

STATISTICAL REASONING AND METHODS



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**STATISTICAL
REASONING AND
METHODS**

To my students—for making statistics an interesting and exciting career

R. A. J.

To my parents, Chan-Wing Tsui and Wai-King Yeung

K. W. T.

Preface

Statistics plays an important role in our daily lives. It helps us understand the nature of situations that appear to be unpredictable and aid decision making when events are not certain to occur. Statistics concerns the collection, description, and making of generalizations from data. It plays an increasing role in almost all fields of activity and is used extensively in government reports and by professionals in many fields.

The aims of this book are to introduce basic statistical concepts and methods and to show how they enhance critical thinking and reasoning. The first chapter emphasizes critical thinking regarding data collection. Throughout the book, a number of cases are presented where statistics are improperly used and, consequently, the corresponding conclusions are invalid. Special material in each chapter, set out in boxes labeled *Statistical Reasoning*, presents and reinforces key ideas of statistical reasoning in story settings. Exercises help students learn to question the quality of data and think about what factors produce variation in the response.

Many intuitive and simple explanations motivate the key statistical concepts and terms. Numerous examples illustrate the methods in the context of situations most students experience in their everyday lives.

We present probability, a somewhat difficult subject, as it will be used to help make decisions and to make inferences. This novel treatment, including many examples, makes clear how the key ideas of probability underpin statistical concepts and inferences.

This book can be used for any first elementary one-quarter or one-semester statistics course. The mathematical level is elementary and suitable for students in two-year and four-year colleges as well as universities. The topics covered are the most important ones common to any first course in statistics for freshmen and sophomores. By concentrating on the main concepts and methods, we place more emphasis on reasoning than is currently available in most texts.

Responding to the current trend of having students perform experiments and collect data, we have included suggested class projects in most chapters. They help deepen students' understanding of how to collect good data to answer well posed questions and the role of randomization in comparative experiments.

Numerous computer outputs are included with the examples in the text. This makes it possible for an instructor to place less emphasis on the formulas if de-

sired. Additional MINITAB commands and output are included in the many computer-based exercises.

To the students, we hope that you will become critical consumers of statistical information provided by others. Further, we hope that you will learn how to collect good data and assemble the information in a way that leads to better understanding of a product, service, or the environment.

DISTINGUISHING FEATURES

- Brief text focuses on extensive coverage of key topics. Extended intuitive explanations of the basic concepts and methods, beginning with sample mean, median, and variance, reinforce their importance.
- This book shows how basic statistical concepts and methods enhance critical thinking and reasoning. Chapter 1 helps students distinguish “good” samples from “bad” samples and criticize data. Statistical Reasoning boxes, throughout, describe reasoning in settings familiar to students.
- Chapter 4 develops probability from the point of view of its use in statistics. The laws of probability are applied to solve probability and inference problems of the kind encountered by students in their everyday activities.
- Early emphasis on designing the data collection process, in Chapters 1–3, allows the interested instructor to assign student projects early in the course. These can help students understand the key elements of statistical design, and students can refer to their own data when inference procedures are discussed.
- Special charts show the general themes connecting various inference procedures. These link statistical methods that students might otherwise assume are unrelated.

ORGANIZATION

The text can be divided into five parts.

1. Collecting and describing data (Chapters 1–3). Determining good and bad samples and understanding graphical and numerical summaries.
2. The ideas underlying inference (Chapters 4–7). Probability and its role; discrete and continuous distributions; the notions of sampling and sampling distributions.
3. Inferences about means (Chapters 8–10). Clear, simple explanations of confidence intervals and tests of hypotheses.
4. Inferences about proportions (Chapter 11). One-sample and two-sample inference procedures with some coverage of chi-square tests.
5. Regression (Chapter 12). Fitting a straight line to data; and associated inferences and model checking.

COURSE SEQUENCING

Most of the first nine chapters can be taught in sequence in a one-semester course. Chapter 3 could be skipped if student projects are not a priority. We typically finish with some inference about proportions and fitting a straight line.

SUPPLEMENTS

- An *Instructor's Manual* includes complete worked solutions to all of the exercises in the text.
- A *Test Bank* provides true–false, fill-in-the-blank, multiple-choice, and computational exercises for every chapter in the book. It is available in both hard copy and IBM-PC version formats.
- A *Data Disk* contains files for all large data sets in ASCII format. The data disk is available from the Wiley website at www.wiley.com/college. If unable to access the Internet, adopters of the text may also request a copy of the disk by contacting your local Wiley sales representative.

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Contents

CHAPTER 1 ABOUT STATISTICS 1

- 1 Why Study Statistics? 2
- 2 Statistics in Our Lives 3
- 3 Variation and the Role of Statistics in Learning 4
- 4 Two Basic Concepts—Population and Sample 5
- 5 Elements of Data Collection 9
- 6 The Purposeful Collection of Data 16
- 7 Objectives of Statistics 18
- 8 *Review Exercises* 20

CHAPTER 2 ORGANIZING DATA AND DESCRIBING PATTERNS 25

- 1 Introduction 26
- 2 Variables and Data 26
- 3 Describing Data by Tables and Graphs 28
- 4 Numerical Measures of Center 50
- 5 Numerical Measures of Variation 62
- 6 Checking the Stability of the Observations Over Time 77
- 7 *Review Exercises* 82

CHAPTER 3 DESCRIBING BIVARIATE DATA 97

- 1 Introduction 98
- 2 Summarization of Bivariate Categorical Data 99
- 3 A Designed Experiment for Making a Comparison 105
- 4 Scatter Diagram of Bivariate Measurement Data 107
- 5 The Correlation Coefficient—A Measure of Linear Relation 111

6 Designing Comparative Experiments—Student Projects	122
7 <i>Review Exercises</i>	126

CHAPTER 4 PROBABILITY—THE BASIS FOR INFERENCE 133

1 Introduction and Overview of Probability	134
2 The Probability of an Event	138
3 Event Relations and Two Laws of Probability	153
4 Conditional Probability	165
5 Independence	170
6 Another Counting Tool: The Rule of Combinations	183
7 <i>Review Exercises</i>	190

CHAPTER 5 RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS 197

1 Introduction	198
2 Random Variables and Their Probability Distributions	199
3 Expectation (Mean) and Standard Deviation	212
4 Successes and Failures—Bernoulli Trials	222
5 The Binomial Distribution	227
6 The Binomial Distribution in Context	240
7 <i>Review Exercises</i>	244

CHAPTER 6 NORMAL DISTRIBUTIONS 253

1 Introduction	254
2 Probability Model for a Continuous Random Variable	254
3 The Normal Distributions—General Features	260
4 The Standard Normal Distribution	263
5 Probability Calculations with Normal Distributions	270
6 The Normal Approximation to the Binomial	275
7 Checking Normality	282
8 <i>Review Exercises</i>	286

CHAPTER 7 VARIATION IN REPEATED SAMPLES—SAMPLING DISTRIBUTIONS 291

- 1 Introduction 292
- 2 The Sampling Distribution of a Statistic 294
- 3 Distribution of the Sample Mean and the Central Limit Theorem 302
- 4 *Review Exercises* 315

CHAPTER 8 INFERENCES ABOUT MEANS—LARGE SAMPLES 319

- 1 Introduction 320
- 2 Point Estimation of a Population Mean 322
- 3 Confidence Interval for a Population Mean 330
- 4 Testing Hypotheses Concerning a Population Mean 339
- 5 *Review Exercises* 358

CHAPTER 9 SMALL SAMPLE INFERENCES FOR NORMAL POPULATIONS 365

- 1 Introduction 366
- 2 Student's t Distribution 366
- 3 Inferences about the Mean—Small Sample Size 370
- 4 The Relation Between Tests and Confidence Intervals 382
- 5 Inferences about the Standard Deviation (The Chi-Square Distribution) 385
- 6 *Review Exercises* 392

CHAPTER 10 COMPARING TWO TREATMENTS 397

- 1 Introduction 398
- 2 Independent Random Samples from Two Populations 402
- 3 Randomization and Its Role in Inference 428
- 4 Matched Pair Comparisons 430
- 5 *Review Exercises* 444

CHAPTER 11 ANALYZING COUNT DATA 451

- 1 Introduction 452
- 2 Inferences about a Proportion 453
- 3 Comparing Two Population Proportions 464
- 4 Comparing Populations with a Chi-Square Test 475
- 5 Contingency Table with Neither Margin Fixed (Test of Independence) 487
- 6 *Review Exercises* 496

CHAPTER 12 REGRESSION ANALYSIS—SIMPLE LINEAR REGRESSION 503

- 1 Introduction 504
- 2 Formulating a Model for a Straight-Line Regression 506
- 3 Fitting a Line by the Method of Least Squares 511
- 4 Inferences for a Straight-Line Model 518
- 5 Checking the Adequacy of the Straight-Line Model 532
- 6 *Review Exercises* 540

APPENDIX A SUMMATION NOTATION 545

APPENDIX B TABLES 551

- Table 1 Random Digits 552
- Table 2 Cumulative Binomial Probabilities 555
- Table 3 Standard Normal Probabilities 562
- Table 4 Percentage Points of t Distributions 564
- Table 5 Percentage Points of χ^2 Distributions 565
- Table 6 Percentage Points of $F(\nu_1, \nu_2)$ Distributions 566

DATA BANK 569

ANSWERS TO SELECTED ODD-NUMBERED EXERCISES 579

INDEX 587

CHAPTER 1

About Statistics



Chapter Objectives

After reading this chapter, you should be able to

- ▶ Distinguish between populations and samples.
- ▶ Use a table of random digits to select a random sample.
- ▶ Begin to develop a critical viewpoint toward the collection of data.
- ▶ Give a clear statement of purpose for the collection of data.

1. WHY STUDY STATISTICS?

You are undoubtedly already familiar with the term *statistics* as it applies to the numbers of unemployed, college students in various disciplines, or the cost of living index, all of which are based on data collected by the government. In its earliest applications, government rulers numbered their subjects as the basis for collecting taxes and determining the size of armies they could raise. In fact, the word *statistics* is derived from the Latin word “status,” meaning “state.”

Gigantic strides made during the twentieth century have brought the discipline of statistics to prominence by providing ways for reaching reasoned conclusions based on data. Statistics now serves as an investigative tool and a guide to the unknown. Statistics helps provide

a systematic approach for obtaining reasoned answers together with some assessment of their reliability

in situations where complete information is unobtainable or not available in a timely manner.

Answers provided by statistical approaches can provide the basis for making decisions or choosing actions. For example, city officials might examine whether the level of lead in the water supply is within safety standards. Because not all of the water can be checked, answers must be based on the partial information from samples of water that are collected for this purpose. As another example, a store wants to determine which days of the week and times of day they should expect the most shoppers in a typical month. Data on the number of customers at various times of day, collected daily for a few weeks, can provide a reference.

When information is sought, statistical ideas suggest a typical process with four crucial steps:

- (a) Set clearly defined goals for the investigation.
- (b) Make a plan of what data to collect and how to collect them.
- (c) Apply appropriate methods to extract information from the data.
- (d) Interpret the information and draw conclusions.

These indispensable steps will provide a frame of reference throughout as we develop the key ideas and methods of reasoning statistically.

In summary,

Statistics as a subject provides a body of principles and methodology for designing the process of data collection, summarizing and interpreting the data, and drawing conclusions or generalities.

2. STATISTICS IN OUR LIVES

Fact finding through the collection and interpretation of data is not confined to professional researchers. In our attempts to understand the issues of air and water quality, improvement of the mass transit system we ride, or the performance of our favorite football team, numerical information and figures need to be reviewed and interpreted.

Our sources of information range from individual experience to reports in the news media, government records, and technical articles. As consumers of these reports, citizens need to be aware of the quality of the data and to have some understanding of statistical reasoning to properly interpret the data and evaluate the conclusions. Statistical criteria determine which conclusions are supported by the data and which are not.

The following examples illustrate the diversity of applications of statistics.

Repairing library books. Librarians need to estimate the number of their books that need repairing each year. Records from previous years provide one source of data. Librarians use their estimate to budget repair costs and any extra staff required.

Election time brings the pollsters into the limelight.

Gallup Poll. This, the best known of the national polls, produces estimates of the percentage of popular vote for each candidate based on interviews with a minimum of 1500 adults. Beginning several months before the presidential election, results are regularly published. These reports help predict winners and track changes in voter preferences.

Statistical approaches are also key to improving any type of manufacturing or service process.

Quality and productivity improvement. In the past 30 years, the United States has faced increasing competition in the world marketplace. An international revolution in quality and productivity improvement has heightened the pressure on the U.S. economy. The ideas and teaching of W. Edwards Deming helped rejuvenate Japan's industry in the late 1940s and 1950s. In the 1980s and 1990s, Deming stressed to American executives that, in order to survive, they must mobilize their work force to make a continuing commitment to quality improvement. His ideas have also been applied to government. The city of Madison, Wisconsin, has implemented quality improvement projects in the police department, and in bus repair and scheduling. In each case, the project goal was better service at less cost. Treating citizens as the customers of government services, the first step was to collect information from them to identify situations that needed improvement. One end result was the strategic placement of a new police substation

and a subsequent increase in the number of foot patrolpersons to interact with the community.

Once a candidate project is selected for improvement, data must be collected to assess the current status and then more data collected on the effects of possible changes. At this stage, statistical skills in the collection and presentation of summaries are not only valuable but necessary for all participants.

In an industrial setting, statistical training for all employees—production line and office workers, supervisors, and managers—is vital to the quality transformation of American industry.

Monitoring advertising claims. The public is constantly bombarded with commercials that claim the superiority of one product brand compared with others. When such comparisons are founded on sound experimental evidence, they serve to educate the consumer. Not infrequently however, misleading advertising claims are made due to insufficient experimentation, faulty analysis of data, or even blatant manipulation of experimental results. Government agencies and consumer groups must be prepared to verify the comparative quality of products by using adequate data collection procedures and proper methods of statistical analysis.

3. VARIATION AND THE ROLE OF STATISTICS IN LEARNING

Why do pine trees grow in one part of the country and not in another? Is it because of weather conditions, the chemical composition of the soil, or the existence of special live organisms? For our forests to remain renewable resources, these questions need to be answered. Experiments can be conducted to determine the major factors that influence the growth of pine trees. We can use statistical methods to efficiently search for the important factors. After careful considerations, one can formulate a tentative hypothesis about a factor that is influential for pine tree growth. Statistical procedures can guide the collection of appropriate data for supporting or refuting the hypothesis. Essentially, one carries out the four steps (a)–(d), presented in Section 1, for a typical statistical investigation. If this check confirms agreement between the data and what is suggested by the hypothesis, the hypothesis is verified to the extent it provides an explanation of the data. Learning has occurred.

If the check reveals disagreement, another hypothesis can be formulated and the investigative procedure repeated to examine its validity. The searching continues until, with good fortune, the important factors are identified.

In the context of a manufacturing process, what are the factors or materials that yield products that are insensitive to variation in the environment? A portable tape player is expected to work well in all temperatures as well as in very high or very low humidity conditions. If equipment is too sensitive to variation in the environment, its usefulness is questionable and it will not sell well over the long term.

STATISTICS REQUIRED TO EXPLAIN VARIATION

Whenever we wish to explain an observed pattern of variation, we are required to carefully examine the situation and then propose a potential theory or hypothesis. Statistics can play a key role in the verification step. It can provide guidance on how to design an experiment or survey to collect data efficiently. Then, following the methods described in later chapters, it provides the only methods for assessing the agreement between hypotheses and data that exhibit variation.

As another example, we notice that there is a wide variation of income among individuals. Naturally, we would like to search for explanations of this variation. Several hypotheses have been proposed. Many persons think that the variation is due to an age factor and educational levels. An older person has had more time to accumulate wealth or to attain a senior position.

As we learn more about a situation, how to complete a task, or play a sport, we typically get better results. The learning process can be made more efficient by applying a logical procedure for reasoning from available evidence. A conjecture is made and data are collected. Typically, an analysis of the data leads to a modified conjecture. New data are collected and the cycle continues until a satisfactory conjecture, theory, or explanation is obtained. This approach, called the **scientific method**, is similar to the investigative procedure described in Section 1. It is represented by the sequence of steps conjecture–experiment–verification. These steps are located on a circle in Figure 1 to emphasize the fact that one pass is usually not enough but that learning continues to take place as we cycle through the steps.

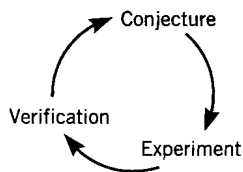


Figure 1. The conjecture–experiment–verification cycle for learning:
The scientific method

Keep in mind that no conjecture is apt to be perfect so the verification step really asks only whether it provides an adequate explanation of the data currently available.

In any given application, the steps may run together and be supplemented by trial-and-error, good guesswork, and luck. Rarely, if ever, do we get it right the first time, so expect to repeat the cycle of steps. Do you remember your experiences when learning to swim or ride a bike?

4. TWO BASIC CONCEPTS—POPULATION AND SAMPLE

The examples above, where the evaluation of factual information is essential for acquiring new knowledge, motivate the development of statistical reasoning and