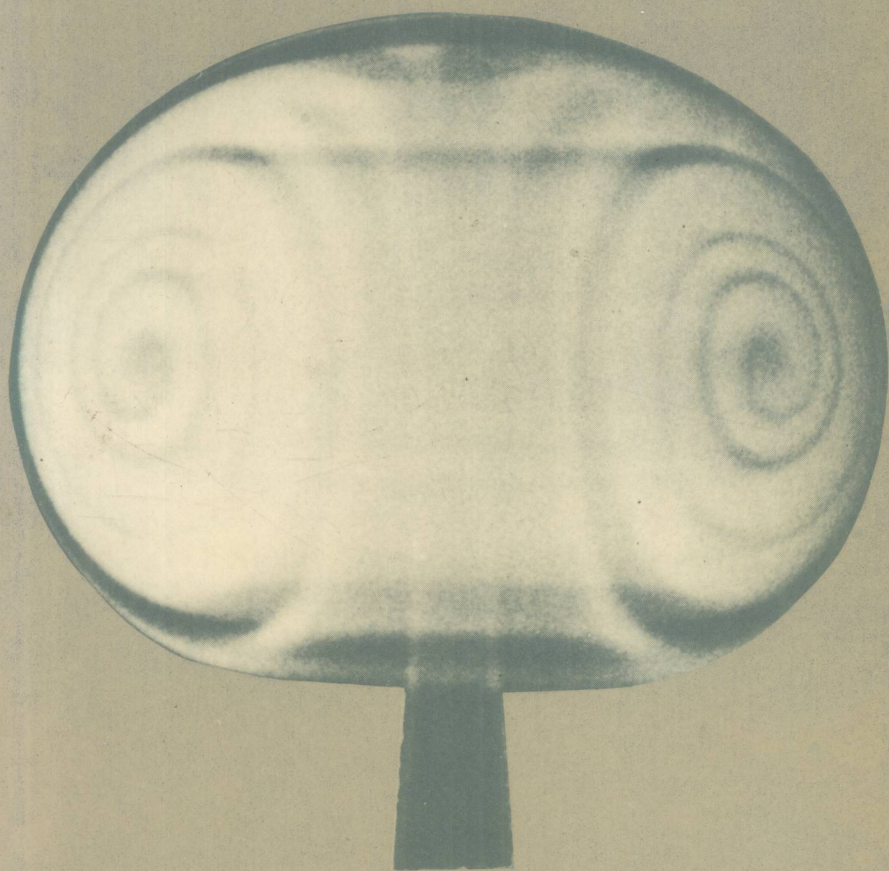


An Introduction to
**Fluid
Dynamics**



G.K. Batchelor

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AN INTRODUCTION TO FLUID DYNAMICS

BY

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PREFACE

While teaching fluid dynamics to students preparing for the various Parts of the Mathematical Tripos at Cambridge I have found difficulty over the choice of textbooks to accompany the lectures. There appear to be many books intended for use by a student approaching fluid dynamics with a view to its application in various fields of engineering, but relatively few which cater for a student coming to the subject as an applied mathematician and none which in my view does so satisfactorily. The trouble is that the great strides made in our understanding of many aspects of fluid dynamics during the last 50 years or so have not yet been absorbed into the educational texts for students of applied mathematics. A teacher is therefore obliged to do without textbooks for large parts of his course, or to tailor his lectures to the existing books. This latter alternative tends to emphasize unduly the classical analytical aspects of the subject, and the mathematical theory of irrotational flow in particular, with the probable consequence that the students remain unaware of the vitally important physical aspects of fluid dynamics. Students, and teachers too, are apt to derive their ideas of the content of a subject from the topics treated in the textbooks they can lay hands on, and it is undesirable that so many of the books on fluid dynamics for applied mathematicians should be about problems which are mathematically solvable but not necessarily related to what happens in real fluids.

I have tried therefore to write a textbook which can be used by students of applied mathematics and which incorporates the physical understanding and information provided by past research. Despite its bulk this book is genuinely an introduction to fluid dynamics; that is to say, it assumes no previous knowledge of the subject and the material in it has been selected to introduce a reader to the important ideas and applications. The book has grown out of a number of courses of lectures, and very little of the material has not been tested in the lecture room. Some of the material is old and well known, some of it is relatively new; and for all of it I have tried to devise the presentation which appears to be best from a consistent point of view. The book has been prepared as a connected account, intended to be read and worked on as a whole, or at least in large portions, rather than to be referred to for particular problems or methods.

I have had the needs of second-, third- and fourth-year students of applied mathematics in British universities particularly in mind, these being the needs with which I am most familiar, although I hope that engineering students will also find the book useful. The true needs of applied mathe-

maticians and engineers are nowadays not far apart. Both require above all an understanding of the fundamentals of fluid dynamics; and this can be achieved without the use of advanced mathematical techniques. Anyone who is familiar with vector analysis and the notation of tensors should have little difficulty with the purely mathematical parts of this work. The book is fairly heavily weighted with theory, but not with mathematics.

Attention is paid throughout the book to the correspondence between observation and the various conceptual and analytical models of flow systems. The photographs of flow systems that are included are an essential part of the book, and will help the reader, I hope, to develop a sense of the reality that lies behind the theoretical arguments and analysis. This is particularly important for students who do not have an opportunity of seeing flow phenomena in a laboratory. The various books and lectures by L. Prandtl seem to me to show admirably the way to keep both theory and observation continually in mind, and I have been greatly influenced by them. Prandtl knew in particular the value of a clear photograph of a well-designed experimental flow system, and many of the photographs taken by him are still the best available illustrations of boundary-layer phenomena.

A word is necessary about the selection of topics in this book and the order in which they have been placed. My original intention was to provide between two covers an introduction to all the main branches of fluid dynamics, but I soon found that this comprehensiveness was incompatible with the degree of thoroughness that I also had in mind. I decided therefore to attempt only a partial coverage, at any rate so far as this volume is concerned. The first three chapters prepare the ground for a discussion of any branch of fluid dynamics, and are concerned with the physical properties of fluids, the kinematics of a flow field, and the dynamical equations in general form. The purpose of these three introductory chapters is to show how the various branches of fluid dynamics fit into the subject as a whole and rest on certain idealizations or assumptions about the nature of the fluid or the motion. A teacher is unlikely to wish to include all this preliminary material in a course of lectures, but it can be adapted to suit a specialized course and will I hope be useful as background. In the remaining four chapters the fluid is assumed to be incompressible and to have uniform density and viscosity. I regard flow of an incompressible viscous fluid as being at the centre of fluid dynamics by virtue of its fundamental nature and its practical importance. Fluids with unusual properties are fashionable in research, but most of the basic dynamical ideas are revealed clearly in a study of rotational flow of a fluid with internal friction; and for applications in geophysics, chemical engineering, hydraulics, mechanical and aeronautical engineering, this

is still the key branch of fluid dynamics. I regret that many important topics such as gas dynamics, surface waves, motion due to buoyancy forces, turbulence, heat and mass transfer, and magneto-fluid dynamics, are apparently ignored, but the subject is simply too large for proper treatment in one volume. If the reception given to the present book suggests that a second volume would be welcome, I may try later to make the coverage more nearly complete.

As to the order of material in chapters 4 to 7, the description of motion of a viscous fluid and of flow at large Reynolds number precedes the discussion of irrotational flow (although the many purely kinematical properties of an irrotational velocity distribution have a natural place in chapter 2) and of motion of an inviscid fluid with vorticity. My reason for adopting this unconventional arrangement is not that I believe the 'classical' theory of irrotational flow is less important than is commonly supposed. It is simply that results concerning the flow of inviscid fluid can be applied realistically only if the circumstances in which the approximation of zero viscosity is valid are first made clear. The mathematical theory of irrotational flow is a powerful weapon for the solution of problems, but in itself it gives no information about whether the whole or a part of a given flow field at large Reynolds number will be approximately irrotational. For that vital information some understanding of the effects of viscosity of a real fluid and of boundary-layer theory is essential; and, whereas the understanding was lacking when Lamb wrote his classic treatise *Hydrodynamics*, it is available today. I believe that the first book, at least in English, to show how so many common flow systems could be understood in terms of boundary layers and separation and vorticity movement was *Modern Developments in Fluid Dynamics*, edited by Sydney Goldstein. That pioneering book published in 1938 was aimed primarily at research workers, and I have tried to take the further step of making the understanding of the flow of real fluids accessible to students at an early stage of their study of fluid dynamics.

Desirable though it is for study of the flow of viscous fluids to precede consideration of an inviscid fluid and irrotational flow, I appreciate that a lecturer may have his hand forced by the available lecturing time. In the case of mathematics students who are to attend only one course on fluid dynamics, of length under about 30 lectures, it would be foolish to embark on a study of viscous fluid flow and boundary layers in preparation for a description of inviscid-fluid flow and its applications, since too little time would be left for this topic; the lecturer would need to compromise with scientific logic, and could perhaps take his audience from chapters 2 and 3

to chapter 6, with some of the early sections of chapters 5 and 7 included. It is a difficulty inherent in the teaching of fluid dynamics to mathematics undergraduates that a partial introduction to the subject is unsatisfactory, tending to leave them with analytical procedures and results but no information about when they are applicable. Furthermore, students do take some time to grasp the principles of fluid dynamics, and I suggest that 40 to 50 lectures are needed for an adequate introduction of the subject to non-specialist students. However, a book is not subject to the same limitations as a course of lectures. I hope lecturers will agree that it is desirable for students to be able to see all the material set out in logical order, and to be able to improve their own understanding of the subject by reading, even if in a course of lectures many important topics such as boundary-layer separation must be ignored.

Exercises are an important part of the process of understanding and mastering so analytical a subject as fluid dynamics, and the reading of this text should be accompanied by the working of illustrative exercises. I should have liked to be able to provide many suitable questions and exercises, but a search among those already published in various places did not produce many in keeping with the approach adopted in this book. Moreover, the published exercises are concentrated on a small number of topics. The lengthy task of devising and compiling suitable exercises over the whole field of 'modern' fluid dynamics has yet to be undertaken. Consequently only a few exercises will be found at the end of sections. To some extent exercises ought to be chosen to suit the particular background and level of the class for which they are intended, and it may be that a lecturer can turn into exercises for his class many portions of the text not included explicitly in his course of lectures, as I have done in my own teaching.

It is equally important that a course of lectures on the subject matter of this book should be accompanied by demonstrations of fluid flow. Here the assistance of colleagues in a department of engineering may be needed. The many films on fluid dynamics that are now available are particularly valuable for classes of applied mathematicians who do not undertake any laboratory work. By one means or another, a teacher should show the relation between his analysis and the behaviour of real fluids; fluid dynamics is much less interesting if it is treated largely as an exercise in mathematics.

I am indebted to a large number of people for their assistance in the preparation of this book. Many colleagues kindly provided valuable comments on portions of the manuscript, and enabled me to see things more clearly. I am especially grateful to Philip Chatwin, John Elder, Emin

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G. K. B.

Cambridge
April 1967

CONVENTIONS AND NOTATION

Bold type signifies vector character.

\mathbf{x}, \mathbf{x}' position vectors; $|\mathbf{x}| = r$

$\mathbf{s} = \mathbf{x} - \mathbf{x}'$ relative position vector

\mathbf{u} velocity at a specified time and position in space; $|\mathbf{u}| = q$

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ operator giving the material derivative, or rate of change at a point moving with the fluid locally; applies only to functions of \mathbf{x} and t

System	Co-ordinates	Velocity components
Rectilinear	x, y, z or x_1, x_2, x_3	u, v, w or u_1, u_2, u_3
Polar, two dimensions	r, θ	u, v or u_r, u_θ
Spherical polar	r, θ, ϕ	u, v, w or u_r, u_θ, u_ϕ
Cylindrical	x, σ, ϕ ($\sigma^2 = y^2 + z^2$)	u, v, w or u_x, u_σ, u_ϕ

$\Delta = \nabla \cdot \mathbf{u}$ rate of expansion (fractional rate of change of volume of a material element)

$\boldsymbol{\omega} = \nabla \times \mathbf{u}$ vorticity (twice the local angular velocity of the fluid)

$e_{ij} = \frac{1}{2} \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)$ rate-of-strain tensor

ϕ scalar potential of an irrotational velocity distribution ($\mathbf{u} = \nabla \phi$)

\mathbf{B} vector potential of a solenoidal velocity distribution ($\mathbf{u} = \nabla \times \mathbf{B}$)

ψ stream function for a solenoidal velocity distribution;

(a) two-dimensional flow: $\mathbf{B} = (0, 0, \psi)$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad \text{or} \quad u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad u_\theta = -\frac{\partial \psi}{\partial r}$$

(b) axisymmetric flow:

$$\text{cylindrical co-ordinates } B_\phi = \frac{\psi}{\sigma}, \quad u_x = \frac{1}{\sigma} \frac{\partial \psi}{\partial \sigma}, \quad u_\sigma = -\frac{1}{\sigma} \frac{\partial \psi}{\partial x}$$

$$\text{polar co-ordinates } B_\phi = \frac{\psi}{r \sin \theta}, \quad u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, \quad u_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}$$

\mathbf{n} unit normal to a surface, usually outward if the surface is closed

$\delta V, \mathbf{n} \delta A, \delta \mathbf{x}$ volume, surface and line elements with a specified position in space

$\delta \tau, \mathbf{n} \delta S, \delta \mathbf{l}$ material volume, surface and line elements

σ_{ij} stress tensor; $\sigma_{ij} n_j \delta A$ is the i -component of the force exerted across the surface element $\mathbf{n} \delta A$ by the fluid on the side to which \mathbf{n} points

$\mathbf{F} = -\nabla \Psi$ conservative body force per unit mass

Inertia force (per unit mass) minus the local acceleration

Vortex-line line whose tangent is parallel to $\boldsymbol{\omega}$ locally

Line vortex singular line in vorticity distribution round which the circulation is non-zero

Books which may provide collateral reading are cited in detail in the text, usually in footnotes. A comparatively small number of original papers are also referred to, sometimes for historical interest, sometimes because a precise acknowledgement is appropriate, and sometimes, although only rarely, as a guide to further reading on a particular topic. These papers are cited in the text as 'Smith (1950)', and the full references for both papers and books are listed at the end of the book.

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