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Discrete Mathematics and Its Appli[illegible]ations

FIFTH EDITION

Discrete Mathematics and Its Applications

Fifth Edition

Kenneth H. Rosen

AT&T Laboratories

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About the Author

Kenneth H. Rosen is a Distinguished Member of the Technical Staff in AT&T Laboratories in Middletown, New Jersey.

Dr. Rosen received his B.S. in Mathematics from the University of Michigan, Ann Arbor (1972), and his Ph.D. in Mathematics from M.I.T. (1976), where he wrote his thesis in the area of number theory under the direction of Harold Stark. Before joining Bell Laboratories in 1982, he held positions at the University of Colorado, Boulder; the Ohio State University, Columbus; and the University of Maine, Orono, where he was an associate professor of mathematics. While working at AT&T Labs, Ken has taught at Monmouth University, teaching courses in discrete mathematics, coding theory, and data security.

Dr. Rosen has published numerous articles in professional journals in the areas of number theory and mathematical modeling. He is the author of the textbooks *Elementary Number Theory and Its Applications,* currently in its fourth edition, published by Addison-Wesley, and *Discrete Mathematics and Its Applications,* in its fifth edition, published by McGraw-Hill. Both books have been used extensively at hundreds of universities. He is coauthor of *UNIX: The Complete Reference, UNIX System V Release 4: An Introduction,* and *Best UNIX Tips Ever*, published by Osborne McGraw-Hill. These books have sold more than 100,000 copies, with translations into Chinese, German, Spanish, and Italian. Ken is also the editor of the *Handbook of Discrete and Combinatorial Mathematics,* published in 2000 by CRC Press, and he is the advisory editor of the CRC series of books in discrete mathematics. Ken is also interested in integrating mathematical software into the educational and professional environments and has worked on projects with Waterloo MAPLE software in both these areas.

At Bell Laboratories, and now AT&T Laboratories, Dr. Rosen has worked on a wide range of projects, including operations research studies and product line planning for computers and data communications equipment. He has helped plan AT&T's future products and services in the area of multimedia, including video communications, speech recognition and synthesis, and image networking. He has evaluated new technology for use by AT&T. He has also invented many new services, and holds or has submitted more than 65 patents. One of his more interesting projects involved helping evaluate technology for the AT&T attraction at EPCOT Center.

Preface

In writing this book, I was guided by my long-standing experience and interest in teaching discrete mathematics. For the student, my purpose was to present material in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. My goal was to show the relevance and practicality of discrete mathematics to students, who are often skeptical. I wanted to give students studying computer science all the mathematical foundations they need for their future studies; I wanted to give mathematics students an understanding of important mathematical concepts together with a sense of why these concepts are important for applications. And I wanted to accomplish these goals without watering down the material.

For the instructor, my purpose was to design a flexible, comprehensive teaching tool using proven pedagogical techniques in mathematics. I wanted to provide instructors with a package of materials that they could use to teach discrete mathematics effectively and efficiently in the most appropriate manner for their particular set of students. I hope that I have achieved these goals.

I have been extremely gratified by the tremendous success of this text. The many improvements in the fifth edition have been made possible by the feedback and suggestions of a large number of instructors and students at many of the more than 500 schools where this book has been successfully used. There are many enhancements in this edition. The ancillary package has been enriched, and a companion website provides helpful material, making it easier for students and instructors to achieve their goals.

This text is designed for a one- or two-term introductory discrete mathematics course to be taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only explicit prerequisite.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; more importantly, such a course should teach students how to think mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, algorithmic thinking, and applications and modeling. A successful discrete mathematics course should carefully blend and balance all five themes.

1. *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.

2. *Combinatorial Analysis:* An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems, not on applying formulae.
3. *Discrete Structures:* A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Algorithmic Thinking:* Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.
5. *Applications and Modeling:* Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, botany, zoology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises.

Changes in the Fifth Edition

The fourth edition of this book has been used successfully at over 500 schools in the United States, dozens of Canadian universities, and at universities throughout Europe, Asia, and Oceania. Although the fourth edition has been an extremely effective text, many instructors, including longtime users, have requested changes designed to make this book more effective. I have devoted a significant amount of time and energy to satisfy these requests.

The result is a fifth edition that offers both instructors and students much more than the fourth edition did. Most significantly, an improved organization of topics has been implemented in this fifth edition, making the book a more effective teaching tool. Substantial enhancements to the material devoted to logic, method of proof, and proof strategies are designed to help students master mathematical reasoning. Additional explanations and examples have been added to clarify material where students often have difficulty. New exercises, both routine and challenging, have been inserted into the exercise sets. Highly relevant applications, including many related to the Web and computer science, have been added. The companion website has benefited from extensive development activity and now provides tools students can use to master key concepts and explore the world of discrete mathematics.

Improved Organization

- Coverage of mathematical reasoning is concentrated in Chapter 1, flowing from propositional and predicate logic to rules of inference and basic proof techniques.
- Big-O and related notation is discussed immediately before the complexity of algorithms.
- Sequences and sums are treated immediately before the section on mathematical induction.

- Binomial coefficients are covered in a separate section.
- Probability theory is covered in its own chapter.
- Sorting is introduced in Chapter 2, with more sorting algorithms addressed.

Logic

- Implications receive in-depth coverage, with additional treatment of the converse, inverse, and contrapositive of an implication.
- A subsection on logic puzzles has been added.
- Quantifiers are now covered in two sections.
- More explanation is provided on how to translate from English and mathematical statements to logical expressions and vice versa.
- Negations of quantifications are covered in greater depth.
- Resolution and the use of predicate logic in Prolog are treated.
- The application of logic to system specifications—a topic of interest to system, hardware, and software engineers, is described.

Writing and Understanding Proofs

- Methods of proof are now introduced in Chapter 1, allowing this material to be used explicitly throughout early chapters.
- Uniqueness proofs are now explicitly discussed.
- Section 3.1 further explores proof strategies, expanding on the introductory treatment of proof strategies in Chapter 1.
- Coverage of mathematical induction and strong induction have been enhanced with additional explanations and new examples.
- Structural induction is covered explicitly.
- Proving the correctness of a recursive algorithm is now treated.

Algorithms

- Greedy algorithms are introduced in Chapter 2.
- The coverage of recursive algorithms has been expanded.
- Coverage of depth-first search and breadth-first search algorithms has been increased.
- The complexity of more algorithms is analyzed or discussed.
- Treatment of divide-and-conquer algorithms and recurrence relations used to study their complexity has been expanded.
- Algorithms for fast modular exponentiation, solving the closest-pair problem, Huffman coding, and the greedy algorithm for making change have been added.

Applications

- Graph models added include the Web graph, telephone call graphs, the Hollywood graph, acquaintanceship graphs, and collaboration graphs.
- Search techniques used by Web spiders are now discussed.
- The use of Gray codes in K-maps is now described.
- Coverage of Backus–Naur form has been increased.
- The treatment of n-ary relations and relational databases has been expanded.

Number Theory, Combinatorics, and Probability Theory

- Number theory relevant to public key cryptography, including pseudoprimes, Carmichael numbers, and probabilistic primality testing, is covered in greater depth.
- The material on conversions between different base expansions has been expanded.
- Coverage of binomial coefficients and the binomial theorem is now in a separate section.
- Coverage of probability theory has been expanded and is now in a separate chapter with expanded treatments of many key topics.

Graphs and Trees

- An explanation of how to inductively construct n-cubes has been added.
- Sufficient conditions for the existence of Hamilton circuits are now covered in greater detail.
- Game trees and minmax strategies are discussed.
- Huffman coding is now introduced.
- Depth-first and breadth-first search now receive expanded coverage.

Exercise Sets

- Over 600 new exercises have been added, ranging from routine to challenging, with a special emphasis on new exercises related to logic and proof, including mathematical induction.
- Exercises have been added to balance odd-numbered exercises with answers and even-numbered exercises.

Additional Biographies and Historical Notes

- Biographies have been added for Aristotle, Sheffer, Smullyan, Rivest, Shamir, Adleman, Carmichael, McCarthy, and Huffman.
- Images of people described in the biographies are now included, except in the few cases where no image is available.
- Many biographies found in the previous edition have been enhanced.
- New historical notes have been added.

The Website (www.mhhe.com/rosen)

- Expanded annotated links to hundreds of Web resources have been developed.
- Extra examples in key areas are hosted.
- More detailed explanation of certain examples and proofs can be accessed.
- Tools for self-assessment of key topics including implications, quantifiers, proof methods, functions, big-O notation, mathematical induction, and counting problems are available.
- Icons are placed in the book indicating the type of associated content on the Web.
- Interactive demos of key algorithms have been developed for integrated use with the text.

Special Features

ACCESSIBILITY This text has proven to be easily read and understood by beginning students. There are no mathematical prerequisites beyond college algebra for almost all of this text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructors can easily pace their lectures using these blocks.

WRITING STYLE The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Care has been taken to balance the mix of notation and words in mathematical statements.

EXTENSIVE CLASSROOM USE This book has been used at over 500 schools, and more than 400 have used it more than once. The feedback from instructors and students at many of the schools has helped make this fifth edition an even more successful teaching tool than previous editions.

MATHEMATICAL RIGOR AND PRECISION All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. Recursive definitions are explained and used extensively.

WORKED EXAMPLES Over 700 examples are used to illustrate concepts, relate different topics, and introduce applications. In most examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. This text includes applications to a wide variety of areas, including computer science, data networking, psychology, chemistry, engineering, linguistics, biology, business, and the Internet.

ALGORITHMS Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix A.2. The computational complexity of the algorithms in the text is also analyzed at an elementary level.

HISTORICAL INFORMATION The background of many topics is succinctly described in the text. Brief biographies of more than 60 mathematicians and computer scientists, accompanied by photos or images, are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics and images of these contributors are displayed. In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text.

KEY TERMS AND RESULTS A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, not every term defined in the chapter.

EXERCISES There are over 3500 exercises in the text. There are many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and many challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions, with exercises, that develop new concepts not covered in the text, enabling students to discover new ideas through their own work.

Exercises that are somewhat more difficult than average are marked with a single star; those that are much more challenging are marked with two stars. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the symbol ☞. Answers or outlined solutions to all odd-numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS Each chapter is followed by a set of computer projects. The approximately 150 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

COMPUTATIONS AND EXPLORATIONS A set of computations and explorations is included at the conclusion of each chapter. These exercises (approximately 100 in total) are designed to be completed using existing software tools, such as programs that students or instructors have written or mathematical computation packages such as MAPLE or Mathematica. Many of these exercises give students the opportunity to uncover new facts and ideas through computation. (Some of these exercises are discussed in the book, *Exploring Discrete Mathematics with MAPLE.*)

WRITING PROJECTS Each chapter is followed by a set of writing projects. To do these projects students need to consult the mathematical literature. Some of these projects are historical in nature and may involve looking up original sources. Others are designed to serve as gateways to new topics and ideas. All are designed to expose students to ideas not covered in depth in the text. These projects tie together mathematical concepts and the writing process and help expose students to possible areas for future study. (Suggested references for these projects can be found in the *Student Solutions Guide.*)

APPENDIXES There are two appendixes to the text. The first covers exponential and logarithmic functions, reviewing some basic material used heavily in the course; the second specifies the pseudocode used to describe algorithms in this text.

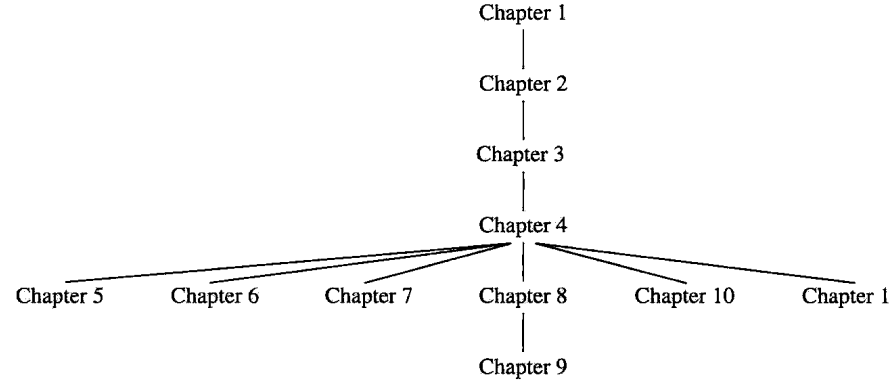
SUGGESTED READINGS A list of suggested readings for each chapter is provided in a section at the end of the text. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published.

How to Use This Book

This text has been carefully written and constructed to support discrete mathematics courses at several levels and with differing foci. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the instructor. A two-term introductory course could include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections.

Chapter	*Core Sections*	*Optional Computer Science Sections*	*Optional Mathematics Sections*
1	1.1–1.8 (as needed)		
2	2.1–2.4, 2.7 (as needed)	2.5	2.6
3	3.1–3.4	3.5, 3.6	
4	4.1–4.3	4.6	4.4, 4.5
5	5.1	5.3	5.2
6	6.1, 6.5	6.3	6.2, 6.4, 6.6
7	7.1, 7.3, 7.5	7.2	7.4, 7.6
8	8.1–8.5		8.6–8.8
9	9.1	9.2, 9.3	9.4, 9.5
10		10.1–10.4	
11		11.1–11.5	

Instructors using this book can adjust the level of difficulty of their course by choosing either to cover or to omit the more challenging examples at the end of sections, as well as the more challenging exercises. The dependence of chapters on earlier chapters is shown in the following chart.



Ancillaries

STUDENT SOLUTIONS GUIDE This student manual, available separately, contains *full* solutions to all the odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem can be solved in several different ways. Suggested references for the writing projects found at the end of each chapter are also included in this volume. The guide contains a guide to writing proofs and an extensive description of common mistakes students make in discrete mathematics. It also includes sample tests and a sample crib sheet for each chapter, both designed to help students prepare for exams. Students find this guide extremely useful.

INSTRUCTOR'S RESOURCE GUIDE This manual contains full solutions to even-numbered exercises in the text. It also provides suggestions on how to teach the material in each chapter of the book, including the points to stress in each section and how to put the material into perspective. Furthermore, the manual contains a test bank of sample examination questions for each chapter, including some sample tests as well as the solutions to the sample questions. Finally, sample syllabi are presented.

APPLICATIONS OF DISCRETE MATHEMATICS This ancillary, available in print format or downloadable from the website, can be used either in conjunction with the text or independently. It contains more than 20 chapters (each with its own set of exercises) written by instructors who have used the text. Following a common format similar to that of the text, the chapters in this book can be used as a text for a separate course, for a student seminar, or for a student doing independent study.

TEST BANK An extensive test bank of more than 1600 questions is available for use on Windows or Macintosh systems. Instructors can use this software to create their own tests by selecting questions of their choice or by random selection. Instructors can add their own headings and instructions, print scrambled versions of the same test, and edit the existing questions or add their own. A printed version of this test bank, including the questions and their answers, is included in the Instructor's Resource Guide.

EXPLORING DISCRETE MATHEMATICS AND ITS APPLICATIONS WITH MAPLE This ancillary is designed to help students use the MAPLE computer algebra system to do a wide range of computations in discrete mathematics. For each chapter of this text, this ancillary includes the following: a description of relevant MAPLE functions and how they are used, MAPLE programs that carry out relevant computations, suggestions and examples showing how MAPLE can be used for the computations and explorations at the end of each chapter, and exercises that can be worked using MAPLE.

Acknowledgments

I would like to thank the many instructors and students at a variety of schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. I especially want to thank Jerrold Grossman and John Michaels for their technical reviews of the fifth edition and their "eagle eyes," which have helped ensure the accuracy of this book. I also appreciate the help provided by all those who have submitted comments via the website.

I thank the reviewers of this fifth and the four previous editions. These reviewers have provided much helpful criticism and encouragement to me. I hope this edition lives up to their high expectations.

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Kenneth H. Rosen

The Companion Website

An extensive companion website has been developed and will be maintained and improved on a continuing basis. You can use this site in many ways to enhance your experience studying discrete mathematics. The address of this site is:

www.mhhe.com/rosen

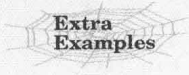
Following this address takes you to the home page for the site. On this site you will find links for:

- Information Center
- Student Center
- Instructor Center

THE INFORMATION CENTER The Information Center contains some basic information about the book and its ancillaries. Instructors can find out about the *Page Out*™ system they can use to build their own course Web pages. They can also learn about custom publishing options. An overview of the website for this book can be reached by following the appropriate link from the Information Center.

THE STUDENT CENTER The Student Center contains a wealth of resources available for student use, including the following resources tied into the text and for which a special icon is shown in the text.

- **The Web Resources Guide** This guide provides annotated links to hundreds of external websites containing relevant material. You can browse these links or access them by page number in the text or by keywords. These links will take you to websites containing historical and biographical information, puzzles and problems, discussions, applets, programs, and other types of resources.



- **Extra Examples** You can find a large number of additional examples on the site. These examples are concentrated in areas where students often ask for additional material. Although most of these examples amplify the basic concepts, some more challenging examples can also be found here.

- **Additional Steps** Further explanations are provided to help you understand some troublesome points in the text, especially in some proofs and examples.

- **Assessments** You can assess your understanding of seven key concepts. Each assessment provides a question bank where each question includes a brief tutorial, followed by a multiple-choice question. If you select an incorrect answer, advice is provided to help you understand your error. Using these assessments, you should be able to diagnose your problems and focus on available remedies.

- **Interactive Demonstrations** We have developed eight interactive demonstrations which you can use to explore how important algorithms work. These demonstrations are keyed to material in the text.

From the Student Center you can access *Net Tutor*™, which provides online tutorial help. You can ask questions relating to this text and receive answers in real-time during regular, scheduled hours or ask questions and later receive answers.

The Student Center also supports a *Bulletin Board* on which you can post messages. You can post questions and respond to messages from other students using this facility.

Additional resources in the Student Center include:

- A Guide to Writing Proofs
- Common Mistakes in Discrete Mathematics
- Advice on Writing Projects
- MAPLE software

THE INSTRUCTOR CENTER This part of the website provides links to the resources in the Student Center and the Information Center, as well as:

- Sample syllabi
- Teaching suggestions
- Transparency masters
- Several chapters from the book *Applications of Discrete Mathematics*

To the Student

What is discrete mathematics? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here *discrete* means consisting of distinct or unconnected elements.) The kind of problems solved using discrete mathematics include:

- How many ways are there to choose a valid password on a computer system?
- What is the probability of winning a lottery?
- Is there a link between two computers in a network?
- What is the shortest path between two cities using a transportation system?
- How can a list of integers be sorted so that the integers are in increasing order?
- How many steps are required to do such a sorting?
- How can it be proved that a sorting algorithm correctly sorts a list?
- How can a circuit that adds two integers be designed?
- How many valid Internet addresses are there?

You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite (or countable) sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

WHY STUDY DISCRETE MATHEMATICS? There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity, that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills.

Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Discrete mathematics provides the mathematical foundations for many computer science courses, including data structures, algorithms, database theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math. One student has sent me an electronic mail message to tell me that she used the contents of this book in every computer science course she took!

Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject).