Nonlinear Microwave Circuit Design

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Preface

Nonlinear microwave circuits is a field still open to investigation; however, many basic concepts and design guidelines are already well established. Many researchers and design engineers have contributed in the past decades to the development of a solid knowledge that forms the basis of the current powerful capabilities of microwave engineers.

This book is composed of two main parts. In the first part, some fundamental tools are described: nonlinear circuit analysis, nonlinear measurement, and nonlinear modeling techniques. In the second part, basic structure and design guidelines are described for some basic blocks in microwave systems, that is, power amplifiers, oscillators, frequency multipliers and dividers, and mixers. Stability in nonlinear operating conditions is also addressed.

A short description of fundamental techniques is needed because of the inherent differences between linear and nonlinear systems and because of the greater familiarity of the microwave engineer with the linear tools and concepts. Therefore, an introduction to some general methods and rules proves useful for a better understanding of the basic behaviour of nonlinear circuits. The description of design guidelines, on the other hand, covers some well-established approaches, allowing the microwave engineer to understand the basic methodology required to perform an effective design.

The book mainly focuses on general concepts and methods, rather than on practical techniques and specific applications. To this aim, simple examples are given throughout the book and simplified models and methods are used whenever possible. The expected result is a better comprehension of basic concepts and of general approaches rather than a fast track to immediate design capability. The readers will judge for themselves the success of this approach.

Finally, we acknowledge the help of many colleagues. Dr. Franco Di Paolo has provided invaluable help in generating simulation results and graphs. Prof. Tom Brazil, Prof. Aldo Di Carlo, Prof. Angel Mediavilla, and Prof. Andrea Ferrero, Dr. Giuseppe Ocera and Dr. Carlo Del Vecchio have contributed with relevant material. Prof. Giovanni Ghione and Prof. Fabrizio Bonani have provided important comments and remarks, although responsibility for eventual inaccuracies must be ascribed only to the authors. To all these people goes our warm gratitude.

Authors' wives and families are also acknowledged for patiently tolerating the extra work connected with writing a book.

Franco Giannini
Giorgio Leuzzi

Contents

1

Nonlinear Analysis Methods

1.1 INTRODUCTION

In this introduction, some well-known basic concepts are recalled, and a simple example is introduced that will be used in the following paragraphs for the illustration of the different nonlinear analysis methods.

Electrical and electronic circuits are described by means of voltages and currents. The equations that fulfil the topological constraints of the network, and that form the basis for the network analysis, are Kirchhoff's equations. The equations describe the constraints on voltages (mesh equations) or currents (nodal equations), expressing the constraint that the sum of all the voltages in each mesh, or, respectively, that all the currents entering each node, must sum up to zero. The number of equations is one half of the total number of the unknown voltages and currents. The system can be solved when the relation between voltage and current in each element of the network is known (constitutive relations of the elements). In this way, for example, in the case of nodal equations, the currents that appear in the equations are expressed as functions of the voltages that are the actual unknowns of the problem. Let us illustrate this by means of a simple example (Figure 1.1).

$$i_s + i_g + i_C = 0 \qquad \text{Nodal Kirchhoff's equation} \tag{1.1}$$

$$\begin{aligned} i_s &= i_s(t) \\ i_g &= g \cdot v \\ i_C &= C \cdot \frac{dv}{dt} \end{aligned} \qquad \text{Constitutive relations of the elements} \tag{1.2}$$

where $i_s(t)$ is a known, generic function of time. Introducing the constitutive relations into the nodal equation we get

$$C \cdot \frac{dv(t)}{dt} + g \cdot v(t) + i_s(t) = 0 \tag{1.3}$$

Since in this case all the constitutive relations (eq. (1.2)) of the elements are linear and one of them is differential, the system (eq. (1.3)) turns out to be a linear differential

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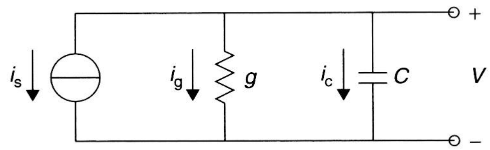

Figure 1.1 A simple example circuit

system in the unknown $v(t)$ (in this case a single equation in one unknown). One of the elements ($i_s(t)$) is a known quantity independent of voltage (known term), and the equation is non-homogeneous. The solution is found by standard solution methods of linear differential equations:

$$v(t) = v(t_0) \cdot e^{-\frac{g}{C}\cdot(t-t_0)} - \int_{t_0}^{t} \frac{e^{-\frac{g}{C}\cdot(t-\tau)}}{C} \cdot i_s(\tau) \cdot d\tau \tag{1.4}$$

More generally, the solution can be written in the time domain as a convolution integral:

$$v(t) = v(t_0) + \int_{t_0}^{t} h(t-\tau) \cdot i_s(\tau) \cdot d\tau \tag{1.5}$$

where $h(t)$ is the impulse response of the system.

The linear differential equation system can be transformed in the Fourier or Laplace domain. The well-known formulae converting between the time domain and the transformed Fourier domain, or frequency domain, and vice versa, are the Fourier transform and inverse Fourier transform respectively:

$$V(\omega) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} v(t) \cdot e^{-j\omega t} \cdot dt \tag{1.6}$$

$$v(t) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} V(\omega) \cdot e^{j\omega t} \cdot d\omega \tag{1.7}$$

By Fourier transforming eq. (1.3), after simple manipulation (Appendix A.1) we have

$$V(\omega) = H(\omega) \cdot I_s(\omega) \tag{1.8}$$

where $H(\omega)$ and $I_s(\omega)$ are obtained by Fourier transformation of the time-domain functions $h(t)$ and $i_s(t)$; $H(\omega)$ is the transfer function of the circuit.

We can describe this approach from another point of view: if the current $i_s(t)$ is sinusoidal, and we look for the solution in the permanent regime, we can make use of phasors, that is, complex numbers such that

$$v(t) = \mathrm{Im}[V \cdot e^{j\omega t}] \tag{1.9}$$

and similarly for the other electrical quantities; the voltage phasor V corresponds to the $V(\omega)$, defined above. Then, by replacing in eq. (1.3) we get

$$j\omega C \cdot V + g \cdot V + I_s = (g + j\omega C) \cdot V + I_s = Y \cdot V + I_s = 0 \tag{1.10}$$

and the solution is easily found by standard solution methods for linear equations:

$$V = \frac{I_s}{Y} \qquad \frac{1}{Y(\omega)} = H(\omega) = \frac{1}{g + j\omega C} \tag{1.11}$$

Let us now introduce nonlinearities. Nonlinear circuits are electrical networks that include elements with a nonlinear relation between voltage and current; as an example, let us consider a nonlinear conductance (Figure 1.2) described by

$$i_g(v) = i_{max} \cdot tgh\left(\frac{g \cdot v}{i_{max}}\right) \tag{1.12}$$

that is, a conductance saturating to a maximum current value i_{max} (Figure 1.3).

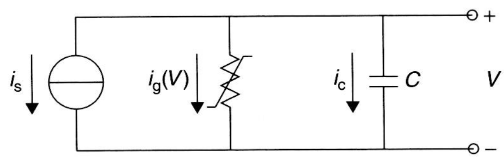

Figure 1.2 The example circuit with a nonlinear conductance

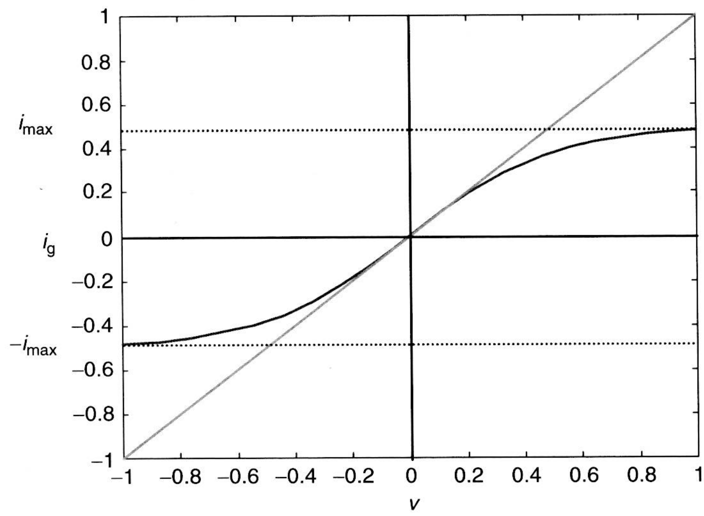

Figure 1.3 The current–voltage characteristic of the nonlinear conductance

When we introduce this relation in Kirchhoff's equation, we have a nonlinear differential equation (in general, a system of nonlinear differential equations)

$$C \cdot \frac{\mathrm{d}v(t)}{\mathrm{d}t} + i_{\max} \cdot tgh\left(\frac{g \cdot v(t)}{i_{\max}}\right) + i_{\mathrm{s}}(t) = 0 \tag{1.13}$$

that has no explicit solution. Moreover, contrary to the linear case, transformation into the Fourier or Laplace domain is not applicable.

Practical solutions to this type of problems fall into two main categories: direct numerical integration in the time domain, and numerical solution through series expansion; they are described in some detail in the following paragraphs.

1.2 TIME-DOMAIN SOLUTION

In this paragraph, the solution of the nonlinear differential Kirchhoff's equations by direct numerical integration in the time domain is described. Advantages and drawbacks are described, together with some improvements to the basic approach.

1.2.1 General Formulation

The time-domain solution of the nonlinear differential equations system that describes the circuit (Kirchhoff's equations) can be performed by means of standard numerical integration methods. These methods require the discretisation of the time variable, and likewise the sampling of the known and unknown time-domain voltages and currents at the discretised time instants.

The time variable, in general a real number in the interval $[t_0, \infty]$, is discretised, that is, considered as a discrete variable:

$$t = t_k \qquad k = 1, 2, \ldots \quad t \in [t_0, \infty] \tag{1.14}$$

All functions of time are evaluated only at this set of values of the time variable. The differential equation becomes a finite-difference equation, and the knowledge of the unknown function $v(t)$ is reduced to the knowledge of a discrete set of values:

$$v_k = v(t_k) \qquad k = 1, 2, \ldots \quad t \in [t_0, \infty] \tag{1.15}$$

Similarly, the known function $i_{\mathrm{s}}(t)$ is computed only at a discrete set of values:

$$i_{\mathrm{s},k} = i_{\mathrm{s}}(t_k) \qquad k = 1, 2, \ldots \quad t \in [t_0, \infty] \tag{1.16}$$

The obvious advantage of this scheme is that the derivative with respect to time becomes a finite-difference incremental ratio:

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = \frac{v_k - v_{k-1}}{t_k - t_{k-1}} \tag{1.17}$$

Let us apply the discretisation to our example. Equation (1.13) becomes

$$C \cdot \left(\frac{v_k - v_{k-1}}{t_k - t_{k-1}}\right) + i_{\max} \cdot tgh\left(\frac{g \cdot v_k}{i_{\max}}\right) + i_{s,k} = 0 \qquad k = 1, 2, \ldots \tag{1.18}$$

where we have replaced the derivative with respect to time, defined in the continuous time, with the incremental ratio, defined in the discrete time. In this formulation, the discrete derivative is computed between the current point k, where also the rest of the equation is evaluated, and the previous point $k-1$. There is, however, another possibility:

$$C \cdot \left(\frac{v_k - v_{k-1}}{t_k - t_{k-1}}\right) + i_{\max} \cdot tgh\left(\frac{g \cdot v_{k-1}}{i_{\max}}\right) + i_{s,k-1} = 0 \qquad k = 1, 2, \ldots \tag{1.19}$$

In the second case, the rest of the equation, including the nonlinear function of the voltage, is evaluated in the previous point $k-1$. In both cases, if an initial value is known for the problem, that is, if the value $v_0 = v(t_0)$ is known, then the problem can be solved iteratively, time instant after time instant, starting from the initial time instant $t_0$at $k = 0$. In the case of our example, the initial value is the voltage at which the capacitance is initially charged.

The two cases of eq. (1.18) and eq. (1.19) differ in complexity and accuracy. In the case of eq. (1.19), the unknown voltage v_k at the current point k appears only in the finite-difference incremental ratio; the equation can be therefore easily inverted, yielding

$$v_k = v_{k-1} - \frac{(t_k - t_{k-1})}{C} \cdot i_{\max} \cdot tgh\left(\frac{g \cdot v_{k-1}}{i_{\max}}\right) + i_{s,k-1} \qquad k = 1, 2, \ldots \tag{1.20}$$

This approach allows the explicit calculation of the unknown voltage v_k at the current point k, once the solution at the previous point $k-1$ is known. The obvious advantage of this approach is that the calculation of the unknown voltage requires only the evaluation of an expression at each of the sampling instants t_k. A major disadvantage of this solution, usually termed as 'explicit', is that the stability of the solution cannot be guaranteed. In general, the solution found by any discretised approach is always an approximation; that is, there will always be a difference between the actual value of the exact (unknown) solution $v(t)$ at each time instant t_k and the values found by this method

$$v(t_k) \neq v_k \qquad v(t_k) - v_k = \Delta v_k \qquad k = 1, 2, \ldots \tag{1.21}$$

because of the inherently approximated nature of the discretisation with respect to the originally continuous system. The error Δv_k due to an explicit formulation, however, can increase without limits when we proceed in time, even if we reduce the discretisation step $t_k - t_{k-1}$, and the solution values can even diverge to infinity. Even if the values do not diverge, the error can be large and difficult to reduce or control; in fact, it is not guaranteed that the error goes to zero even if the time discretisation becomes arbitrarily dense and the time step arbitrarily small. In fact, for simple circuits the explicit solution is usually adequate, but it is prone to failure for strongly nonlinear circuits. This explicit formulation is also called 'forward Euler' integration algorithm in numerical analysis [1, 2].

In the case of the formulation of eq. (1.18), the unknown voltage v_k appears not only in the finite-difference incremental ratio but also in the rest of the equation, and in particular within the nonlinear function. At each time instant, the unknown voltage v_k must be found as a solution of the nonlinear implicit equation:

$$C \cdot \left(\frac{v_k - v_{k-1}}{t_k - t_{k-1}} \right) + i_{\max} \cdot tgh \left(\frac{g \cdot v_k}{i_{\max}} \right) + i_{s,k} = F(v_k) = 0 \qquad k = 1, 2, \ldots \qquad (1.22)$$

This equation in general must be solved numerically, at each time instant t_k. Any zero-searching numerical algorithm can be applied, as for instance the fixed-point or Newton–Raphson algorithms. A numerical search requires an initial guess for the unknown voltage at the time instant t_k and hopefully converges toward the exact solution in a short number of steps; the better the initial guess, the shorter the number of steps required for a given accuracy. As an example, the explicit solution can be a suitable initial guess. The iterative algorithm is stopped when the current guess is estimated to be reasonably close to the exact solution. This approach is also called 'backward Euler' integration scheme in numerical analysis [1, 2].

An obvious disadvantage of this approach w.r.t. the explicit one is the much higher computational burden, and the risk of non-convergence of the iterative zero-searching algorithm. However, in this case the error Δv_k can be made arbitrarily small by reducing the time discretisation step $t_k - t_{k-1}$, at least in principle. Numerical round-off errors due to finite number representation in the computer is however always present.

The discretisation of the t_k can be uniform, that is, with a constant step Δt, so that

$$t_{k+1} = t_k + \Delta t \qquad t_k = t_0 + k \cdot \Delta t \qquad k = 1, 2, \ldots \qquad (1.23)$$

This approach is not the most efficient. A variable time step is usually adopted with smaller time steps where the solution varies rapidly in time and larger time steps where the solution is smoother. The time step is usually adjusted dynamically as the solution proceeds; in particular, a short time step makes the solution of the nonlinear eq. (1.22) easier. A simple procedure when the solution of eq. (1.22) becomes too slow or does not converge at all consists of stopping the zero-searching algorithm, reducing the time step and restarting the algorithm.

There is an intuitive relation between time step and accuracy of the solution. For a band-limited signal in permanent regime, an obvious criterion for time discretisation is given by Nykvist's sampling theorem. If the time step is larger than the sampling time required by Nykvist's theorem, the bandwidth of the solution will be smaller than that of the actual solution and some information will be lost. The picture is not so simple for complex signals, but the principle still holds: the finer the time step, the more accurate the solution. Since higher frequency components are sometimes negligible for practical applications, a compromise between accuracy and computational burden is usually chosen. In practical algorithms, more elaborate schemes are implemented, including modified nodal analysis, advanced integration schemes, sophisticated adaptive time-step schemes and robust zero-searching algorithms [3–7].

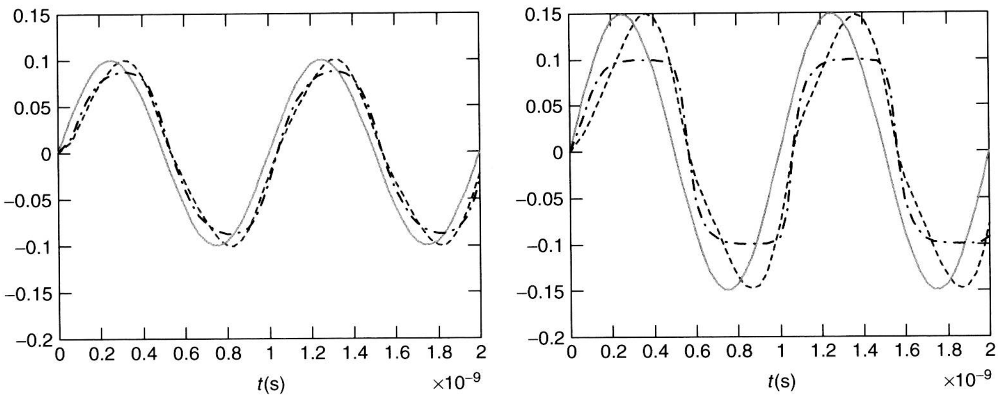

Figure 1.4 Currents and voltages in the example circuit for two different amplitudes of a sinusoidal input current

With the view to illustrate, the time-domain solution of our example circuit is given for a sinusoidal input current, for the following values of the circuit elements (Figure 1.4):

$$g = 10\ \text{mS} \qquad C = 500\ \text{fF} \qquad f = 1\ \text{GHz} \tag{1.24}$$

A simple implicit integration scheme is used, with a uniform time step of $\Delta t =$ 33.3 ps (30 discretisation points per period). The plots show the input current i_s (———), the voltage v (- - - -) and the current in the nonlinear resistor i_g (-·-·-·), for an input current of $i_{s,max} = 100$ mA (a) and for a larger input of $i_{s,max} = 150$ mA (b).

As an additional example, the response of the same circuit to a 1 mA input current step is shown in Figure 1.5, where a uniform time step of $\Delta t = 10$ ps is used.

Time-domain direct numerical integration is very general. No limitation on the type or stiffness of the nonlinearity is imposed. Transient as well as steady state behaviour are computed, making it very suitable, for instance, for oscillator analysis, where the determination of the onset of the oscillations is required. Instabilities are also well predicted, provided that the time step is sufficiently fine. Also, digital circuits are easily analysed.

1.2.2 Steady State Analysis

Direct numerical integration is not very efficient when the steady state regime is sought, especially when large time constant are present in a circuit, like those introduced by the bias circuitry. In this case, a large number of microwave periods must be analysed before the reactances in the bias circuitry are charged, starting from an arbitrary initial state. Since the time step must be chosen small enough in order that the microwave voltages and currents are sufficiently well sampled, a large number of time steps must be computed before the steady state is reached. The same is true when the spectrum of the signal includes components both at very low and at very high frequencies, as in the case