J. S. Bell

Speakable and Unspeakable in Quantum Mechanics

SECOND EDITION

With a new Introduction by Alain Aspect

CAMBRIDGE

Collected papers on quantum philosophy

Speakable and Unspeakable in Quantum Mechanics

J. S. BELL
CERN

With an Introduction by Alain Aspect

CAMBRIDGE
UNIVERSITY PRESS

Speakable and Unspeakable in Quantum Mechanics

John Bell FRS was one of the leading expositors and interpreters of modern quantum theory. He is particularly famous for his discovery of the crucial difference between the predictions of conventional quantum mechanics and the implications of local causality, a concept insisted on by Einstein. John Bell's work has played a major role in the development of our current understanding of the profound nature of quantum concepts and of the fundamental limitations they impose on the applicability of the classical ideas of space, time, and locality.

This book includes all of John Bell's published and unpublished papers on the conceptual and philosophical problems of quantum mechanics, including two papers that appeared after the first edition was published. All the papers have been reset, the references put in order and minor corrections made. The book includes a short preface written by the author for the first edition, and also an introduction by Alain Aspect that puts into context John Bell's enormous contribution to the quantum philosophy debate.

This collection will be of interest to graduate students and research workers in physics with an interest in the conceptual foundations of quantum theory. It will also be of value to philosophers of science working in this area.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 1987
First paperback edition 1988
Reprinted 1989 (twice), 1991, 1993, 1994, 1997

Revised edition first published 2004

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times New Roman 10/13pt. *System* QuarkXPress® [UPH]

A catalog record for this book is available from the British Library

ISBN 0 521 81862 1 hardback
ISBN 0 521 52338 9 paperback

To my Mother and Father

J.S. Bell: Papers on quantum philosophy

On the problem of hidden variables in quantum mechanics. *Reviews of Modern Physics* **38** (1966) 447–52.

On the Einstein–Podolsky–Rosen paradox. *Physics* **1** (1964) 195–200.

The moral aspect of quantum mechanics. (with M. Nauenberg) In *Preludes in Theoretical Physics*, edited by A. De Shalit, H. Feshbach, and L. Van Hove. North Holland, Amsterdam, (1966) pp 279–86.

Introduction to the hidden-variable question. *Foundations of Quantum Mechanics*. Proceedings of the International School of Physics 'Enrico Fermi', course IL, New York, Academic (1971) pp 171–81.

On the hypothesis that the Schrödinger equation is exact. TH-1424-CERN October 27, 1971. Contribution to the International Colloquium on Issues in Contemporary Physics and Philosophy of Science, and their Relevance for our Society, Penn State University, September 1971. Reproduced in *Epistemological Letters*, July 1978, pp 1–28, and here in revised form as 15. Omitted.

Subject and Object. In *The Physicist's Conception of Nature* Dordrecht-Holland, D. Reidel (1973) pp 687–90.

On wave packet reduction in the Coleman–Hepp model. *Helvetica Physica Acta* **48** (1975) 93–8.

The theory of local beables. TH-2053-CERN, 1975 July 28. Presented at the Sixth GIFT Seminar, Jaca, 2–7 June 1975, and reproduced in *Epistemological Letters*, March 1976.

Locality in quantum mechanics: reply to critics. *Epistemological Letters*, Nov. 1975, pp 2–6.

How to teach special relativity. *Progress in Scientific Culture*, Vol 1, No 2, summer 1976.

Einstein–Podolsky–Rosen experiments. *Proceedings of the Symposium on Frontier Problems in High Energy Physics*, Pisa, June 1976, pp 33–45.

The measurement theory of Everett and de Broglie's pilot wave. In *Quantum Mechanics, Determinism, Causality, and Particles*, edited by M. Flato *et al.* Dordrecht-Holland, D. Reidel, (1976) pp 11–17.

Free variables and local causality. *Epistemological Letters*, Feb. 1977.

Atomic-cascade photons and quantum-mechanical nonlocality. *Comments on Atomic and Molecular Physics* **9** (1980) pp 121–6. Invited talk at the Conference of the European Group for Atomic spectroscopy, Orsay-Paris, 10–13 July, 1979.

de Broglie-Bohm, delayed-choice double-slit experiment, and density matrix. *International Journal of Quantum Chemistry*: Quantum Chemistry Symposium 14 (1980) 155–9.

Quantum mechanics for cosmologists. In *Quantum Gravity* 2, editors C. Isham, R. Penrose, and D Sciama. Oxford, Clarendon Press (1981) pp 611–37. Revised version of 'On the hypothesis that the Schrödinger equation is exact' (see above).

Bertlmann's socks and the nature of reality. *Journal de Physique*, Colloque C2, suppl. au numero 3, Tome 42 (1981) pp C2 41–61.

On the impossible pilot wave. *Foundations of Physics* **12** (1982) pp 989–99.

Speakable and unspeakable in quantum mechanics. Introduction remarks at Naples–Amalfi meeting, May 7, 1984.

Quantum field theory without observers. Talk at Naples–Amalfi meeting, May 11, 1984. (Preliminary version of 'Beables for quantum field theory'.) Omitted.

Beables for quantum field theory. 1984 Aug 2, CERN-TH. 4035/84.

Six possible worlds of quantum mechanics. *Proceedings of the Nobel Symposium 65: Possible Worlds in Arts and Sciences*. Stockholm, August 11–15, 1986.

EPR correlations and EPW distributions. In *New Techniques and Ideas in Quantum Measurement Theory* (1986). New York Academy of Sciences.

Are there quantum jumps? In *Schrödinger. Centenary of a polymath* (1987). Cambridge University Press.

Against 'measurement'. In *62 Years of Uncertainty: Erice, 5–14 August* 1989. Plenum Publishers.

La nouvelle cuisine. In *Between Science and Technology*, editors A. Sarlemijn and P. Kroes. Elsevier Science Publishers (1990).

Preface to the first edition

Simon Capelin, of Cambridge University Press, suggested that I send him my papers on quantum philosophy and let him make them into a book. I have done so. The papers, from the years 1964–1986, are presented here in the order, as far as I now can tell, in which they were written. But of course that is not the order, if any, in which they should be read.

Papers 18 and 20, 'Speakable and unspeakable in quantum mechanics' and 'Six possible worlds of quantum mechanics', are nontechnical introductions to the subject. They are meant to be intelligible to nonphysicists. So also is most of paper 16, 'Bertlmann's socks and the nature of reality', which is concerned with the problem of apparent action at a distance.

For those who know something of quantum formalism, paper 3, 'The moral aspect of quantum mechanics', introduces the infamous 'measurement problem'. I thank Michael Nauenberg, who was co-author of that paper, for permission to include it here. At about the same level, paper 17, 'On the impossible pilot wave', begins the discussion of 'hidden variables', and of related 'impossibility' proofs.

More elaborate discussions of the 'measurement problem' are given in paper 6, 'On wavepacket reduction in the Coleman–Hepp model', and in 15, 'Quantum mechanics for cosmologists'. These show my conviction that, despite numerous solutions of the problem 'for all practical purposes', a problem of principle remains. It is that of locating precisely the boundary between what must be described by wavy quantum states on the one hand, and in Bohr's 'classical terms' on the other. The elimination of this shifty boundary has for me always been the main attraction of the 'pilot-wave' picture.

Of course, despite the unspeakable 'impossibility proofs', the pilot-wave picture of de Broglie and Bohm exists. Moreover, in my opinion, all students should be introduced to it, for it encourages flexibility and precision of thought. In particular, it illustrates very explicitly Bohr's insight that the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems to me that full appreciation of this would have aborted most of the 'impossibility proofs', and most of 'quantum logic'. Papers 1 and 4, as well as 17, dispose of 'impossibility proofs'. More constructive expositions of various aspects of the pilot-wave picture are contained in papers 1, 4, 11, 14, 15, 17, and 19. Most of this is for nonrelativistic quantum mechanics, but the last paper, 19, 'Beables for quantum field theory', discusses relativistic extensions. While the usual predictions are obtained for experimental tests of special relativity, it is lamented that a preferred frame of reference is involved behind the phenomena. In this connection one paper, 9, 'How to teach special relativity', has been included although it has no particular reference to quantum mechanics. I think that it may be helpful as regards the preferred frame, at the fundamental level, in 19. Many students never realize, it seems to me, that this primitive attitude, admitting a special system of reference which is experimentally inaccessible, is consistent... if unsophisticated.

Any study of the pilot-wave theory, when more than one particle is considered, leads quickly to the question of action at a distance, or 'nonlocality', and the Einstein–Podolsky–Rosen correlations. This is considered briefly in several of the papers already mentioned, and is the main concern of most of the others. On this question I suggest that even quantum experts might begin with 16, 'Bertlmann's socks and the nature of reality', not skipping the slightly more technical material at the end. Seeing again what I have written on the locality business, I regret never having written up the version of the locality inequality theorem that I have been mostly using in talks on this subject in recent years. But the reader can easily reconstruct that. It begins by emphasizing the need for the concept 'local beable', along the lines of the introduction to 7. (If local causality in some theory is to be examined, then one must decide which of the many mathematical entities that appear are supposed to be real, and really here rather than there). Then the simpler locality condition appended to 21 is formulated (rather than the more elaborate condition of 7). With an argument modelled on that of 7 the factorization of the probability distribution again follows. The Clauser–Holt–Horne–Shimony inequality is then obtained as at the end of 16.

My attitude to the Everett–de Witt 'many world' interpretation, a rather negative one, is set out in paper 11, 'The measurement theory of Everett and de Broglie's pilot wave', and in 15, 'Quantum mechanics for cosmologists'. There are also some remarks in paper 20.

There is much overlap between the papers. But the fond author can see something distinctive in each. I could bring myself to omit only a couple which were used again later with slight modifications. The later versions are included as 15 and 19.

For reproduction here, some trivial slips have been corrected, and references to preprints have been replaced by references to publications where possible.

In the individual papers I have thanked many colleagues for their help. But I here renew very especially my warm thanks to Mary Bell. When I look through these papers again I see her everywhere.

J. S. Bell, Geneva, March, 1987.

Acknowledgements

1 On the problem of hidden variables in quantum theory. *Rev. Mod. Phys.* **38** (1966) 447–52. Reprinted by permission of The American Physical Society.

2 On the Einstein–Podolsky–Rosen paradox. *Physics* **1** (1964) 195–200. Reprinted by permission of The American Physical Society.

3 The moral aspect of quantum mechanics. (with M. Nauenberg) In *Preludes in Theoretical Physics*, edited by A. De Shalit, H. Feshbach, and L. Van Hove, North Holland, Amsterdam (1966) 279–86. Reprinted by permission of North-Holland Physics Publishing, Amsterdam.

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16 Bertlmann's socks and the nature of reality. *Journal de Physique*, Colloque C2, suppl. au numero 3, Tome 42 (1981) C2 41–61. Reprinted by permission of Les Editions de Physique.

17 On the impossible pilot wave. *Foundations of Physics* **12** (1982) 989–99. Reprinted by permission of Plenum Publishing Corporation.

18 Beables for quantum field theory. 1984 Aug 2, CERN-TH.4035/84. Reprinted by permission of Routledge & Kegan Paul.

19 Six possible worlds of quantum mechanics. *Proceedings of the Noble Symposium 65: Possible Worlds in Arts and Sciences.* Stockholm, August 11–15, 1986, edited by Sture Allén. Reprinted by permission of The Nobel Foundation.

20 EPR correlations and EPW distributions. *In New Techniques and Ideas in Quantum Measurement Theory* (1986). Reprinted by permission of the New York Academy of Sciences.

23 Against 'measurement'. In 62 years of Uncertainty: Erice 5–14 August 1989. Reprinted by permission of Plenum Publishing Corporation.

Introduction: John Bell and the second quantum revolution

Alain Aspect

1 The quantum revolutions: from concepts to technology

The development of quantum mechanics in the beginning of the twentieth century was a unique intellectual adventure, which obliged scientists and philosophers to change radically the concepts they used to describe the world[1]. After these heroic efforts, it became possible to understand the stability of matter, the mechanical and thermal properties of materials, the interaction between radiation and matter, and many other properties of the microscopic world that had been impossible to understand with classical physics. A few decades later, that *conceptual revolution* enabled a *technological revolution*, at the root of our information-based society. It is indeed with the quantum mechanical understanding of the structure and properties of matter that physicists and engineers were able to invent and develop the transistor and the laser – two key technologies that now permit the high-bandwidth circulation of information, as well as many other scientific and commercial applications.

After such an accumulation of conceptual – and eventually technological – successes, one might think that by 1960 all the interesting questions about quantum mechanics had been raised and answered. However, in his now-famous paper of 1964[2] – one of the most remarkable papers in the history of physics – John Bell drew the attention of physicists to the extraordinary features of entanglement: quantum mechanics describes a pair of entangled objects as a single global quantum system, impossible to be thought of as two individual objects, even if the two components are far apart. John Bell demonstrated that there is no way to understand entanglement in the framework of the usual ideas of a physical reality localized in space-time and obeying causality. This result was opposite to the expectations of Einstein, who had first pointed out, with his collaborators Podolsky and Rosen, the strong correlations between entangled particles, and analyzed these correlations in the framework of ideas of a local physical reality. The most remarkable feature of Bell's work was undoubtedly the possibility it offered to determine *experimentally* whether or not Einstein's ideas could be kept. The experimental tests of *Bell inequalities* gave an unambiguous answer: entanglement *cannot* be understood as usual correlations, whose interpretation relies on the existence of common properties, originating in a common preparation, and remaining attached to each individual object after separation, as components of their physical reality[a]. A few decades after the 1964 paper, the physics of entanglement is flourishing, and thousands of papers, theoretical and experimental, are found when one types 'Bell inequalities' on a search engine.

Starting in the 1970s, another concept has progressively become more and more important in quantum physics: the description of *single objects*, in contrast to the statistical use of quantum mechanics to describe only properties of large ensembles (for instance the fluorescence of an atomic vapor). That question had, like the EPR problem, been a subject of debate between Bohr and Einstein[3], but it was the development of experimental abilities to isolate and observe single microscopic objects like photons, electrons, ions and atoms that prompted physicists to take quantum mechanical dynamics of single objects, including 'quantum jumps', seriously. The experimental observation of quantum jumps (in the fluorescence light from a single ion) inspired new theoretical approaches, the so-called 'Quantum Monte-Carlo Wave Function' simulations, primarily used to describe 'elementary' microscopic objects like ions, atoms, and small molecules. Recently, progress in nanofabrication, as well as experimental breakthroughs, have allowed physicists to create mesoscopic systems (e.g., electric and magnetic devices, and gaseous Bose–Einstein condensates) which push the border of the quantum world to larger and larger systems that still need to be described as single quantum objects.

As a witness of that period I would like to argue that John Bell also played, indirectly, an important role in the emergence of the new theoretical approaches clarifying the quantum description of individual objects. Before the realization of the importance of Bell's theorem, which happened only in the 1970s, the conventional wisdom among physicists was that the 'founding fathers' of quantum mechanics had settled all the conceptual questions. Bell's work on entanglement did not

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[a] An example of usual correlations is the identity of the eye colours of twin brothers, linked to their identical chromosome sets. Correlations in entangled twin photons are different in nature, as we explain later.

cast any doubt on the validity of quantum mechanics as a predictive tool. To the contrary, experiments found that nature definitely follows the quantum mechanical predictions even in those weird situations. But there was a lesson to be drawn: questioning the 'orthodox' views, including the famous 'Copenhagen interpretation', might lead to an improved understanding of the quantum mechanics formalism, even though that formalism remained impeccably accurate. It is my claim that Bell's example helped physicists to free themselves from the belief that the conceptual understanding that had been achieved by the 1940s was the end of the story.

I think it is not an exaggeration to say that the realization of the importance of entanglement and the clarification of the quantum description of single objects have been at the root of a *second quantum revolution*, and that John Bell was its prophet. And it may well be that this once purely intellectual pursuit will also lead to a *new technological revolution*. Indeed, we should have no doubt that the advances in the quantum concepts used to describe single objects will certainly rejoin and play a key role in the ongoing *revolution of nanotechnology*. Even more amazing, physicists have endeavoured to apply entanglement to '*quantum computation*', and most of the systems that are being experimentally tested as elementary quantum processors are entangled quantum systems, such as a few interacting ions. Whether or not the second revolution will have an impact on our societies is a premature question. But who would have imagined the ubiquitous presence of integrated circuits when the first transistor was invented?

2 The first quantum revolution

Searching for a consistent explanation of the black-body radiation spectrum at both high and low frequencies, M. Planck introduced in 1900 the quantization of energy exchange between light and matter[4]. A. Einstein took a step further in 1905 by proposing the quantization of light itself to understand the photoelectric effect[5]. The properties he deduced were then tested by R. A. Millikan in 1914[6]. At the same epoch convincing evidence of the existence of molecules – doubted until the beginning of the twentieth century – was provided by various observations, including Einstein's explanation of Brownian motion[7]. Together with many other experiments, these observations convinced physicists and philosophers to accept the granularity of matter and quantization of energy in the microscopic world and led to the development of quantum mechanics.

In addition to rendering an account of experimental data, the foundation of quantum mechanics resolved basic problems. For instance, N. Bohr's

1913 model of the atom explained both the absorption spectra of atomic gases and the stability of matter: without quantum mechanics, the Rutherford atom, composed of orbiting particles with opposite (i.e., attracting) charges, should radiate and collapse.

The first comprehensive paradigm of quantum mechanics centred about the Heisenberg and Schrödinger formalisms of 1925. The latter was a wave equation for matter, completing a beautiful duality: like light, matter can behave as either a particle or a wave. The wave–particle duality was originally L. de Broglie's 1924 proposition[8], and remains incomprehensible to the classical way of thinking. Within twenty years of its birth, the quantum mechanical formalism could explain chemical bonds, electrical properties, and thermal properties of matter at a microscopic level. Continuing progress in physics was pushing along different directions: towards the incredibly small, with particle physics, or into the domain of more exotic properties of matter, such as superconductivity (the absence of resistance in some conductors at low temperatures), or superfluidity (the absence of viscosity of liquid helium at low temperatures). Studies in light–matter interaction were refined by orders of magnitude, thanks to experimental breakthroughs made possible by advances in microwave technology[9]. All this progress took place perfectly within the quantum mechanical framework, which had been refined to be applied both in the elementary phenomenon (Quantum Electrodynamics) as well as in complex situations encountered in condensed matter. But in the early 1950s, quantum mechanics still appeared as a game to be played by physicists only for the sake of progress in knowledge, without any impact on everyday life.

The electronics and information age: quantum mechanics applied

Even if the public is not always aware, the applications of quantum physics are all around us in electronics and photonics. In technologies today, quantum mechanics is required to understand material properties (electrical, mechanical, optical, etc.) and the behaviour of elementary devices at the root of many technological achievements.

The transistor was invented in 1948 by a brilliant group of solid state physicists after fundamental reflection about the quantum nature of electrical conduction[10]. This invention and its descendents, microfabricated integrated circuits[11], clearly had a monumental impact. Like the steam engine over a century earlier, the transistor changed our lives and gave birth to a new era, the information age.

The second technological progeny of quantum mechanics is the laser, developed in the late 1950s[12]. Some of its applications are present in everyday life: bar code readers, CD readers and players, medical tools, etc. Less visible but perhaps more important is the use of laser light in telecommunications, where it dramatically boosts the flow of information: terabits (millions of millions of information units) per second can be transmitted across the oceans through a single optical fiber. These information highways connect us to the stored knowledge and active computation distributed around the world. Starting from the few bits per second of the first telegraph operators, we have come a long way.

The quantum mechanical understanding of atom–photon interactions has also continued to develop, and eventually led to applications. For example, in 1997 a Nobel prize was given to S. Chu, C. Cohen-Tannoudji, and W. D. Phillips, for the development of laser cooling and trapping of atoms. Here as well, fundamental research soon led to a spectacular application, cold atom clocks, which have already allowed the accuracy of time measurement to reach a level of 1 in 10^{15} (one second accuracy in thirty million years!). More is to come with cold atom or ion *optical* clocks. Atomic clocks are used in the global positioning system (GPS), and their replacement with cold atom clocks will eventually permit an improved positioning precision. Coming full circle, that improved technology of clocks can be applied to fundamental questions, such as tests of general relativity, or the search for slow variation in fundamental physical constants. The first quantum revolution, with its interplay between basic questions and applications, is still at work.

3 Entanglement and Bell's theorem

The Bohr–Einstein debate

Quantum mechanics was constructed at the price of several radical – and sometimes painful – revisions of classical concepts. For instance, to take into account particle–wave duality, quantum mechanics had to renounce the idea of a classical trajectory. This renunciation is best stated in the celebrated Heisenberg uncertainty principle, which describes quantitatively the impossibility of defining precisely and simultaneously the position and velocity of a particle. One can also illustrate this renunciation of classical trajectories by remarking that in an interference experiment the particle 'follows many paths at once'.

In fact, such renunciations were so radical that several, including Einstein and de Broglie, could not admit their inevitability, and differed