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INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

WHYS AND HOWS IN UNCERTAINTY MODELLING

PROBABILITY, FUZZINESS AND ANTI-OPTIMIZATION

EDITED BY

ISAAC ELISHAKOFF

SpringerWienNewYork

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PREFACE

Recognition of the need to introduce the ideas of uncertainty reflects in part some of the profound changes in engineering over the last two decades. The natural question arises: how to deal with uncertainty? Since one of the meanings of uncertainty is randomness, a natural answer to this question was and often appears to be to apply the theory of probability and random processes. This idea was spectacularly successful, and many scientists even refer to a "probabilistic revolution".

However, as was established recently the probabilistic methods are accompanied with serious difficulties during their implementation process in engineering applications. This gave rise to other, alternative treatments of uncertainty. This volume deals with both probabilistic and non-probabilistic aspects of uncertainty modelling. In particular, the fuzzy-set-based analysis and the anti-optimization methods are pursued, in addition to recent developments in the stochastics theory and applications. The emphasis is placed on applications in civil, mechanical and aerospace engineering.

The aim of this volume is to present to researchers, engineers and graduate students working on and interested in the problems concerned with the mechanics of solids and structures, a unified view on uncertainty. The current state of the development and applications of uncertainty modelling and analysis will be reviewed from alternative and seemingly opposing points of view. Indeed, the present state of affairs can be characterised as that of Tower of Babel: There is no discussion between various researchers, actively embracing different points of view.

The book aims to disseminate recent stochastic and, especially, non-stochastic approaches to model the uncertainty. It poses questions which many engineers were interested to ask, but were afraid to do. Answers to these questions and in-depth discussions should appeal in the first place to practicing engineers who must constantly widen their perspectives, researchers will find the lecturers most illuminating for the book accommodated alternative and often opposing points of view. Pragmatic approach allows one to choose modern techniques which most suit to the practical situations. This is not just one more book about uncertainty. This is a collection of the latest thoughts of engineers and scientists from all over the world about many, often overlooked and yet to be recognized facets of ever-present uncertainty.

It was a pleasure to edit this volume. The cooperation of all authors is gratefully appreciated. Most unfortunately, Dr. Rudolf Lohner was unable to submit his contribution on the interval analysis. We were most fortunate, that Professor Ulrich Kulisch graciously agreed to contribute to this volume, on this topic. We record our heartfelt thanks to CISM International Centre for

Mechanical Sciences for both approving the course and providing the most pleasant atmosphere during its conducting in the beatiful palace in Udine. Rare courtesies extended by the Secretary General, Professor Giovanni Bianchi and Rector, Professor Sandor Kaliszky must be recorded. Reliable and encouraging assistance as well as the extremely kind patience of Professor Carlo Tasso of the University of Udine are most kindly acknowledged. The staff of Mrs. Elsa Venir Burti provided most reliable and timely service throughout the course, and created an atmosphere of exceptional dedication and work including nearly instant copying service, efficient communication between the lecturers and the attendees, library facilities, and solution of numerous problems of various magnitudes; all of these seemingly small gestures and good will and courtesy truly make CISM a unique and great institution that must be cherished, preserved and possibly expanded for the benefit of engineering sciences. Last but not least, our thanks are to all the attendees of the course: While teaching them, all lecturers learned a lot, not excluding the ways how the material should be presented more efficiently. The responsibility on any misconceptions or typographical errors lies upon ourselves, the authors themselves. We will be most indebted if you could communicate them to us by electronic mail at ielishak@me.fau.edu. or by the FAX of 561-297-2825.

Isaac Elishakoff

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CHAPTER 1

HOW TO UTILIZE THE ANTI-OPTIMIZATION ANALYSIS TO TREAT UNCERTAINTY IN SEISMIC EXCITATION?

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ABSTRACT

In the paper an approach to treat uncertainty in the response of structures under the action of earthquakes is presented. The main problem is focused in the unpredictability of the seismic accelerograms, and the research effort points at identifying the set of admisible quakes by a few basic parameters (duration, peak acceleration, gross information on the power spectrum, ...). The influence of the details of ground shaking, that really have a very significant influence on the structure's performance, is approached by the institution of a worst-case scenario. The basic idea is to build up a consistent model for the seismic hazard at the site, and to set up some rules which ground shaking must fit. Hence, the worst combination of details of the quake is sought by a search procedure, aiming at identifying the earthquake producing the extremum of some response parameters, in the set of admissible quakes. In the paper a number of r5eference models are set up, and results referred to a particular seismic area, the Campania region in Italy, are drawn, proving that the procedure is efficient and, to some extent, practical.

1) INTRODUCTION

The safety and economics of structures in seismic areas are highly dependent on the calibration of the appropriate seismic action. This is certainly one of the most interesting subjects in aseismic engineering. The extremely unpredictable variability of the proceeding of an earthquake and its effects at a given site, does not allow to define the solution of the problem in a definite, unique, way in a deterministic context. Uncertainty involves basically, from a structural point of view, the time-history of ground oscillation, since it, apart from some overall features, seems quite unpredictable while, on the other side, the structures' response is very sensitive even to small and apparently not significant details of the base vibration (see i.e. Ref.[1], where it is proved that a 2÷10% random perturbation of the ordinates of an accelerogram yields 50% and more variation in the response of th P-Δ elastic-plastic oscillator). The intrinsically hazardous nature of the problem suggests that a *probabilistic* or *possibilistic* point of view must be taken, aiming at keeping the hazard as small as possible, as far as this is economically compatible. On the other hand, the lack of instrumental records of significant seismic events at any site has addressed the researchers to make reference to some abstract model as a reference point for the synthesis of earthquake-type accelerograms. This approach requires that fundamental characters of seismic shaking at the site are known or, at least, can be reasonably predicted. In this regard, it has been recognised all over the world that some properties of recorded accelerograms, like peak acceleration, duration, build-up and decay, power distribution on the frequency range and so on, are strictly related to the localization of the epicenter and to the geological constitution of the site. This means that some *central characters* of the forcing function can be identified at every site, while a lot of many *other details* remain free to vary in a less controllable way, practically *random or indeterminate* to within some given, possibly *fuzzy*, boundary. Therefore, methods for the synthesis of earthquake-type accelerograms mostly rely on the simulation of a large number of random parameters correlated to the accelerogram ordinates, plus a finite, smaller number of *local* constants which represent the parameters conditioning the basic properties of the accelerograms to be generated. According to this philosophy, design accelerograms are taken more or less randomly from the whole population that can be potentially generated on the basis of the rationale one refers to; safety relies mainly on the statistical managing of the results and can be guaranteed only up to a given extent.

The problem that is illustrated in the present chapter is concerned with the case when *high reliability* is required; in this case one may need to select the most severe shape of the forcing function in the set of site-compatible accelerograms, reserving randomness, and consequent statistical treatment of uncertainty, to basic properties like peak acceleration and energy. Baratta and Zuccaro (see Baratta [2,3,4], Zuccaro [5], Baratta and Zuccaro [6,7,8]) developped a technique to produce the maximum theoretical values of the structural response under seismic load at given site. They pursued the goal combining available techniques for synthesis of random accelerograms (Ruiz-Penzien [9,10]) with optimization procedures capable to maximize some parameters, significant for aseismic design, in the respect of the constraints represented by the basic values characterizing the shaking properties at the site, as will be explained in Sec. 4. The introduction of an optimal criterion for the selection of time-histories that are the most dangerous with respect to a given response parameter, leads to the idea that the compatibility criterion for accelerograms to be synthesized can be formulated in some simpler way, that lends itself to optimization more naturally than random generators do. The first proposals on this line were advanced by Drenick [11,12,13,14] and Shinozuka [15] in the early 70's, and later on by Elishakoff and Pletner [16] and Baratta et al. [17,18]. All these approaches utilized an alternative, non-probabilistic, avenue. Drenick [11,12] used a constraint on the total energy which the earthquake is likely to develop at a certain site, as a description of uncertainty. He used the Cauchy-Schwarz inequality to determine the maximum response of the system to such an excitation. In the opinion of several investigators such a bound was too conservative.

Shinozuka [15] has suggested to characterize the earthquake uncertainty by specifying an envelope of the Fourier amplitude spectrum. Numerical calculations have demonstrated that the maximum response of the structure predicted by this method is less than that predicted in Refs. [11,12]. Elishakoff and Pletner [16] investigated the modification of the response prediction when the global information on the excitation is increased. In particular, the maximum possible response, which the structure may develop, was evaluated under the assumption that only the bound on base acceleration is known; then the maximum response was modified under the assumption that in addition to the base acceleration bound, the bounds on base velocity and/or displacement were specified. Finally, an application to earthquake engineering of *ellipsoidal modelling*, proposed as a general model for uncertainty .in mechanical problems by Elishakoff and Ben-Haim in 1990 [19], and first introduced in the theory of control by Schweppe [20] and previously applied to deal with geometrical imperfections in structural analysis [21], is developed by Baratta et al. in Refs [17, 18], and is fully illustrated in Sec.5. The term *antioptimization* was coined by Elishakoff (1991) for more general, than convex, uncertainties.

2) BASIC EQUATIONS AND BEHAVIOUR OF THE SDOF SHEAR-TYPE FRAME

2.1) ELASTIC BEHAVIOUR

Consider the simple linear SDOF shear frame in Fig. 2.1, with natural frequency ω_o, damping ratio ζ, and inertial mass m, and let u be the horizontal displacement of the beam with respect to the base of the column, (positive as shown). under the action of a ground acceleration a(t).

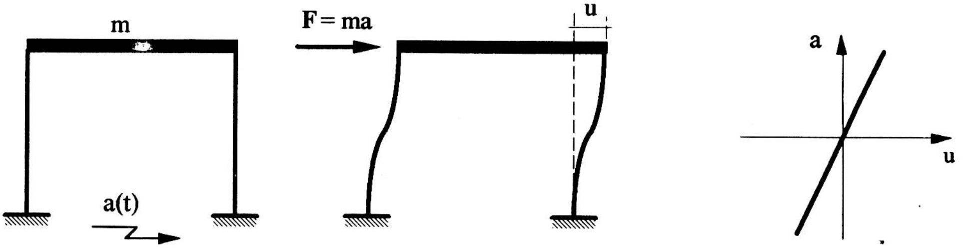

Fig. 2.1: The sdof shear-frame in the elastic range

The equation of motion is

$$\ddot{u}+2\zeta\omega_o\dot{u}+\omega_o^2 u=a(t) \tag{2.1}$$

with initial conditions

$$u(0) = 0 \quad ; \quad \dot{u}(0) = 0 \tag{2.2}$$

The solution, as well known, is given by

$$u(t) = \int_0^t a(\tau)\, h(t-\tau)\, d\tau \tag{2.3}$$

where h(·) is the impulse response function

$$h(x) = \begin{cases} \dfrac{1}{\omega_d} \exp(-\zeta\omega_o x)\sin(\omega_d x) & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases} \tag{2.4}$$

with $\omega_d = \omega_o\sqrt{1-\zeta^2}$.

2.2) Perfectly plastic behaviour with ductility control

2.2.1) Basic statements

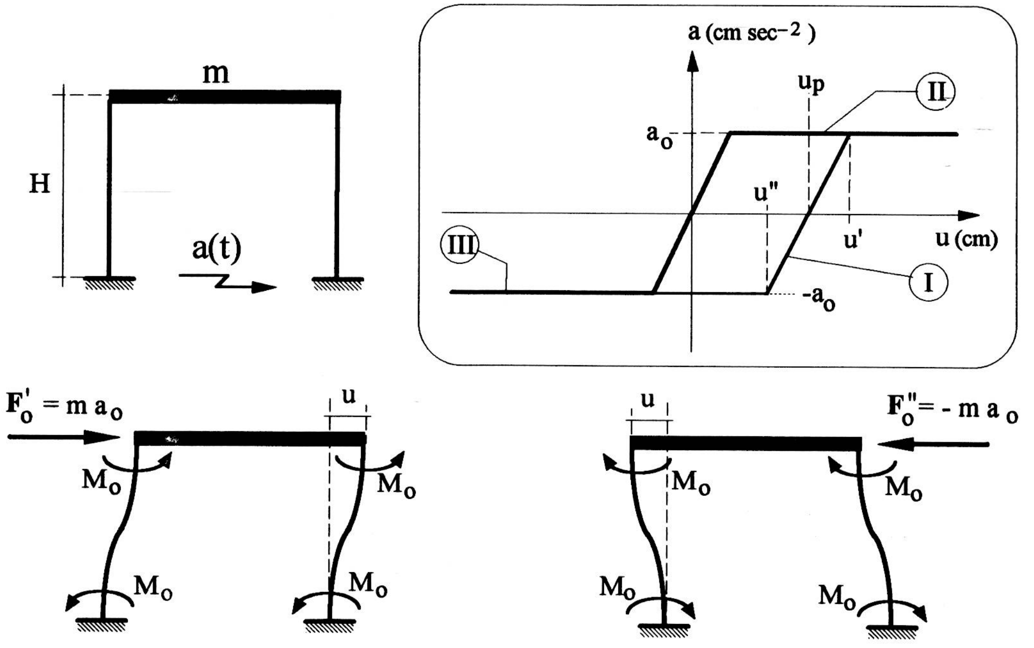

Fig. 2.2: The shear type sdof system under consideration

One considers the shear type frame (STF) in Fig. 2.2. under the action of a ground acceleration $a(t)$, and denotes by M_o the limit plastic moment in the columns.

The condition of the rotational equilibrium leads to

$$M = \frac{FH}{4} \tag{2.5}$$

where M denotes the bending moment at the top and at the bottom of the piles, and F denotes the total horizontal force on the transverse beam.

Putting $M = M_o$ in eq. (2.5), one gets the total limit shear force

$$F_o = \frac{4M_o}{H} = ma_o \tag{2.6}$$

where the strength of the system is expressed in term of acceleration by

$$a_o = \frac{F_o}{m} \tag{2.7}$$

The characteristic diagram of the structure is shown in Fig.2.2; where u_p is the instantaneous value of the plastic displacement after the elastic thresholds $a'_o = a_o > 0$ and/or $a''_o = -a_o < 0$ have been violated.

The equations of the lines I, II and III shown in Fig. 2.2 are

$$\begin{aligned} &\text{line I: } a_o(u) = \omega_0^2(u - u_p) \\ &\text{line II: } a_o(u) = a'_o \\ &\text{line III: } a_o(u) = a''_o \end{aligned} \tag{2.8}$$

2.2.2 The elastic-plastic equation of motion

One studies the dynamic behaviour of the system considered by the following elastic-plastic equation of the motion

$$\ddot{u}(t) + f_r(u, \dot{u}, u_p, \dot{u}_p) = a(t) \tag{2.9}$$

where $f_r(u, \dot{u}, u_p, \dot{u}_p)$ denotes the restoring force that the numerical procedure updates during the elastic-plastic hysteresis; it can be expressed in the following form:

$$f_r(u,\dot u,u_p,\dot u_p)=\begin{cases} 2\zeta\omega_0(\dot u-\dot u_p)+\omega_0^2(u-u_p) & \text{if} \begin{cases} a''_o<\omega_o^2\left[u(t)-u_p(t)\right]<a'_o \\ \omega_o^2\left[u(t)-u_p(t)\right]=a'_o \text{ and } \dot u(t)\le 0 \\ \omega_o^2\left[u(t)-u_p(t)\right]=a''_o \text{ and } \dot u(t)\ge 0 \end{cases} \quad (2..10) \\ 2\zeta\omega_0(\dot u-\dot u_p)+a'_o & \text{if } \omega_o^2\left[u(t)-u_p(t)\right]=a'_o \text{ and } \dot u(t)>0 \quad (2..11) \\ 2\zeta\omega_0(\dot u-\dot u_p)+a''_o & \text{if } \omega_o^2\left[u(t)-u_p(t)\right]=a''_o \text{ and } \dot u(t)<0 \quad (2..12) \end{cases}$$

where ζ is the damping coefficient and u_p and $\dot u_p$ are given by

$$\dot u_p(t)=\begin{cases} \dot u \text{ if } \left[\omega_o^2\left(u-u_p\right)=a'_o \text{ and } \dot u(t)>0\right] \text{ or } \left[\omega_o^2\left(u-u_p\right)=a''_o \text{ and } \dot u(t)<0\right] \\ 0 \qquad \text{otherwise} \end{cases} \quad (2.13)$$

$$u_p(t)=\int_0^t \dot u_p(\tau)\,d\tau \quad (2.14)$$

2.3) Perfectly plastic behaviour with P-Δ effect and ductility control

2.3.1) Basic statements

One considers the shear type frame in Fig. 2.3 under the action of a ground acceleration $a(t)$ and of the vertical loads W given by

$$W=mg/2 \quad (2.15)$$

with g denoting the gravity acceleration. The condition of the rotational equilibrium leads to

$$M=\frac{Wu}{2}+\frac{FH}{4} \quad (2.16)$$

Substituting $u = 0$ and $M = M_o$ in eq. (2.16), one gets the total limit shear force in the absence of P-Δ effect

$$F_o=\frac{4M_o}{H}=ma_o \quad (2.17)$$

Alternatively, putting $F = 0$ and $M = M_o$ in eq. (2.16), one gets the condition for collapse by purely geometrical effect

$$\frac{W u}{2} = M_o \tag{2.18}$$

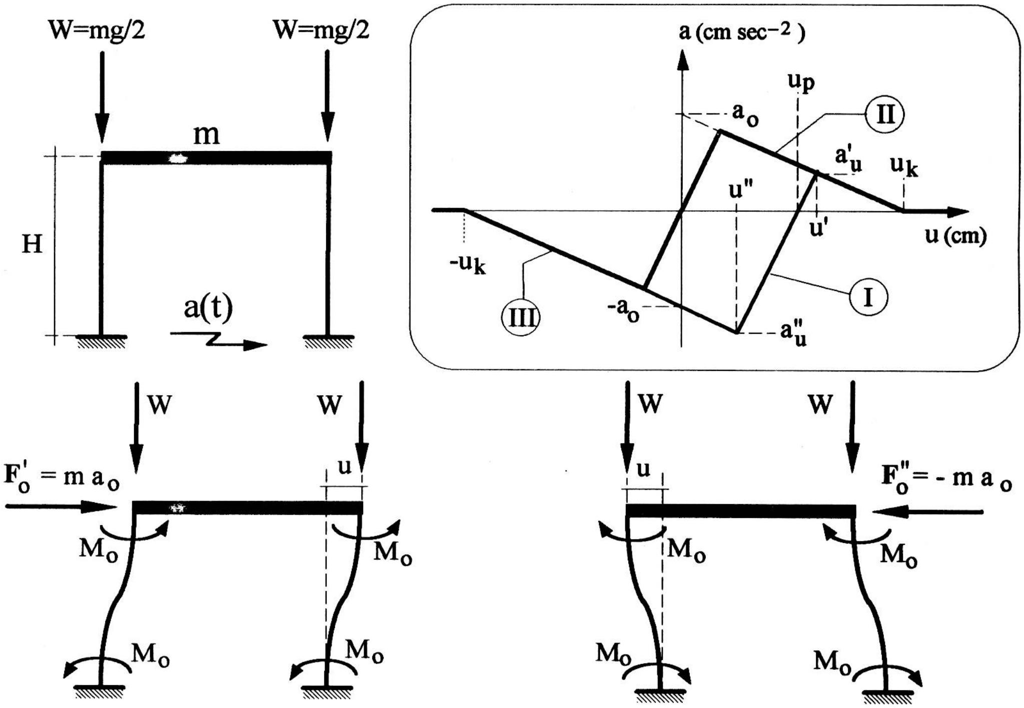

Fig. 2.3 The shear type sdof system under consideration

Substitution of eqs. (2.17) and (2.15) into eq. (2.18) yields

$$m g u = 4M_o = F_o H \tag{2.19}$$

whence, introducing $\vartheta = g / H$, one defines the *collapse displacement*

$$u_k = F_o / (\vartheta\, m) \tag{2.20}$$

corresponding to the limit value F_o of the shear force F at which the residual lateral strength of the frame is zero.

The virgin strength of the system can be expressed in term of acceleration by $a_o = F_o/m$, whence

$$u_k = \frac{a_o}{\vartheta}. \tag{2.21}$$

The characteristic diagram of the structure is shown in Fig.2.3; where a'_u and a''_u represent respectively the elastic limit for $u - u_p > 0$ and for $u - u_p < 0$ once the assigned elastic thresholds $a'_o = a_o > 0$ and/or $a''_o = -a_o < 0$ are violated, u_p being the instantaneous value of the plastic displacement.

The equations of the lines I, II and III shown in Fig. 2.3 are

$$\begin{aligned} &\text{line I: } a_o(u) = \omega_0^2(u - u_p) \\ &\text{line II: } a_o(u) = a'_o - \vartheta u \\ &\text{line III: } a_o(u) = a''_o - \vartheta u \end{aligned} \tag{2.22}$$

where the elastic component of the displacement in P-Δ effect has been neglected.
By intersections of the line I with II and II with III one gets the values of a'_u and a''_u and of the corresponding displacements:

$$a'_u = \omega_0^2(u' - u_p) \quad \text{with} \quad u' = \frac{a'_0 + \omega_0^2 u_p}{\omega_0^2 + \vartheta} \tag{2.23}$$

and

$$a''_u = \omega_0^2(u'' - u_p) \quad \text{with} \quad u'' = \frac{a''_0 + \omega_0^2 u_p}{\omega_0^2 + \vartheta} \tag{2.24}$$

2.3.2 The elastic-plastic equation of motion with P-Δ effect

Neglecting the elastic component of the displacement in P-Δ effect, one studies the dynamic behaviour of the system considered by formally the same equation of the motion (2.9) where $f_r(u, \dot{u}, u_p, \dot{u}_p)$ is expressed in the following form:

$$f_r(u,\dot{u},u_p,\dot{u}_p) = \begin{cases} 2\zeta\omega_0(\dot{u}-\dot{u}_p)+\omega_0^2(u-u_p) & \text{if} \begin{cases} u''(t) < u(t) < u'(t) \\ u(t) = u'(t) \text{ and } \dot{u}(t) \le 0 \\ u(t) = u''(t) \text{ and } \dot{u}(t) \ge 0 \end{cases} & (2.25) \\ 2\zeta\omega_0(\dot{u}-\dot{u}_p)+a'_o-\vartheta u' & \text{if } u(t)=u'(t) \text{ and } \dot{u}(t)>0 & (2.26) \\ 2\zeta\omega_0(\dot{u}-\dot{u}_p)+a''_o-\vartheta u'' & \text{if } u(t)=u''(t) \text{ and } \dot{u}(t)<0 & (2.27) \end{cases}$$

where u' and u'' are given respectively by eqs. (2.23) and (2.24), and u_p and $\dot{u}_p$ are given by

$$\dot{u}_p(t) = \begin{cases} \dfrac{(\omega_o^2 + \vartheta)}{\omega_0^2}\dot{u} & \text{if} \quad u(t) = u'(t) \text{ and } \dot{u}(t) > 0 \text{ or } u(t) = u''(t) \text{ and } \dot{u}(t) < 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.28}$$

$$u_p(t) = \int_0^t \dot{u}_p(\tau)\, d\tau \tag{2.29}$$

3) BASIC TEST ACCELEROGRAMS

The approach that is presented in this chapter is calibrated with reference to the following accelerographic records obtained on the occasion of the Campano-Lucano Earthquake of Nov. 23, 1980, in Italy, not far from city of Naples. The magnitude of the event was estimated in 6.5, and the epicentral intensity was set at the 7.5 degree of the MKS scale. The event was rather atypical, mainly because of its duration that lasted up to almost two minutes in some sites. Anyway, only the duration of the structural-significant motion will be considered here, that varies from about 52 to more than 86 secs.

Information on the local characters of ground motion is derived from direct inspection of recorded accelerograms, limiting the analysis, for simplicity, to only the NS component. The considered records are summarized in Table 3.I

Recorded ordinates of all accelerograms are converted to cm sec^{-2}, and all earthquakes are preliminarly reduced to the same norm [the *energy*, eq.(5.1)] as the one recorded in Torre del Greco, the most close site to Naples, that is the largest town in the region, so that every record possesses energy $E_0 = 79.46$ cm $sec^{-3/2}$.

Harmonic (Fourier) analysis is performed for every earthquake, thus obtaining for each one an expansion of the type (5.10) with the $a_i(t)$ given as in eq. (5.11). The analysis is carried on to include n = 400 sine and cosine waves for Torre del Greco, Sturno and Calitri earthquakes, while this number is increased up to n= 800 waves for the accelerograms recorded in Brienza and Bagnoli Irpino, that exhibit power spectra scattered on a wider range of frequencies.

A plot of each accelerogram, with the approximation resulting from its Fourier expansion and its power spectrum is quoted in Figs. 3.1.1 ÷ 3.1.5.