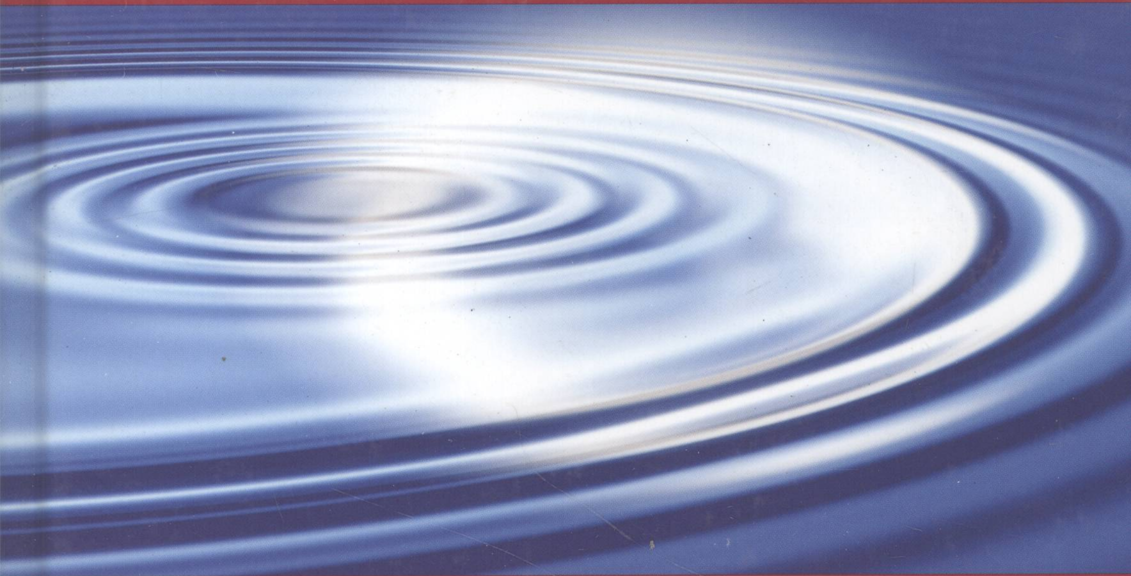

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Models and Numerical Techniques

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Wave Propagation in Fluids

Introduction

What is wave propagation?

In a kitchen or in a bathroom, the number of times we turn a tap every day is countless. So is the number of times we watch the liquid stream impacting the sink. The circular flow pattern where the fast and shallow water film diverging from the impact point changes into a deeper, bubbling flow is too familiar to deserve attention. Very few people looking at the circular, bubbling pattern – referred to as a hydraulic jump by the specialists of hydraulics – are aware that they are contemplating a shock wave.

Turning off the tap too quickly may result in a thud sound. This is the audible manifestation of the well-known water hammer phenomenon, a train of pressure waves propagating in the metal pipes as fast as hundreds to thousands of meters per second. The water hammer phenomenon is known to cause considerable damage to hydropower duct systems or water supply networks under the sudden operation of valves, pumps or turbines. The sound is heard because the vibrations of the duct system communicate with the ambient atmosphere, and from there with the operator's ears.

Everyone has once thrown stones into the water in a pond, watching the concentric ripples propagate on the surface. Less visible and much slower than the ripples is the moving groundwater that displaces a pollutant front in a journey that may last for years.

As ubiquitous and familiar as wave propagation may be, the phenomenon is often poorly understood. The reason why intuition so often fails to grasp the mechanisms of wave propagation may lie in the commonly shared, instinctive perception that waves are made of matter. This, however, is not true. In the example of the hydraulic jump in the sink, the water molecules move across an immobile wave. In the example of the ripples propagating at the free surface of a pond, the waves travel while the water remains immobile.

Waves appear when an object or a system (e.g. the molecules in a fluid, a rigid metallic structure) reacts to a perturbation and transmits it to its neighbors. In many cases, as in the example of the water ripples, the initially perturbed system returns to its initial equilibrium state, while the waves keep propagating. In this respect, waves may be seen as information. The ripples propagating in a pond are a sign that the water molecules "inform" their neighbors that the equilibrium state has been disturbed. A sound is nothing more than information about a perturbation occurring in the atmosphere.

Numerical techniques for wave propagation simulation have been the subject of intensive research over the last 50 years. The advent of fast computers has led to the development of efficient numerical techniques. Engineers and consultants now use simulation software packages for wave propagation on a daily basis. Whether it is for the purpose of acoustics, aerodynamics, flood wave propagation or contaminant transport studies, computer-based simulation tools have become indispensable to almost all domains of engineering. Such tools, however, remain instruments operated by human beings to execute tedious, repetitive operations previously carried out by hand. They cannot, and hopefully never will, replace the expert's judgment and experience. Human presence remains necessary to the sound assessment of the relevance and accuracy of modeling results. Such an assessment, however, is possible only provided that the very specific type of reasoning required for the correct understanding of wave propagation phenomena has been acquired.

The main purpose of this book is to contribute to a better understanding of wave propagation phenomena and the most commonly used numerical techniques for its simulation. The first three chapters deal with the physics and mathematics of wave propagation. Chapters 4 and 5 provide an insight into more theoretical notions, used in specific numerical techniques. Chapters 6 and 7 are devoted to finite difference and finite volume techniques respectively. Basic notions of linear algebra and numerical methods are presented in Appendices A to C. The various formulae used in the present book are summarized in Appendix D.

What is the intended readership for this book?

This book is intended for the students of professional and research master programs and those engaged in doctoral studies, the curriculum of which contains hydraulics and/or fluid mechanics-related subjects. Engineers and developers in the field of fluid mechanics and hydraulics are also a potential target group. This book was written with the following objectives:

1) To introduce the physics of wave propagation, the governing assumptions and the derivation of the governing equations (in other words, the modeling process) in various domains of fluid mechanics. The application fields are as diverse as contaminant transport, open channel and free surface hydraulics, or aerodynamics.

2) To explain how the behavior of the physical systems can be analyzed using very simple mathematical techniques, thus allowing practical problems to be solved.

3) To introduce the main families of numerical techniques used in most simulation software packages. As today's practising engineers cannot afford not to master modeling packages, a basic knowledge of the existing numerical techniques appears as an indispensable engineering skill.

How should this book be read?

The chapters are divided into three parts:

– The first part is devoted to the theoretical notions applied in the remainder of the chapter.

– The second part deals with the application of these theoretical notions to various equations of hydraulics and fluid mechanics.

– The third part provides a summary of the key points developed in the chapter, as well as suggestions of application exercises.

The main purpose of the application exercises is to test the reader's ability to reuse the notions developed in the chapter and apply them to practical problems. The solution principle of the exercises may be accessed from the following URL: http://vincentguinot.free.fr/waves/exercises.htm.

Try to resist the temptation to read the solution immediately. Solving the exercise by yourself should be the primary objective. The solution text is provided only as a help, in case you cannot find a way to start and for you to check the validity of your reasoning after completing the exercise.

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Chapter 1

Scalar Hyperbolic Conservation Laws in One Dimension of Space

1.1. Definitions

1.1.1. *Hyperbolic scalar conservation laws*

A one-dimensional hyperbolic scalar conservation law is a Partial Differential Equation (PDE) that can be written in the form

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = S \qquad [1.1]$$

where t and x are respectively the time- and space-coordinates, U is the so-called conserved variable, F is the flux and S is the source term. Equation [1.1] is said to be the conservation form of the conservation law. The following definitions are used:

– The flux F is the amount of U that passes at the abscissa x per unit time due to the fact that U (also called the transported variable) is being displaced.

– The source term S is the amount of U that appears per unit time and per unit volume, irrespective of the amount transported via the flux F. If U represents the concentration in a given chemical substance, the source term may express degradation phenomena, or radioactive decay. S is positive when the conserved variable appears in the domain, and is negative if U disappears from the domain.

– The conservation law is said to be scalar because it deals with only one dependent variable. When several equations in the form [1.1] are satisfied