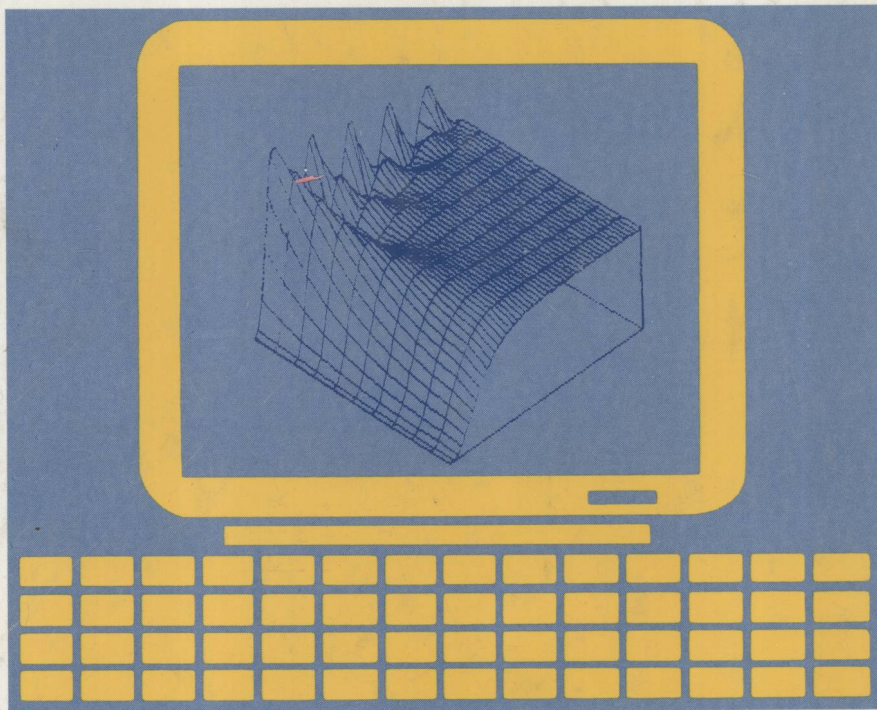


# *Computer-Aided Analysis and Design* of LINEAR CONTROL SYSTEMS



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MAHMOUD TAROKH  
BAHRAM SHAFAI

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# *Computer-Aided Analysis and Design of Linear Control Systems*

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To

*My sisters, Parvin, Mahin, Pooran, and Giti  
and brothers, M. Hossein, M. Hassan, Ahmad,  
Mahmoud, and Abolghasem*

*MJ*

*My wife Ency*

*MT*

*My wife Soheila, my daughter Ghazaleh,  
and my son Shahin*

*BS*

# *Foreword*

The mathematical theory of linear systems, such as linear differential, integral, and difference equations, comprises a rich and dynamic part of mathematics. A major portion of the related work on the fundamental structure and methodology of this field was developed in the nineteenth and twentieth centuries. Presently the research in linear mathematics is still growing at a rapid pace.

In most problems of technological and societal origin we are confronted with a study of a dynamical system, that is, a system such that its behavior evolves with time. From the early part of this century, scientists have attempted to develop analytic procedures for representing, approximating, and estimating the behavior of dynamical systems via linear mathematical models.

Since the nineteen twenties, a new breed of engineers and scientists has succeeded in developing a semi-formal liaison between the physical phenomena and the mathematical theory of linear systems. Their efforts have given birth to a number of engineering disciplines such as linear network, estimation, filtering, signal processing, feed-back and control theories. These new disciplines have exercised a major impact on the advancement of technology, and will continue to play a central role in the future.

Modern control theory embraces the study and the optimal design of systems of diversified origin. The broad spectrum of its applications includes various problems of manufacturing, maneuvering, air craft guidance, space vehicle tracking and robotics.

This book deals with the control theory of linear dynamical systems. The authors

present a comprehensive coverage of the analysis and design of control systems at a graduate level. There is an abundance of examples of applications, with an emphasis on computer-aided design (CAD).

The growth of mathematical research on linear systems opens new avenues for the study of physical and societal problems. The liaison between mathematics and technological applications requires continuous updating. In this respect, the authors should be commended for incorporating the new and very important use of CAD in their models.

The value of books of this caliber depends on the depth of the mathematical substance, the diversity of methods of application, and how the new tools of technology are implemented. To maintain an optimal balance between the advancement of abstract research and the progress of technology, a continuous re-evaluation of the field is indispensable. It is hoped that the authors of the present comprehensive work will continue with their up-to-date aims and efforts in the future editions.

The publication of this book coincides with a cultural event pertinent to the authors' origin and backgrounds. The year 1990 is highlighted by United Nations Educational Scientific and Cultural Organizations (UNESCO) as the millennium of Shahnameh, the great masterpiece of Persian Epic Poetry.

Inspired by this event, this volume is dedicated to the brilliant contributions of early Persian scholars to the fundamentals of mathematics. Here, we recognize but a few of those mathematicians and philosophers who laid down foundations of algebra and trigonometry about a thousand years ago. It is to be noted that during this period most scientific treatises of Persian scholars were written in Arabic.

**KHOWARIZMI** (*Mohammed ibn Musa, died between 835–845.*

Author of the first known book bearing the name of Algebra. His mathematical and astronomical works were translated into Latin under the title *Algoritmi de numero indorum*. The commonly used word “algorithm” is derived from the name Al-Khowarizmi.

**RAZI** (*Mohammed ibn Zarariya, known in the West as Rhazes, died about 932.*

Celebrated physician, discoverer of alcohol and author of books on geometry and astronomy.

**BEIRUNI** (*Abu Rihan, 973–1048, approximately.*

Mathematician-astronomer and historian. His work comprises major contributions to mathematics, astronomy and history. He also systematically used the Balance as an early computing tool for solving algebraic problems.

**IBN-SINA** (*known in the west as Avicenna, 980–1037 approximately.*

Famous physician, philosopher, who wrote extensively on many subjects including music, medicine, geometry and the theory of numbers. The Latin translation of



Avicenna's contribution to medicine, "Canon" was a major medical source book in Europe for several centuries.

The development of science has been undoubtedly enriched by the efforts of all nations at one time or other. Fortunately, this pattern of global scientific contributions prevails with growing intensity. In time, science will transcend all subjective boundaries.

*Faziollah Reza\**  
Ottawa, ONT.  
Canada

\* Faziollah Reza, a prominent scientist, engineer, and Persian literary personality, is internationally recognized for his contributions to the theory of Electrical Circuits, Systems and Communications. He has lectured and conducted research during four decades at many major academic centers in the United States of America, in Europe and in Canada. He has numerous scientific publications including the first book on information theory, (published by McGraw Hill in 1961). He is currently a visiting Professor at Concordia University and an honorary Professor at McGill University in Montreal.

# Preface

This text is intended to give an introduction to modern control systems with a special emphasis on **design**. The book has been written for students who have had one course in conventional feedback control systems with or without an introduction to the concepts of state space and state variables. The book has two primary goals. One is to give an introduction to the multivariable control systems design, and the other is the introduction of computer-aided control system design tools to students, engineers, and researchers alike. For the latter goal of the book, the reader needs only minimal knowledge of computers. A mere familiarity with the use of a personal computer or a terminal of a main frame is sufficient.

The text has, in part, evolved over a number of years of the authors' courses on computer-aided analysis and design of control systems. The text is intended to bridge a gap between a first course in classical/modern control and theoretically-oriented graduate courses such as optimal control, large-scale systems, and an advanced course in multivariable control. Both continuous and discrete-time formulations of linear systems are presented. Wherever appropriate, algorithms are given to follow the theoretical concepts and are followed by an analysis and a design example on the computer. Two classes of numerical examples prevail throughout the book. One class is intended to illustrate the theoretical concepts, while the other is presented to explain solutions within a computer-aided design (CAD) environment. The latter class is designated as *CAD EXAMPLES*. These two classes of problems are also so designated in the *PROBLEMS* section at the end of each chapter. More advanced topics are

designated in both the table of contents and within the text by an asterisk (\*). The reader for an introductory control systems design course may choose to skip these sections.

Only a few years ago, the tools for analysis and synthesis available to the control engineer were paper, pencil, slide rule, spirule, and analog computer. The tools and the methods were simple enough that an engineer could easily master them in a relatively short time. However, over the past thirty years, control theory has evolved to a state where the digital computer has now become a requirement for the control system engineer, and Computer-Aided Control System Design (CACSD) environment has emerged as an indispensable tool.

A good CACSD program or package draws on expertise from many disciplines including aspects of computer engineering, computer science, applied mathematics (e.g., numerical analysis and optimization), as well as control systems engineering and theory. The need for such a breadth of knowledge is partially responsible for the paucity of high quality CACSD software, and indeed, CACSD can be considered still in its infancy. There has been, however, a certain degree of maturity among a number of CACSD packages. This point along with a historical view on the subject with an overview of 22 such programs is presented in Appendix B.

Chapter 1 serves as an introduction to the text. It provides a brief introduction to the design of multivariable systems and CACSD as a new discipline. A concise account of the evolution of modern control systems is presented.

Chapters 2 and 3 present the analysis of linear control systems. In Chap. 2, the state space and transfer function descriptions of continuous and discrete-time systems are reviewed. The various solutions of the state equation are also given. The properties of linear systems are discussed in Chap. 3, where the three primary properties—controllability, observability, and stability—are reviewed. Both conventional and Lyapunov stability concepts are covered. Other topics such as minimal realization, frequency domain criterion for controllability, duality, and state transformations are also discussed.

Chapters 4 to 7 constitute the main theme of this text—**Design**. Chapter 4 is concerned with feedback and observer design. In this chapter, state feedback pole placement (eigenvalue assignment) of linear time-invariant systems is treated. Both single-input and multi-input systems using unity or full rank feedback matrices are treated. The other main themes of this chapter are full and reduced order observer design and feedback control using observers.

Chapter 5 is concerned with output feedback and compensator design. Among topics discussed here are static output feedback, two and three term (PD, PI, and PID) output feedback, and dynamic compensator design. Optimal control design is the subject of Chap. 6. The optimal control problem has been treated through both the sufficiency conditions (Hamilton-Jacobi equation) and necessary conditions (the Minimum Principle). Both continuous-time and discrete-time optimal control problems are considered. Ample discussions are made on the state regulator problem and solutions to the matrix Riccati equations.

In Chap. 7, large-scale systems theory is reviewed briefly. Aggregated models

of large-scale systems and near-optimum design based on aggregated models are discussed first. The celebrated control of large-scale systems via hierarchical and decentralized control constitute the next two themes of this chapter.

Appendix A provides a brief review of linear algebra. Topics discussed here are spaces, independence, basis, inner product, norms, eigenvalues, eigenvectors, functions of a square matrix, Schur transformation, singular-value decomposition, and quadratic forms. The final presentation of the book is devoted to computer-aided control systems design. Appendix B provides a detail and, as much as possible, an up to date overview of CACSD programs available today. Some 22 programs have been mentioned in this appendix. The programs have been divided into two main groups. The first group are those based on MATLAB (original matrix laboratory software developed by Cleve Moler) and the second are those which have been developed independent of MATLAB. In this appendix, four of the seven MATLAB-based programs are reviewed in some detail. These are: **CTRL\_C** (c) Systems Control Technology, Inc., **MATRIXx** (c) Integrated Systems, Inc., **CONTROL.lab** (c) University of New Mexico's CAD Laboratory, and **PC\_MATLAB** (c) the Mathworks, Inc.

The notable non-MATLAB software programs reviewed here are **FREDOM—TIMDON—LSSPAK** (c), developed at University of New Mexico and **CADACS (KEDDC)** at Bochum University, West Germany, and **L-A-S** developed by Professor S. Bingularac of Virginia Polytechnic and State University. There are many more proven CACSD programs available such as **CC**, developed by Peter Thompson, **LUND**, developed by Lund Institute of Technology, Sweden. However, unfortunately because of lack of space, we could not cover all noteworthy CACSD programs in this book. Appendix B finishes with a brief survey and a discussion on the future of CACSD programs.

As a guide to potential instructors of this textbook, we offer the following suggestions: For a design-dominated course on control systems, cover the following chapters and sections: *Chapters 1, 2, 3, followed by Sec. 4.3, Chapter 5, Secs. 6.2, 7.3 to 7.4, and Appendix A.* For a CAD-dominated course on control systems, follow *the same chapters and sections, but with a great influence from Appendix B.* On the other hand, for a first course on linear multivariable systems, the following order of chapters and sections is recommended: *Chapters 1–4, followed by Secs. 5.2, 6.2, and Appendix A.* For advanced research, the following list of sections is recommended: *Sections 1.4, 3.8 to 3.11, 4.5, 5.2, 5.4, 6.3, 6.4, 7.1 to 7.4, and B.5.*

We wish to acknowledge the many colleagues, graduate students, and staff members of the University of New Mexico, San Diego State University, University of California at San Diego, and Northeastern University who have encouraged us or contributed to the development of this book. In particular, we wish to express our appreciation to Dr. R. H. Seacat (former Chair), Dr. N. Ahmed (current Chair) of the Department of Electrical and Computer Engineering, and Dr. J. E. Thompson, Dean of Engineering at the University of New Mexico. We wish to express our appreciation to Dr. D. Short, Dean of College of Sciences and Dr. J. Elwin, Chair of Department of Mathematical Sciences at San Diego State University, Dr. J. Arnold,

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## PREFACE TO CAD\_MCS SOFTWARE

We have incorporated a set of design and analysis modules for linear time-invariant systems into an **IBM\_PC**® based software package called **CAD\_MCS**. Although this software would not be able to implement every design technique described in the book, it can help solve a great many problems which would expand student's knowledge. User's guide for CAD\_MCS is available. This program allows analysis, design, and simulation of linear systems. This software program is basically for educational use. We strongly suggest that the readers use some CACSD package while studying this book.

This program can run on the IBM PC and its compatible computers (PC, XT, or AT), with two floppy or hard disk drives. Some versions of it will also run on Apple MacIntosh computers as well. A plotting capability would allow the user to plot the results. However, a graphics card is necessary.

The program **CAD\_MCS** is available from the first author on double-sided density 5¼" disks. An order form for CAD\_MCS, TIMDOM, LSSPAK, L-A-S, CADACS (KEDDC), as well as newly developed toolboxes for MATLAB on linear systems, large-scale systems, and robotics is included in the back of this book.

**Mohammad Jamshidi**, University of New Mexico, Albuquerque  
**Mahmoud Tarokh**, San Diego State University, San Diego  
**Bahram Shafai**, Northeastern University, Boston

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