ADVANCES IN CHEMICAL PHYSICS VOLUME 119 PART I
I. PRIGOGINE and STUART A. RICE, Series Editors

Edited by
MYRON EVANS

MODERN NONLINEAR OPTICS
Part I

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Part 1

Second Edition

ADVANCES IN CHEMICAL PHYSICS
VOLUME 119

Edited by

Myron W. Evans

Series Editors

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Published simultaneously in Canada.

For ordering and customer service, call 1-800-CALL-WILEY

Library of Congress Catalog Number: 58-9935

ISBN 0-471-38930-7

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

MODERN NONLINEAR OPTICS
Part 1
Second Edition

ADVANCES IN CHEMICAL PHYSICS

VOLUME 119

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Part 1

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INTRODUCTION

Few of us can any longer keep up with the flood of scientific literature, even in specialized subfields. Any attempt to do more and be broadly educated with respect to a large domain of science has the appearance of tilting at windmills. Yet the synthesis of ideas drawn from different subjects into new, powerful, general concepts is as valuable as ever, and the desire to remain educated persists in all scientists. This series, *Advances in Chemical Physics*, is devoted to helping the reader obtain general information about a wide variety of topics in chemical physics, a field that we interpret very broadly. Our intent is to have experts present comprehensive analyses of subjects of interest and to encourage the expression of individual points of view. We hope that this approach to the presentation of an overview of a subject will both stimulate new research and serve as a personalized learning text for beginners in a field.

I. PRIGOGINE
STUART A. RICE

PREFACE

This volume, produced in three parts, is the Second Edition of Volume 85 of the series, *Modern Nonlinear Optics*, edited by M. W. Evans and S. Kielich. Volume 119 is largely a dialogue between two schools of thought, one school concerned with quantum optics and Abelian electrodynamics, the other with the emerging subject of non-Abelian electrodynamics and unified field theory. In one of the review articles in the third part of this volume, the Royal Swedish Academy endorses the complete works of Jean-Pierre Vigier, works that represent a view of quantum mechanics opposite that proposed by the Copenhagen School. The formal structure of quantum mechanics is derived as a linear approximation for a generally covariant field theory of inertia by Sachs, as reviewed in his article. This also opposes the Copenhagen interpretation. Another review provides reproducible and repeatable empirical evidence to show that the Heisenberg uncertainty principle can be violated. Several of the reviews in Part 1 contain developments in conventional, or Abelian, quantum optics, with applications.

In Part 2, the articles are concerned largely with electrodynamical theories distinct from the Maxwell–Heaviside theory, the predominant paradigm at this stage in the development of science. Other review articles develop electrodynamics from a topological basis, and other articles develop conventional or U(1) electrodynamics in the fields of antenna theory and holography. There are also articles on the possibility of extracting electromagnetic energy from Riemannian spacetime, on superluminal effects in electrodynamics, and on unified field theory based on an SU(2) sector for electrodynamics rather than a U(1) sector, which is based on the Maxwell–Heaviside theory. Several effects that cannot be explained by the Maxwell–Heaviside theory are developed using various proposals for a higher-symmetry electrodynamical theory. The volume is therefore typical of the second stage of a paradigm shift, where the prevailing paradigm has been challenged and various new theories are being proposed. In this case the prevailing paradigm is the great Maxwell–Heaviside theory and its quantization. Both schools of thought are represented approximately to the same extent in the three parts of Volume 119.

As usual in the *Advances in Chemical Physics* series, a wide spectrum of opinion is represented so that a consensus will eventually emerge. The prevailing paradigm (Maxwell–Heaviside theory) is ably developed by several groups in the field of quantum optics, antenna theory, holography, and so on, but the paradigm is also challenged in several ways: for example, using general relativity, using O(3) electrodynamics, using superluminal effects, using an extended electrodynamics based on a vacuum current, using the fact that longitudinal waves may appear in vacuo on the U(1) level, using a reproducible and repeatable device, known as the *motionless electromagnetic generator*, which extracts electromagnetic energy from Riemannian spacetime, and in several other ways. There is also a review on new energy sources. Unlike Volume 85, Volume 119 is almost exclusively dedicated to electrodynamics, and many thousands of papers are reviewed by both schools of thought. Much of the evidence for challenging the prevailing paradigm is based on empirical data, data that are reproducible and repeatable and cannot be explained by the Maxwell–Heaviside theory. Perhaps the simplest, and therefore the most powerful, challenge to the prevailing paradigm is that it cannot explain interferometric and simple optical effects. A non-Abelian theory with a Yang–Mills structure is proposed in Part 2 to explain these effects. This theory is known as O(3) *electrodynamics* and stems from proposals made in the first edition, Volume 85.

As Editor I am particularly indebted to Alain Beaulieu for meticulous logistical support and to the Fellows and Emeriti of the Alpha Foundation's Institute for Advanced Studies for extensive discussion. Dr. David Hamilton at the U.S. Department of Energy is thanked for a Website reserved for some of this material in preprint form.

Finally, I would like to dedicate the volume to my wife, Dr. Laura J. Evans.

MYRON W. EVANS

Ithaca, New York

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QUANTUM NOISE IN NONLINEAR OPTICAL PHENOMENA

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CONTENTS

I. INTRODUCTION

More than a century has passed since Planck discovered that it is possible to explain properties of the blackbody radiation by introducing discrete packets of energy, which we now call *photons*. The idea of discrete or quantized nature of energy had deep consequences and resulted in development of quantum mechanics. The quantum theory of optical fields is called *quantum optics*. The construction of lasers in the 1960s gave impulse to rapid development of nonlinear optics with a broad variety of nonlinear optical phenomena that have been

Modern Nonlinear Optics, Part 1, Second Edition, Advances in Chemical Physics, Volume 119, Edited by Myron W. Evans. Series Editors I. Prigogine and Stuart A. Rice.
ISBN 0-471-38930-7

experimentally observed and described theoretically and now are the subject of textbooks [1,2]. In early theoretical descriptions of nonlinear optical phenomena, the quantum nature of optical fields has been ignored on the grounds that laser fields are so strong, that is, the number of photons associated with them are so huge, that the quantum properties assigned to individual photons have no chances to manifest themselves. However, it turned out pretty soon that quantum noise associated with the vacuum fluctuations can have important consequences for the course of nonlinear phenomena. Moreover, it appeared that the quantum noise itself can change essentially when the quantum field is subject to the nonlinear transformation that is the essence of any nonlinear process. The quantum states with reduced quantum noise for a particular physical quantity can be prepared in various nonlinear processes. Such states have no classical counterparts; that is, the results of some physical measurements cannot be explained without explicit recall to the quantum character of the field. The methods of theoretical description of quantum noise are the subject of Gardiner's book [3]. This chapter is not intended as a presentation of general methods that can be found in the book; rather, we want to compare the results obtained with a few chosen methods for the two, probably most important, nonlinear processes: second-harmonic generation and downconversion with quantum pump.

Why have we chosen the second-harmonic generation and the downconversion to illustrate consequences of field quantization, or a role of quantum noise, in nonlinear optical processes? The two processes are at the same time similar and different. Both of them are described by the same interaction Hamiltonian, so in a sense they are similar and one can say that they show different faces of the same process. However, they are also different, and the difference between them consists in the different initial conditions. This difference appears to be very important, at least at early stages of the evolution, and the properties of the fields produced in the two processes are quite different. With these two best-known and practically very important examples of nonlinear optical processes, we would like to discuss several nonclassical effects and present the most common theoretical approaches used to describe quantum effects. The chapter is not intended to be a complete review of the results concerning the two processes that have been collected for years. We rather want to introduce the reader who is not an expert in quantum optics into this fascinating field by presenting not only the results but also how they can be obtained with presently available computer software. The results are largely illustrated graphically for easier comparisons. In Section II we introduce basic definitions and the most important formulas required for later discussion. Section III is devoted to presentation of results for second-harmonic generation, and Section IV results for downconversion. In the Appendixes A and B we have added examples of computer programs that illustrate usage of really existing software and were actually used in our calculations. We draw special attention to symbolic calculations and numerical methods, which can now be implemented even on small computers.

II. BASIC DEFINITIONS

In classical optics, a one mode electromagnetic field of frequency ω, with the propagation vector **k** and linear polarization, can be represented as a plane wave

$$E(\mathbf{r}, t) = 2E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi) \tag{1}$$

where E_0 is the amplitude and φ is the phase of the field. Assuming the linear polarization of the field, we have omitted the unit polarization vector to simplify the notation. Classically, both the amplitude E_0 and the phase φ can be well-defined quantities, with zero noise. Of course, the two quantities can be considered as classical random variables with nonzero variances; thus, they can be noisy in a classical sense, but there is no relation between the two variances and, in principle, either of them can be rendered zero giving the noiseless classical field. Apart from a constant factor, the squared real amplitude, E_0^2, is the intensity of the field. In classical electrodynamics there is no real need to use complex numbers to describe the field. However, it is convenient to work with exponentials rather than cosine and sine functions and the field (1) is usually written in the form

$$E(\mathbf{r}, t) = E^{(+)} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + E^{(-)} e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \tag{2}$$

with the complex amplitudes $E^{\pm} = E_0 e^{\pm i\varphi}$. The modulus squared of such an amplitude is the intensity of the field, and the argument is the phase. Both intensity and the phase can be measured simultaneously with arbitrary accuracy.

In quantum optics the situation is dramatically different. The electromagnetic field E becomes a quantum quantity; that is, it becomes an operator acting in a Hilbert space of field states, the complex amplitudes $E^{\pm}$ become the annihilation and creation operators of the electromagnetic field mode, and we have

$$\hat{E} = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} [\hat{a} e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + \hat{a}^{+} e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega t)}] \tag{3}$$

with the bosonic commutation rules

$$[\hat{a}, \hat{a}^{+}] = 1 \tag{4}$$

for the annihilation ($\hat{a}$) and creation ($\hat{a}^{+}$) operators of the field mode, where ε_0 is the electric permittivity of free space and V is the quantization volume. Because of laws of quantum mechanics, optical fields exhibit an inherent quantum indeterminacy that cannot be removed for principal reasons no matter how smart we are. The quantity

$$\mathscr{E}_0 = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} \tag{5}$$

appearing in (3) is a measure of the quantum optical noise for a single mode of the field. This noise is present even if the field is in the vacuum state, and for this reason it is usually referred to as the *vacuum fluctuations of the field* [4]. Quantum noise associated with the vacuum fluctuations, which appears because of noncommuting character of the annihilation and creation operators expressed by (4), is ubiquitous and cannot be eliminated, but we can to some extent control this noise by 'squeezing' it in one quantum variable at the expense of "expanding" it in another variable. This noise, no matter how small it is in comparison to macroscopic fields, can have very important macroscopic consequences changing the character of the evolution of the macroscopic fields. We are going to address such questions in this chapter.

The electric field operator (3) can be rewritten in the form

$$\hat{E} = \mathscr{E}_0\big[\hat{Q}\cos(\mathbf{k}\cdot\mathbf{r}-\omega t) + \hat{P}\sin(\mathbf{k}\cdot\mathbf{r}-\omega t)\big] \tag{6}$$

where we have introduced two Hermitian quadrature operators, $\hat{Q}$ and $\hat{P}$, defined as

$$\hat{Q} = \hat{a} + \hat{a}^{+}, \qquad \hat{P} = -i(\hat{a} - \hat{a}^{+}) \tag{7}$$

which satisfy the commutation relation

$$[\hat{Q}, \hat{P}] = 2i \tag{8}$$

The two quadrature operators thus obey the Heisenberg uncertainty relation

$$\langle(\Delta\hat{Q})^2\rangle\langle(\Delta\hat{P})^2\rangle \geq 1 \tag{9}$$

where we have introduced the quadrature noise operators

$$\Delta\hat{Q} = \hat{Q} - \langle\hat{Q}\rangle, \qquad \Delta\hat{P} = \hat{P} - \langle\hat{P}\rangle \tag{10}$$

For the vacuum state or a coherent state, which are the minimum uncertainty states, the inequality (9) becomes equality and, moreover, the two variances are equal

$$\langle(\Delta\hat{Q})^2\rangle = \langle(\Delta\hat{P})^2 = 1 \tag{11}$$

The Heisenberg uncertainty relation (9) imposes basic restrictions on the accuracy of the simultaneous measurement of the two quadrature components of the optical field. In the vacuum state the noise is isotropic and the two components have the same level of quantum noise. However, quantum states can be produced in which the isotropy of quantum fluctuations is broken—the uncertainty of one quadrature component, say, $\hat{Q}$, can be reduced at the expense of expanding the uncertainty of the conjugate component, $\hat{P}$. Such states are called *squeezed states* [5,6]. They may or may not be the minimum uncertainty states. Thus, for squeezed states

$$\langle(\Delta\hat{Q})^2\rangle < 1 \quad \text{or} \quad \langle(\Delta\hat{P})^2\rangle < 1 \tag{12}$$

Squeezing is a unique quantum property that cannot be explained when the field is treated as a classical quantity—field quantization is crucial for explaining this effect.

Another nonclassical effect is referred to as *sub-Poissonian photon statistics* (see, e.g., Refs. 7 and 8 and papers cited therein). It is well known that in a coherent state defined as an infinite superposition of the number states

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,|n\rangle \tag{13}$$

the photon number distribution is Poissonian

$$p(n) = |\langle n|\alpha\rangle|^2 = \exp(-|\alpha|^2)\frac{|\alpha|^{2n}}{n!} = \exp(-\langle\hat{n}\rangle)\frac{\langle\hat{n}\rangle^n}{n!} \tag{14}$$

which means

$$\langle(\Delta\hat{n})^2\rangle = \langle\hat{n}^2\rangle - \langle\hat{n}\rangle^2 = \langle\hat{n}\rangle \tag{15}$$

If the variance of the number of photons is smaller than its mean value, the field is said to exhibit the sub-Poissonian photon statistics. This effect is related to the second-order intensity correlation function

$$G^{(2)}(\tau) = \langle{:}\hat{n}(t)\hat{n}(t+\tau){:}\rangle = \langle\hat{a}^{+}(t)\hat{a}^{+}(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle \tag{16}$$

where :: indicate the normal order of the operators. This function describes the probability of counting a photon at t and another one at $t+\tau$. For stationary fields, this function does not depend on t but solely on τ. The normalized