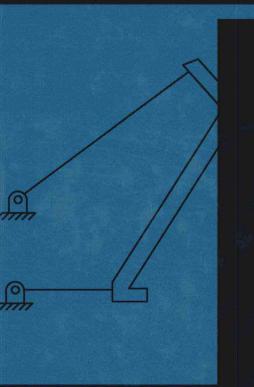
Compliant Mechanisms





Larry L. Howell

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COMPLIANT MECHANISMS

To my wife

Peggy

and my children

Angela, Travis, Nathan, and Matthew

PREFACE

Compliant mechanisms offer great promise in providing new and better solutions to many mechanical-design problems. Since much research in the theory of compliant mechanisms has been done in the last few years, it is important that the abundant information be presented to the engineering community in a concise, understandable, and useful form. The purpose of this book is to fulfill this need for students, practicing engineers, and researchers.

The book presents methods for the analysis and design of compliant mechanisms and illustrates them with examples. The materials in the book provide ideas for engineers to employ the advantages of compliant mechanisms in ways that otherwise may not be possible. The analysis of small deflection devices is addressed, but emphasis is given to compliant mechanisms that undergo large, nonlinear deflections. The pseudo-rigid-body model is introduced as a method which simplifies the analysis of compliant mechanisms that undergo large deflections by modeling them with elements common to traditional mechanisms. This simplification makes it possible to design compliant mechanisms for many types of tasks. The advantages of compliant mechanisms in the emerging area of microelectromechanical systems (MEMS) are also addressed, and several MEMS examples are provided throughout the book.

The chapters are organized to flow from simple to more complex concepts; the book then concludes with the application of the previous materials to specific types of devices. This is done by organizing the chapters into major sections of introduction, fundamentals, analysis, design, and special-purpose mechanisms. In a similar way, simple examples facilitate understanding, followed by more complicated examples that demonstrate how the material can be used in applications.

xvi Preface

Review of essential topics in strength of materials, machine design, and kinematics is provided to create a self-contained book that does not require a lot of additional references to solve compliant-mechanism problems. These reviews can help emphasize important topics the reader has studied previously, or they can be used as a resource for those from other disciplines who are working in the area of MEMS or related areas. The appendixes provide a resource for quick reference to important equations presented in the book.

The area of compliant mechanisms exists thanks to the vision and insight of Professor Ashok Midha. Many have contributed to the knowledge of compliant mechanisms, but Professor Midha may be considered the father of modern compliant mechanisms. His insight and vision have had a profound effect on the field and on those with whom he has associated. I have greatly benefited from both his work in compliant mechanisms and his example and mentorship, and I am grateful for his influence.

The earlier versions of this book were used as notes in compliant mechanisms courses offered at Brigham Young University, Purdue University, and the University of Missouri, Rolla. Students made many helpful comments to improve the quality of the notes.

Several colleagues have graciously volunteered their time and expertise by contributing parts of the book. Professor G. K. Ananthasuresh at the University of Pennsylvania and Professor Mary I. Frecker at Pennsylvania State University wrote Chapter 9. Dr. Morgan D. Murphy of Delphi Automotive Systems contributed Appendix G. Chapter 11 relies heavily on graduate work completed by Brian Jensen when he was at Brigham Young University.

Some of the text and figures in this book are summarized from previous writings, including a number of papers coauthored with graduate students and colleagues and published by the American Society of Mechanical Engineers (ASME) in various conference proceedings and in the *Journal of Mechanical Design*. Work from a number of graduate student theses has also been included. Grateful thanks is extended to all those who have participated in this work: James Derderian, Patrick Opdahl, Brian Edwards, John Parise, and Brian Jensen have generously contributed sections of this book. The contributions of Scott Lyon, Brent Weight, and Greg Roach are also greatly appreciated, as are the efforts of many other students that have made this possible. The valuable assistance of Megan Poppitz is also gratefully acknowledged.

The Mechanical Engineering Department at Brigham Young University has been very supportive of this project and has provided many resources to assist in its completion. The College of Engineering and the administration of Brigham Young University have also supported the author's efforts in many ways.

In addition to the many students who have provided recommendations and encouragement for this work, others are thanked for their helpful reviews and comments to improve the manuscript. Special thanks to Professor G. K. Ananthasuresh, Dr. Morgan D. Murphy, Professor Kenneth W. Chase, and Professor Don Norton of Brigham Young University's English Department, and the university editing service for valuable reviews and comments on the manuscript.

Preface xvii

Much of the fundamental work in compliant mechanisms has been funded by the National Science Foundation (NSF). The resources provided were a wise investment and will have a far-reaching impact for many years to come. The following NSF grants have supported the author's work in the area of compliant mechanisms: DMI-9624574 (CAREER Award), CMS-9978737, ECS-9528238, and DMI-9980835. The Utah Center of Excellence Program is also acknowledged for support of commercialization of compliant mechanism theory through funding of the Center of Excellence in Compliant Mechanisms.

I express my love and gratitude to my wife and children for their continued love, support, and companionship. And my eternal thanks to my parents, for their love and sacrifice. Finally, I humbly acknowledge the gifts from God, for which no words could ever adequately express my gratitude.

LARRY L. HOWELL

Provo, Utah

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CHAPTER 1

INTRODUCTION

A mechanism is a mechanical device used to transfer or transform motion, force, or energy [1, 2]. Traditional rigid-body mechanisms consist of rigid links connected at movable joints; the section of a reciprocating engine shown in Figure 1.1a is an example. The linear input is transformed to an output rotation, and the input force is transformed to an output torque. As another example, consider the Vise Grip pliers shown in Figure 1.1b. This mechanism transfers energy from the input to the output. Since energy is conserved between the input and output (neglecting friction losses), the output force may be much larger than the input force, but the output displacement is much smaller than the input displacement. Like mechanisms, structures may also consist of rigid links connected at joints, but relative rigid-body motion is not allowed between the links.

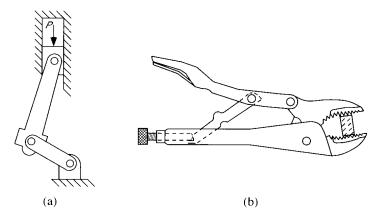


Figure 1.1. Examples of rigid-link mechanisms: (a) part of a reciprocating engine, and (b) Vise Grip.

2 Introduction

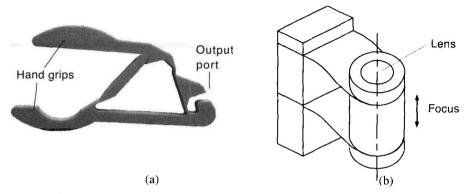


Figure 1.2. Examples of compliant mechanisms: (a) crimping mechanism (from [3]), and (b) parallel-guiding mechanism.

A compliant mechanism also transfers or transforms motion, force, or energy. Unlike rigid-link mechanisms, however, compliant mechanisms gain at least some of their mobility from the deflection of flexible members rather than from movable joints only. An example of a compliant crimping mechanism is shown in Figure 1.2a. The input force is transferred to the output port, much like the Vise Grip, only now some energy is stored in the form of strain energy in the flexible members. Note that if the entire device were rigid, it would have no mobility and would therefore be a structure. Figure 1.2b shows a device that also requires compliant members to focus a lens [4, 5].

1.1 ADVANTAGES OF COMPLIANT MECHANISMS

Compliant mechanisms may be considered for use in a particular application for a variety of reasons. The advantages of compliant mechanisms are considered in two categories: cost reduction (part-count reduction, reduced assembly time, and simplified manufacturing processes) and increased performance (increased precision, increased reliability, reduced wear, reduced weight, and reduced maintenance).

An advantage of compliant mechanisms is the potential for a dramatic reduction in the total number of parts required to accomplish a specified task. Some mechanisms may be manufactured from an injection-moldable material and be constructed of one piece. For example, consider the compliant overrunning clutch shown in Figure 1.3a [6, 7] and its rigid-body counterpart shown in Figure 1.3b. Considerably fewer components are required for the compliant mechanism than for the rigid mechanism. The reduction in part count may reduce manufacturing and assembly time and cost. The compliant crimping mechanism and its rigid-body counterpart illustrated in Figure 1.4 are other examples of part reduction.

Compliant mechanisms also have fewer movable joints, such as pin (turning) and sliding joints. This results in reduced wear and need for lubrication. These are valuable characteristics for applications in which the mechanism is not easily accessible, or for operation in harsh environments that may adversely affect joints.

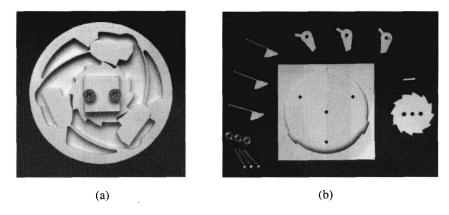


Figure 1.3. (a) Compliant overrunning clutch, and (b) its rigid-body counterpart shown disassembled. (From [6] and [7].)

Reducing the number of joints can also increase mechanism precision, because backlash may be reduced or eliminated. This has been a factor in the design of high-precision instrumentation [8, 9]. An example of a high-precision compliant mechanism is shown in Figure 1.5. Because the motion is obtained from deflection rather than by adjoining parts rubbing against each other, vibration and noise may also be reduced.

An example of a compliant mechanism designed for harsh environments is shown in Figure 1.6. This simple gripping device holds a die (such as a computer chip) during processing. The die must be transported between several different chemicals without becoming damaged. Made of Teflon—inert to the chemicals in which it is placed—the gripper holds the die without external force.

Because compliant mechanisms rely on the deflection of flexible members, energy is stored in the form of strain energy in the flexible members. This stored energy is similar to the strain energy in a deflected spring, and the effects of springs may be integrated into a compliant mechanism's design. In this manner, energy can easily be stored or transformed, to be released at a later time or in a different manner. A bow-and-arrow system is a simple example. Energy is stored in the limbs as the archer draws the bow; strain energy is then transformed to the kinetic energy of

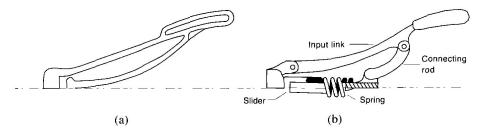


Figure 1.4. (a) Compliant crimping mechanism developed by AMP Inc., and (b) its rigid-body counterpart. Because of symmetry, only half the mechanism is shown. (From [4].)