Proceedings of the XV International Conference on

DIFFERENTIAL GEOMETRIC METHODS IN THEORETICAL PHYSICS

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DIFFERENTIAL GEOMETRIC METHODS IN THEORETICAL PHYSICS

P R E F A C E

This volume consists of the papers presented at the XV. International Conference on Differential Geometric Methods in Mathematical Physics (DGM conference series) August 1986, at the Arnold Sommerfeld Institute for Mathematical Physics, Technical University of Clausthal, Clausthal, FRG.

The main motivation for the DGM conference series is to bring together a large number of prominent researchers, working as well in the more mathematical as in the more physical parts of mathematical physics with special emphasis towards differential geometrical methods and geometrical and algebraical modelling of complex physical systems, as relativistic fields, particles and nonlinear phenomena. The exchange of ideas and the close cooperation between physicists and mathematicians was essential for new developments both in physics and in mathematics, as in particle physics (e.g. gauge and string theories), in the description of nonlinear systems (e.g. application of diffeomorphismgroups), in topology and differential geometry (e.g. exotic differentiable structures on R^4 and invariants on 4-manifolds). The DGM conference series was initiated by K. Bleuler and developed by K. Bleuler and H. D. Doebner after the first conference held 1971 in Bonn. The proceedings in this volume show the continuously growing success of topology, differential geometry, related parts of functional analysis and algebra as building blocks for a modelling of systems and a real mergence of mathematical and physical research.

This volume contains 46 contributions devided in

- structure of classical and quantized fields
- quantization methods and geometrical properties of quantum systems
- superalgebras and superspace
- Clifford algebras
- geometrical modelling of special systems
- differential geometric techniques

The section "structure of classical and quantized fields", including quantum field theory, gauge theories, strings and general relativity and the section "quantization methods and geometrical properties of quantum systems" are traditional fields of the DGM series. The same holds for the section "geometrical modelling of special systems". Clifford algebras and applications are included for the first time.

The papers range from direct physical applications (e.g. A. Bohm, H. R. Petry) to the presentation of mathematical methods and results (V. V. Tsanov, M. Modugno). Some papers are research reports, some are reviews. To cover the field between physics and geometry as completely as possible we accepted also short and updated reviews on already published material.

To give this volume special value to postgraduate students and to physicists and mathematicians who want to enter the field we asked for contributions which contain extended introductions.

Acknowledgements

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H. D. Doebner
J. D. Hennig

The DGM - Conference Series 1971 - 1987

No	year/place	Proceedings	
		Editor	Publisher
I	1971 Bonn	mimeographed lecture notes	
II	1973 Bonn	mimeographed lecture notes	
III	1974 Aix-en-Provence	Souriau	Coll. Int. du Centre Nat. de la Rech. Scient. No. 237 (1975)
IV	1975 Bonn	Bleuler/ Reetz	Lect. Notes in Math. Vol. 570 (1977)
V	1976 Warszawa		Polish Scient. Public. Vol. 12, No. 3, (1977)
VI	1977 Bonn	Bleuler/ Petry/ Reetz	Lect. Notes in Math. Vol. 676 (1978)
VII	1978 Clausthal	Doebner	Lect. Notes in Phys. Vol. 139 (1981)
VIII	1979 Aix-en-Provence/ Salamanca	Souriau/ Garcia/ Perez-Rendon	Lect. Notes in Math. Vol. 836 (1980)
IX	1980 Clausthal	Doebner/ Andersson/ Petry	Lect. Notes in Math. Vol. 905 (1982)
X	1981 Trieste	Denardo/ Doebner	Proceedings World Scientific (1983)
XI	1982 Jerusalem	Sternberg	D. Reidel, Math. Phys. Stud. No. 6 (1984)
XII	1983 Clausthal	Doebner/ Hennig	Lect. Notes in Math. Vol. 1139 (1984)
XIII	1984 Shumen	Doebner/ Palev	Proceedings World Scientific (1986)
XIV	1985 Salamanca	Garcia/ Perez-Rendon	Lect. Notes in Math. Vol. 1251 (1987)
XV	1986 Clausthal	Doebner/ Hennig	Proceedings World Scientific (1987)
XVI	1987 Como	Bleuler	D. Reidel, to be published (1988)

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2. *Gauge Theories and Strings*
3. *General Relativity*

General Covariance and Strict Locality in Quantum Field Theory

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Quantum Field Theory can be considered as the incorporation of the principle of locality (which is at the basis of classical field theory) into quantum theory. According to Haag and Kastler /1/ it can be formulated as an assignement of space time regions O to *-algebras $\mathfrak{A}(O)$ generated by the observables measurable in O. Up to now this program has been carried through only on the level of special relativity: there is still a rigid metric structure of space time, tied via local commutativity with the algebraic structure, and there is the requirement of stability which uses (in form of the spectrum condition) the existence of the global time translation symmetry.

General relativity is, in contrast to this, strictly local in the following sense: space time is a differentiable manifold, the basic objects of the theory are fields (scalar, vector, tensor, ...) related only to the space time points itself and the associated tangent spaces; the physical laws are differential equations for these fields which are invariant under general coordinate transformations.

Therefore, if one trys to include gravity into quantum field theory, one is confronted with the following conceptual problem: can quantum field theory be formulated in a generally covariant and strictly local way?

(*) Heisenberg fellow

This question has been investigated in a recent paper of Rudolf Haag and myself /2/. The answer is essentially affirmative and assures that the mentioned conceptual problem can be overcome. Let me first treat the problem of general covariance. Algebraic relations between observables in different space time regions are closely connected with causal relations between these regions. Since causal relations are not invariant under diffeomorphisms, in general, there should be no a priori relations between algebras of different regions. This goal can be reached if one replaces observables by observable procedures which are descriptions of measurements as operations in space and time /3/. Observables are classes of observable procedures which are equivalent with respect to some state (expectation functional) ω ,

$$A \underset{\omega}{\sim} B \iff \omega'(A) = \omega'(B) \quad \forall\, \omega' \in F_\omega \qquad (1)$$

where F_ω is the so-called folium of the state ω , i.e. the set of vector and density matrix states in the GNS representation $(\pi_\omega, \mathcal{H}_\omega, \Omega_\omega)$ of ω.

The net of observable procedures $\mathcal{M} \supset \mathcal{O} \to \mathfrak{A}(\mathcal{O})$ on the space time manifold $\mathcal{M}$ should have the following properties:

(i) $$\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathfrak{A}(\mathcal{O}_1) \subset \mathfrak{A}(\mathcal{O}_2)$$
$$1 \in \mathfrak{A}(\mathcal{O}) \quad \forall\, \mathcal{O}$$

(ii) "there are no relations between different regions", i.e. if $\gamma_i : \mathfrak{A}(\mathcal{O}_i) \longrightarrow \tilde{\mathfrak{A}}$, $i = 1, 2$ are homomorphisms with $\gamma_1 = \gamma_2$ on $\mathfrak{A}(\mathcal{O}_1 \cap \mathcal{O}_2)$ then there is a homomorphism $\gamma : \mathfrak{A}(\mathcal{O}_1 \cup \mathcal{O}_2) \longrightarrow \tilde{\mathfrak{A}}$ with $\gamma = \gamma_i$ on $\mathfrak{A}(\mathcal{O}_i)$, $i = 1, 2$.

(iii) $\mathfrak{A}(\mathcal{O}_1 \cup \mathcal{O}_2) = \mathfrak{A}(\mathcal{O}_1) \vee \mathfrak{A}(\mathcal{O}_2)$ (the algebra generated by $\mathfrak{A}(\mathcal{O}_1)$ and $\mathfrak{A}(\mathcal{O}_2)$).

(iv) There is a representation of the group of diffeomorphisms of $\mathcal{M}$ by automorphisms of $\mathfrak{A} = \bigcup \mathfrak{A}(\mathcal{O})$

$$\alpha : \mathrm{Diff}(\mathcal{M}) \longrightarrow \mathrm{Aut}(\mathfrak{A}), \quad \chi \longmapsto \alpha_\chi$$

such that $\alpha_\chi(\mathfrak{A}(\mathcal{O})) = \mathfrak{A}(\chi(\mathcal{O}))$.

The requirements (i) to (iv) may be considered as a generally covariant version of the Haag-Kastler axioms. There is a simple model satisfying all these axioms, the so-called Borchers algebra /4/, i.e. the tensor algebra over the space of test functions on the space time manifold The local algebras $\mathfrak{A}(O)$ in this model consist of finite sequences of test functions, $f = (f^{(0)}, f^{(1)}, \ldots f^{(n)}, 0, \ldots)$ with $f^{(0)} \in \mathbb{C}$ and $f^{(k)} \in \mathcal{D}(O^k)$, $k \in \mathbb{N}$. The product is the tensor product of functions,

$$(fg)^{(n)}(x_1, \ldots x_n) = \sum_{k=0}^{n} f^{(k)}(x_1, \ldots x_k)\, g^{(n-k)}(x_{k+1}, \ldots x_n) \quad (2)$$

the *-operation is

$$(f^*)^{(n)}(x_1, \ldots x_n) = \overline{f^{(n)}(x_n, \ldots x_1)} \quad (3)$$

and the action of diffeomorphisms is given by

$$(\alpha_\chi f)^{(n)}(x_1, \ldots x_n) = f^{(n)}(\chi^{-1}(x_1), \ldots \chi^{-1}(x_n)) \quad . \quad (4)$$

States on $\mathfrak{A}(M)$ are sequences of distributions (the n-point (Wightman) functions),

$$\omega = (\omega^{(0)}, \omega^{(1)}, \ldots \omega^{(n)}, \ldots) \quad (5)$$

with $\omega^{(0)} \in \mathbb{C}$ and $\omega^{(k)} \in \mathcal{D}(O^k)'$ and

$$\omega(f) = \sum_{k=0}^{\infty} \omega^{(k)}(f^{(k)}) \quad . \quad (6)$$

which are positive and normalized,

$$\begin{aligned} \omega(f^* f) &\geq 0 \\ \omega(1) &= 1 \end{aligned} \quad (7)$$

The action of diffeomorphisms on the algebra of observable procedures may be used to describe the behavior of detectors in nonuniform motion. Let F and F' be folia of states associated to classical metric fields g and g', respectively, and let γ and γ' denote possible world lines with respect to g and g', respectively. A pointlike detector C on