data for examples and homework problems in this edition of our text. To that list, we wish to add Imperial Chemical Industries PLC; Butterworth's; John Wiley & Sons, Inc.; and the National Academy of Sciences. All of the above courtesies are acknowledged in the appropriate locations in the text. We are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S. to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London for permission to reprint Table XXIII from their book Statistical Tables for Biological, Agricultural and Medical Research (6th Edition, 1974).

We are grateful to SAS Institute Inc. for permission to use and reproduce the SAS programs and other material which are their property. We appreciate the comments we have received on the first edition from many colleagues, reviewers, and numerous students. We have incorporated many of those suggestions in this revision. We thank those students and others who have allowed us to use data from research and laboratory experiments for examples and problems. We especially wish to thank Sue Willis for typing all the revisions and the editorial staff who have been so helpful.

It goes without saying that without the patience and understanding of our families, this revision would never have been completed.

Robert M. Bethea  
Benjamin S. Duran  
Thomas L. Boullion

## PREFACE TO THE FIRST EDITION

This book is intended as a basic introductory text in applied statistical techniques for undergraduate students in engineering and the physical sciences. Theoretical developments and mathematical treatment of the principles involved are included as needed for understanding of the validity of the techniques presented. More intensive treatment of these subjects should be obtained, if desired, from the many theoretical statistics texts available.

The material in this text can be arranged for either two three-credit quarter courses or one three-credit semester course at the option of the instructor. For the former case, the material would be covered in toto with additional time allowed throughout for applications in the disciplines of the students involved. The most logical place for division is at the end of Chapter 7 on statistical inference. For the latter case the material in the first four chapters is presented in three weeks. Chapter 5 is allotted one week; Chapters 6 through 8, two weeks each; Chapters 9 and 10 are covered in four weeks total; and Chapter 11, three weeks. Instructors desiring only one three-credit quarter course in methodology would probably use only Chapters 6 through 9 provided the students have been introduced to basics.

Although many of the example problems are oriented toward chemical engineering and chemistry, by no means is this text limited to those areas. The examples merely illustrate general statistical principles as applicable to all fields, but particularly

engineering and the related physical sciences. The problems at the end of each chapter are graded in difficulty for the convenience of the instructor. They are arranged to logically follow the outline of material presented in the text.

Our deepest appreciation is extended to our good friends and former departmental chairmen, Arnold J. Gully and Patrick L. Odell, for their encouragement, constructive criticism of all parts of this text, and above all for allowing us adequate time for the preparation of the manuscript. Appreciation is also extended to James M. Davenport for his assistance in the tabulation of the F-distribution.

We are indebted to the many students and associates who have contributed to this text by way of suggestions for improvements, problem statements, experimental data, and consultation. We are grateful to Dr. H. T. David and Dr. D. V. Huntsberger of the Department of Statistics at Iowa State University and Dr. F. B. Cady now of the Statistics Department of the University of Kentucky for permission to use as illustrative examples and/or homework problems some of the problems they assigned one of us (RMB) in their respective classes or on preliminary examinations. We are indebted to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Oliver and Boyd, Ltd., Edinburgh, for permission to reprint Table XXIII from their book, Statistical Tables for Biological, Agricultural and Medical Research.

In addition to these, our thanks go to The Iowa State University Press, Addison-Wesley Publishing Co., Prentice-Hall, Inc., and Gulf Publishing Co. for permission to reprint data for use in examples and problems. Thanks are also due to the editors of the American Institute of Chemical Engineers, the Committee of Editors of the American Chemical Society, the American Society for Quality Control, the American Society for Testing and

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Benjamin S. Duran  
Thomas L. Boullion

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