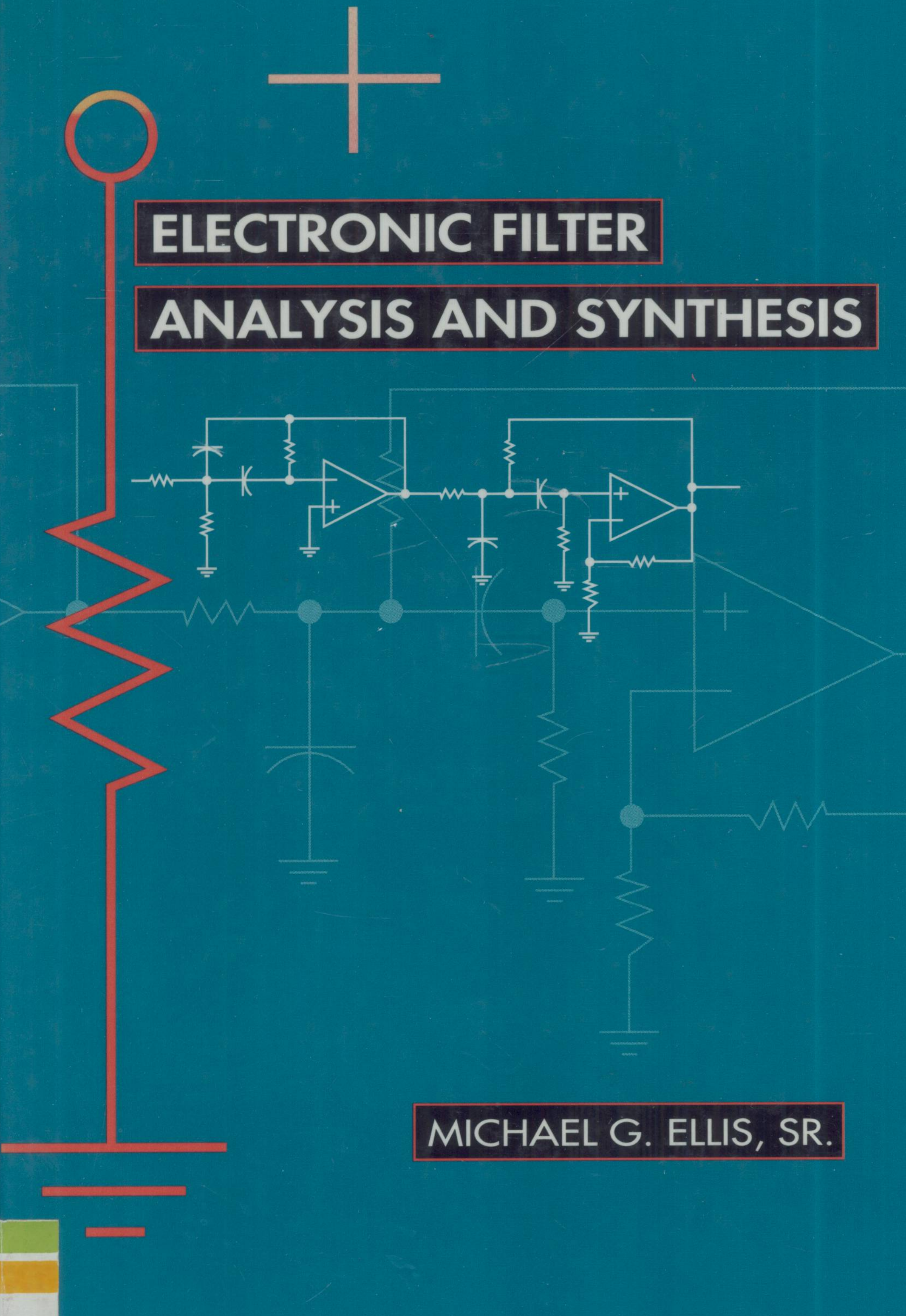
# ELECTRONIC FILTER ANALYSIS AND SYNTHESIS



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# Electronic Filter Analysis and Synthesis

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*To Mary Kay*
*and our children*
*Michael, David, and Sara*

# *Preface*

This text presents the derivation of design techniques for electronic filters in a manner conducive for writing computer-aided design and analysis programs. It is assumed that the reader has an understanding of the basic procedures used to analyze electric circuits and systems, such as taught in an undergraduate network theory series. Some familiarity with numerical analysis and a computer language would be helpful. The text is written at a senior or graduate level for students and practicing engineers who desire to understand the theory behind filter design methodology. To a large extent, tabulation of resistor, inductor, and capacitor component values for filters has been avoided because the formulas given in the text allow the engineer to generate tables of component values for a large variety of circuit configurations. The prototype computer programs given in Appendix B are meant to supplement the derivations in the text by providing a starting point for those interested in programming more exotic designs. A complete set of compiled programs, which illustrate the methods given in this text, are also available from Artech House, Inc.

The book can be divided into four main sections, categorized by passive, active, digital, and switched capacitor realizations. Chapter 1 and Appendix A are intended to be a review of background material from elementary numerical analysis and network theory. Basic methods are given for the analysis of analog and digital filters with regard to magnitude, phase, group delay, impulse and step response.

Chapter 2 provides a foundation for understanding the low-pass Butterworth filter. Simple equations are given for determining L-C component values. The locations of the Butterworth poles are defined and formulas are derived for the order, attenuation, and response of Butterworth functions.

Chapters 3 through 5 derive and define similar concepts for low-pass Tchebyscheff, low-pass inverse Tchebyscheff filters, and low-pass elliptic filters. For the elliptic filter type, two computer programs are given in Appendix B as an aid for those interested in programming elliptic filter solutions.

Chapter 6 deals with the transformation of low-pass filters to other bandpass topologies. Impedance scaling, frequency scaling, duality, reciprocity, network transformations, two-port network parameters, lattice structures, and group delay equalizer topologies for passive filters are covered in detail.

The realization of unevenly terminated passive filters is treated in Chapter 7. The equation for input impedance is derived based solely on network theory and used to generate unequally terminated structures for Butterworth, Tchebyscheff, and elliptic filters. Both classical filter synthesis and the permutation method of Amstutz are discussed in the synthesis of elliptic configurations.

Chapter 8 provides over 16 active filter topologies, complete with derivations. Poles and zeros, determined mathematically from the methods given in Chapters 2–5 and Chapter 7, can be used to generate low-pass, bandpass, bandstop, and high-pass active circuits. Filter types include multiple feedback, voltage controlled voltage source, state-variable, and many other configurations. A method for optimization of the gain of each active filter section in a cascade is presented as a means to maximize dynamic range.

Chapter 9 introduces infinite impulse response digital filters using the bilinear transform to map the analog domain into the digital domain. The effects of frequency warping and aliasing are discussed.

In Chapter 10, the Fourier transform is presented as a design tool for finite impulse response (FIR) filters. Window types are given including triangular, Hamming, Hanning, Kaiser, and Blackman. A brief discussion of sigma-delta is given as it pertains to high-resolution analog-to-digital conversion.

The exchange method is presented in Chapter 11 as an optimization technique useful for determining a minimax polynomial that approximates a desired set of data points with an equiripple error. The Parks-McClellan algorithm is shown to be an extension of the exchange method applied to FIR digital filters to generate a minimax error in both the passband and the stopband.

Chapter 12 deals with switched capacitor filters. The concept of equivalent resistance in a switched capacitor is introduced. Charge conservation is used as an analysis tool for obtaining the $z$-domain transfer function. Several parasitic insensitive switched capacitor circuit topologies are given for use in low-pass, bandpass, bandstop, or high-pass circuits. Consideration is given to the effect of nonideal op-amp characteristics.

Equations shown in bold represent fundamental concepts from which other equations will be derived.

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# Chapter 1
# Basic Network Theory

## 1.1 INTRODUCTION

Modern filter theory is based on many diverse but interrelated discoveries over the past two centuries. In 1785 Laplace [1] wrote the almost modern Laplace transform to solve certain differential equations with variable coefficients. Baron Jean-Baptiste-Joseph Fourier [2], as a prefect in Napoleon's army, invented a method for solving equations that described the conduction of heat in solid bodies, and it was published in 1807. Laplace was intrigued and obtained additional results that in turn inspired Fourier's discovery of his own transform. Many nineteenth century mathematicians did not accept Fourier's claim that any initial temperature distribution could be decomposed into a simple arithmetic sum consisting of a fundamental variation and higher frequency harmonics, and the theoretical basis of the Fourier transform was not formally published until 1822.

In 1829, Carl Gustav Jacob Jacobi derived and reported the main properties of a set of new functions useful for solving elliptic integrals. At that time, the elliptic functions were used to solve problems in finding the length of an arc of an ellipse and in determining the motion of a pendulum.

The first known record of sampling analog signals is found on pages 420–425 of the *Treatise on Electricity and Magnetism* (1873) by James Clark Maxwell [3] in his discussion on the equivalent resistance of a periodically switched capacitor.

Filter technology was officially born in 1915 when K. Wagner (Germany) and G. Campbell (United States), working independently, proposed the basic concept of the filter. Their results evolved from earlier work on loaded transmission lines and the classical theory of vibrating systems.

In 1923, Zobel [4] reported a practical method of designing image parameter filters with an unlimited number of reactances. This filter method is summarized in Figure 1.1 but will not be discussed further because it is a little used technique that gives the designer only a very loose control over the passband and stopband characteristics. However, it was the only known method until 1939 and the only practical method until computers became widespread in the mid-1950s. S. Darlington [5] in the United States and W. Cauer in Germany, both inspired by the work of Norton, produced a theory in 1939 that involved a set of problems relating to modern synthesis procedure. The filters proposed by Darlington and Cauer used the Jacobian elliptic integral to determine element values for very sharp roll-off filters with Tchebyscheff (or equiripple) behavior in both the passband and the stopband.

This behavior takes its name from the Russian mathematician who in the last century developed a powerful method of approximation by means of orthogonal polynomials. The elliptic filters of Darlington and Cauer should not be confused with Tchebyscheff filters because the latter exhibit equiripple behavior in either the passband or the stopband but not both. The importance of polynomial filter synthesis was not recognized immediately because of the extremely heavy burden required on computation. It was not until the mid-1950s that Cauer-Darlington (elliptic) filters came into widespread use.

The theory of sampling analog signals was also well developed by the late 1950s. In the 1960s several schemes were proposed that used switches and capacitors to simulate resistors in filters. It was shown that the filter transmittances had the important property of depending only on capacitor ratios.

The Fairchild uA 709 was introduced in 1965 at $70 and became the first integrated circuit (IC) operational amplifier (op-amp) manufactured with such high yields in volume production that it facilitated the implementation of active RC filters equaling in performance and usefulness its passive counterpart. Although the op-amp was a major component of the filter circuit and eliminated the need for bulky inductors, discrete resistors and capacitors were still required because IC technology did not permit resistors and capacitors to be integrated with a high degree of accuracy.

That same year, James W. Cooley of IBM's Thomas J. Watson Research Center and John W. Tukey [6] of the Bell Telephone Laboratories in Murray Hill, N.J., rediscovered a program known as the fast Fourier transform (FFT), which allowed a speed increase of up to several orders of magnitude in the calculation of the Fourier transform. The FFT has become the basis for the implementation of many digital filters because the computations can be done in real time by high-speed digital signal processor (DSP) chips.

The importance and significance of many of the ideas proposed for analog sampled data signal processing had to wait until technology provided the means of turning ideas into practical reality. In 1972, it was suggested that metal oxide semiconductor (MOS) technology would be applicable for the construction of integrated circuit analog sampled data filters using switched capacitors to simulate resistors in active filters. This suggestion was then followed by a rapid development in the implementation of analog signal processing integrated circuits. The switched capacitor techniques provided very high accuracy in the design of analog IC filters because the filter parameters depended only on the ratios of capacitors and this ratio can be controlled to a very high accuracy in the design of MOS integrated circuits.

## The Zobel Image Parameter Filter Design Method

Let $f_c$ be the cutoff frequency of the low-pass prototype in Hertz. Then

$$\omega_c = 2\pi f_c$$

$$R = \text{termination resistance (equal at both ends)}$$

$$m = 0.6$$

compute

$$L = \frac{2R}{\omega_c} \qquad C = \frac{2}{R \cdot \omega_c}$$

The filter components can now be calculated as

$$L_{a1} = \frac{(1 - m^2) \cdot L}{2 \cdot m} \qquad L_b = \frac{L}{2} \qquad L_{c1} = \frac{x \cdot L}{2} \qquad L_{a1} = \frac{(1 - m^2) \cdot L}{2 \cdot m}$$

$$C_a = \frac{m \cdot C}{2} \qquad C_b = C \qquad C_c = x \cdot C \qquad C_a = \frac{m \cdot C}{2}$$

$$L_{a2} = \frac{m \cdot L}{2} \qquad L_{c2} = \frac{(1 - x^2) \cdot L}{4 \cdot x} \qquad L_{a2} = \frac{m \cdot L}{2}$$

where $x = \sqrt{[1 - (\omega_c/\omega_\infty)^2]}$.

The frequency, $\omega_\infty$, must be specified by the designer in the stopband for each *M*-derived section. The total number of constant-*K* sections, the number of *M*-derived sections, and an $\omega_\infty$ for each *M*-derived section must be specified. The sections are then cascaded to create the entire filter.

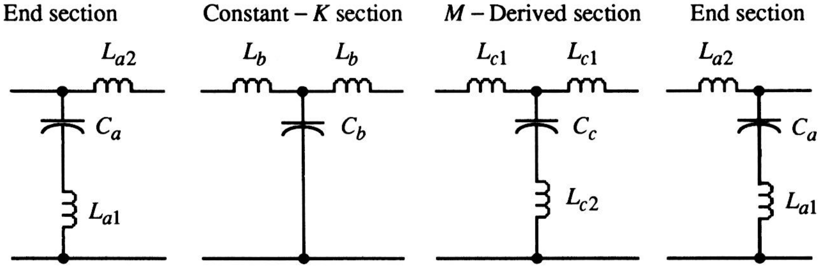


**Figure 1.1** Image parameter filters were popular during the 1940s because they require very simple calculations to design.

This chapter reviews the fundamental theories essential for the design and analysis of electronic filters. The design methods derived later in this text are based on developments over the last 200 years and cover the majority of synthesis techniques in use today. The four major classes of polynomial filters are discussed in sufficient detail to allow practicing engineers flexibility in specifying design parameters. Algorithms for the more advanced designs are given as prototype programs in Appendix B. A complete set of computer aided design and analysis tools, corresponding to the methods used in this text, is also available from Artech House, Inc.

## 1.2 MAGNITUDE AND PHASE RESPONSE OF ANALOG TRANSFER FUNCTIONS

An analog voltage transfer function, $G(s)$, with real poles and zeroes, of the form

$$G(s) = \frac{(s - s_{01})(s - s_{02}) \ldots (s - s_{0M})}{(s - s_1)(s - s_2) \ldots (s - s_N)} \tag{1.1}$$

has a magnitude response as a function of frequency, $\omega$, given by

$$|G(s)| = \frac{\sqrt{\omega^2 + s_{01}^2}\sqrt{\omega^2 + s_{02}^2} \ldots \sqrt{\omega^2 + s_{0M}^2}}{\sqrt{\omega^2 + s_1^2}\sqrt{\omega^2 + s_2^2} \ldots \sqrt{\omega^2 + s_N^2}} \tag{1.2}$$

If the poles (roots of the denominator) and zeroes (roots of the numerator) are complex, then $G(s)$ can be written as

$$G(s) = \frac{(s^2 + A_{01}s + B_{01})(s^2 + A_{02}s + B_{02}) \ldots (s^2 + A_{0M}s + B_{0M})}{(s^2 + A_1 s + B_1)(s^2 + A_2 s + B_2) \ldots (s^2 + A_N s + B_N)} \tag{1.3}$$

with a magnitude response as a function of frequency, $\omega$, as given by equation (1.4):

$$|G(s)| = \frac{\sqrt{(B_{01} - \omega^2)^2 + (A_{01}\omega)^2}\sqrt{(B_{02} - \omega^2)^2 + (A_{02}\omega)^2} \ldots \sqrt{(B_{0M} - \omega^2)^2 + (A_{0M}\omega)^2}}{\sqrt{(B_1 - \omega^2)^2 + (A_1\omega)^2}\sqrt{(B_2 - \omega^2)^2 + (A_2\omega)^2} \ldots \sqrt{(B_N - \omega^2)^2 + (A_N\omega)^2}} \tag{1.4}$$

In both cases the magnitude response was formed by taking the square root of the sum of the squares of the real part and the imaginary part of each section. Similarly the phase response is given by the arc tangent of the imaginary component divided by the real component. For equation (1.1), the phase response becomes

$$\angle G(s) = \tan^{-1}\left(\frac{\omega}{-s_{01}}\right) + \tan^{-1}\left(\frac{\omega}{-s_{02}}\right) + \ldots \tan^{-1}\left(\frac{\omega}{-s_{0M}}\right) - \tan^{-1}\left(\frac{\omega}{-s_1}\right) - \tan^{-1}\left(\frac{\omega}{-s_2}\right) - \ldots \tan^{-1}\left(\frac{\omega}{-s_N}\right) \quad (1.5)$$

whereas for equation (1.3), the phase response is

$$\angle G(s) = \tan^{-1}\left(\frac{A_{01}\omega}{B_{01} - \omega^2}\right) + \tan^{-1}\left(\frac{A_{02}\omega}{B_{02} - \omega^2}\right) + \ldots + \tan^{-1}\left(\frac{A_{0M}\omega}{B_{0M} - \omega^2}\right) - \tan^{-1}\left(\frac{A_1\omega}{B_1 - \omega^2}\right) - \tan^{-1}\left(\frac{A_2\omega}{B_2 - \omega^2}\right) - \ldots - \tan^{-1}\left(\frac{A_N\omega}{B_N - \omega^2}\right) \quad (1.6)$$

## 1.3 TIME DELAY AND GROUP DELAY OF ANALOG FILTERS

The phase response of a filter affects the transient response, which is of particular concern in pulse systems such as radar and digital communications. In these cases, the pulse rise time and overshoot are of interest because they indicate the degree of pulse distortion. If a filter is such that the output pulse is an exact replica of the input pulse, except for a time delay, then the transient response is ideal. The transfer function of this ''ideal'' filter is given by $G(s) = e^{-sT} = e^{-j\omega T}$, where $T$ is the time delay in seconds. The phase of $G(s)$ is given by

$$\angle G(s) = \angle e^{-sT} = \angle[\cos(\omega T) - j\sin(\omega T)] = \tan^{-1}\left(\frac{-\sin(\omega T)}{\cos(\omega T)}\right) = -\omega T \quad (1.7)$$

From equation (1.7) the time delay of a linear phase filter is given by $-\phi/\omega$ seconds; however, in practice a constant group delay is often sufficient to achieve linear phase over a portion of the frequency band of interest. The group delay of a filter is defined by

$$\textbf{group delay} = -\frac{d\phi}{d\omega} \quad \textbf{(1.8)}$$

Using the relation

$$\frac{d}{du}\arctan(u) = \frac{1}{1+u^2}\frac{du}{d\omega} \quad \textbf{(1.9)}$$

the expression for group delay based on equation (1.6) becomes