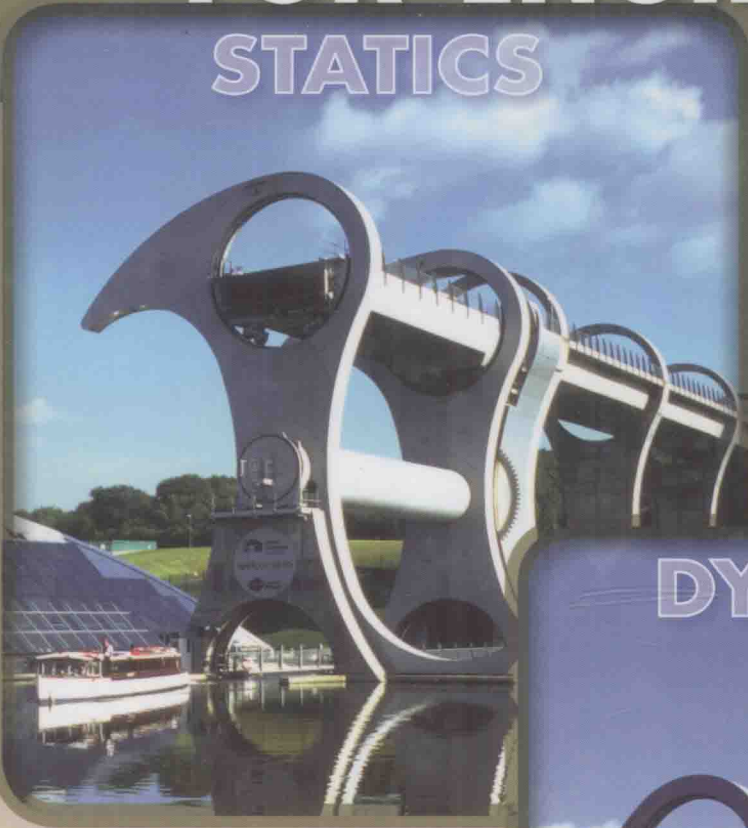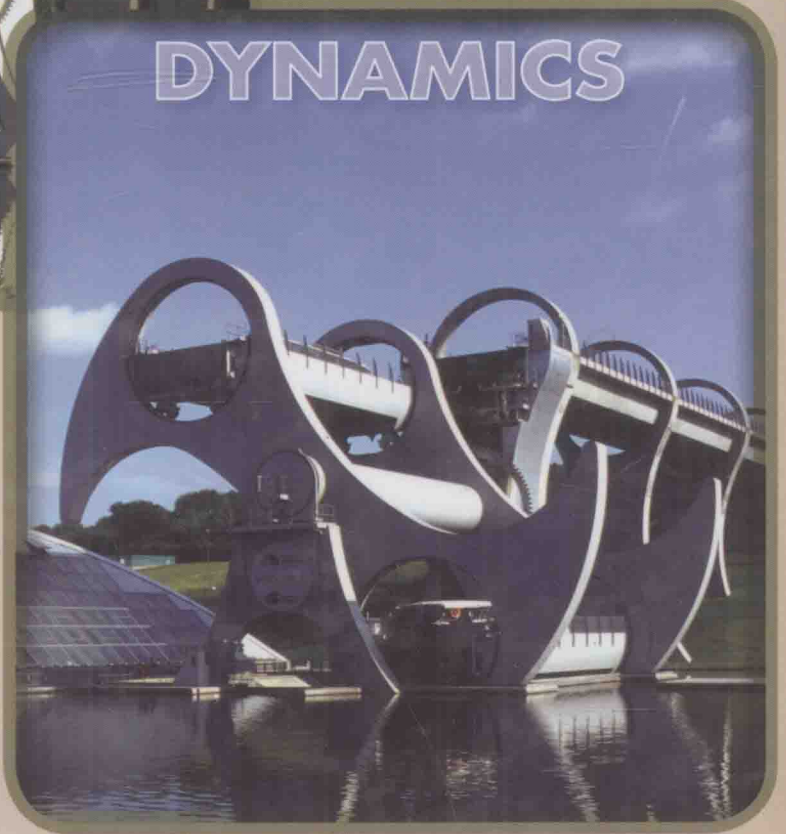# VECTOR MECHANICS FOR ENGINEERS

STATICS

DYNAMICS

BEER | JOHNSTON | MAZUREK | CORNWELL | EISENBERG

Ninth Edition

NINTH EDITION

VECTOR MECHANICS FOR ENGINEERS
Statics and Dynamics

Ferdinand P. Beer
Late of Lehigh University

E. Russell Johnston, Jr.
University of Connecticut

David F. Mazurek
U.S. Coast Guard Academy

Phillip J. Cornwell
Rose-Hulman Institute of Technology

Elliot R. Eisenberg
The Pennsylvania State University

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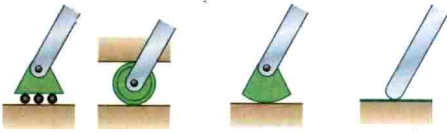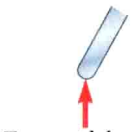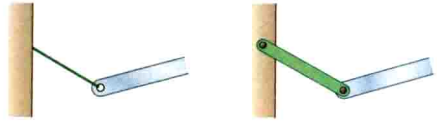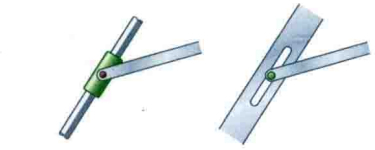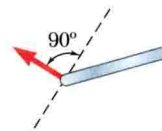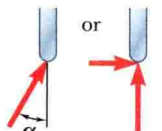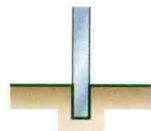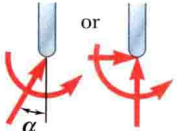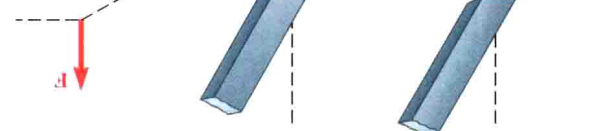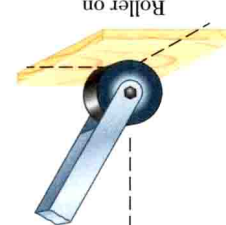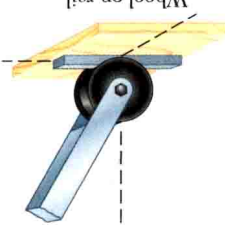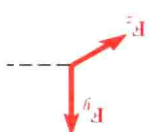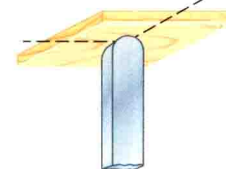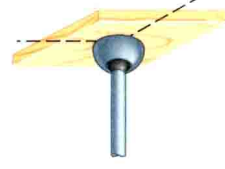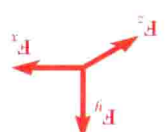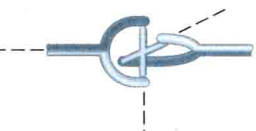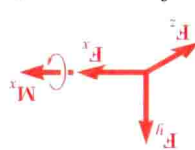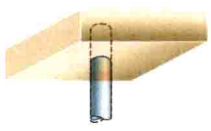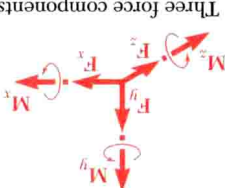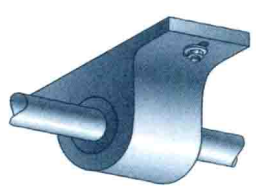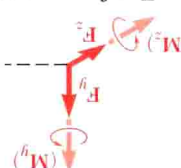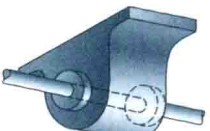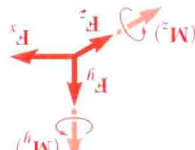
Reactions at Supports and Connections for a Two-Dimensional Structure

Support or Connection	Reaction	Number of Unknowns
Rollers Rocker Frictionless surface	Force with known line of action	1
Short cable Short link	Force with known line of action	1
Collar on frictionless rod Frictionless pin in slot	90° Force with known line of action	1
Frictionless pin or hinge Rough surface	or α Force of unknown direction	2
Fixed support	or α Force and couple	3

The first step in the solution of any problem concerning the equilibrium of a rigid body is to construct an appropriate free-body diagram of the body. As part of that process, it is necessary to show on the diagram the reactions through which the ground and other bodies oppose a possible motion of the body. The figures on this and the facing page summarize the possible reactions exerted on two- and three-dimensional bodies.

As publishers of the books by Ferd Beer and Russ Johnston we are often asked how they happened to write their books together with one of them at Lehigh and the other at the University of Connecticut.

The answer to this question is simple. Russ Johnston's first teaching appointment was in the Department of Civil Engineering and Mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in mechanics.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the mechanics courses. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented to them in a graphic way. Together they wrote lecture notes in statics and dynamics, to which they later added problems they felt would appeal to future engineers, and soon they produced the manuscript of the first edition of *Mechanics for Engineers* that was published in June 1956.

The second edition of *Mechanics for Engineers* and the first edition of *Vector Mechanics for Engineers* found Russ Johnston at Worcester Polytechnic Institute and the next editions at the University of Connecticut. In the meantime, both Ferd and Russ assumed administrative responsibilities in their departments, and both were involved in research, consulting, and supervising graduate students—Ferd in the area of stochastic processes and random vibrations and Russ in the area of elastic stability and structural analysis and design. However, their interest in improving the teaching of the basic mechanics courses had not subsided, and they both taught sections of these courses as they kept revising their texts and began writing the manuscript of the first edition of their *Mechanics of Materials* text.

Their collaboration spanned more than half a century and many successful revisions of all of their textbooks, and Ferd's and Russ's contributions to engineering education have earned them a number of honors and awards. They were presented with the Western Electric Fund Award for excellence in the instruction of engineering students by their respective regional sections of the American Society for Engineering Education, and they both received the Distinguished Educator Award from the Mechanics Division of the same society. Starting in 2001, the New Mechanics Educator Award of the Mechanics Division has been named in honor of the Beer and Johnston author team.

Ferdinand P. Beer. Born in France and educated in France and Switzerland, Ferd received an M.S. degree from the Sorbonne and an Sc.D. degree in theoretical mechanics from the University of Geneva. He came to the United States after serving in the French army during

the early part of World War II and taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Following his service at Williams College, Ferd joined the faculty of Lehigh University where he taught for thirty-seven years. He held several positions, including University Distinguished Professor and chairman of the Department of Mechanical Engineering and Mechanics, and in 1995 Ferd was awarded an honorary Doctor of Engineering degree by Lehigh University.

E. Russell Johnston, Jr. Born in Philadelphia, Russ holds a B.S. degree in civil engineering from the University of Delaware and an Sc. D. degree in the field of structural engineering from the Massachusetts Institute of Technology. He taught at Lehigh University and Worcester Polytechnic Institute before joining the faculty of the University of Connecticut where he held the position of Chairman of the Civil Engineering Department and taught for twenty-six years. In 1991 Russ received the Outstanding Civil Engineer Award from the Connecticut Section of the American Society of Civil Engineers.

David F. Mazurek. David holds a B.S. degree in ocean engineering and an M.S. degree in civil engineering from the Florida Institute of Technology and a Ph.D. degree in civil engineering from the University of Connecticut. He was employed by the Electric Boat Division of General Dynamics Corporation and taught at Lafayette College prior to joining the U.S. Coast Guard Academy, where he has been since 1990. He has served on the American Railway Engineering and Maintenance of Way Association's Committee 15—Steel Structures for the past eighteen years. His professional interests include bridge engineering, tall towers, structural forensics, and blast-resistant design.

Phillip J. Cornwell. Phil holds a B.S. degree in mechanical engineering from Texas Tech University and M.A. and Ph.D. degrees in mechanical and aerospace engineering from Princeton University. He is currently a professor of mechanical engineering at Rose-Hulman Institute of Technology where he has taught since 1989. His present interests include structural dynamics, structural health monitoring, and undergraduate engineering education. Since 1995, Phil has spent his summers working at Los Alamos National Laboratory where he is a mentor in the Los Alamos Dynamics Summer School and does research in the area of structural health monitoring. Phil received an SAE Ralph R. Teetor Educational Award in 1992, the Dean's Outstanding Scholar Award at Rose-Hulman in 2000, and the Board of Trustees Outstanding Scholar Award at Rose-Hulman in 2001.

Elliot R. Eisenberg. Elliot holds a B.S. degree in engineering and an M.E. degree, both from Cornell University. He has focused his scholarly activities on professional service and teaching, and he was recognized for this work in 1992 when the American Society of Mechanical Engineers awarded him the Ben C. Sparks Medal for his contributions to mechanical engineering and mechanical engineering technology education and for service to the American Society for Engineering Education. Elliot taught for thirty-two years, including twenty-nine years at Penn State where he was recognized with awards for both teaching and advising.

Preface

OBJECTIVES

The main objective of a first course in mechanics should be to develop in the engineering student the ability to analyze any problem in a simple and logical manner and to apply to its solution a few, well-understood, basic principles. This text is designed for the first courses in statics and dynamics offered in the sophomore or junior year, and it is hoped that it will help the instructor achieve this goal.†

GENERAL APPROACH

Vector analysis is introduced early in the text and is used throughout the presentation of statics and dynamics. This approach leads to more concise derivations of the fundamental principles of mechanics. It also results in simpler solutions of three-dimensional problems in statics and makes it possible to analyze many advanced problems in kinematics and kinetics, which could not be solved by scalar methods. The emphasis in this text, however, remains on the correct understanding of the principles of mechanics and on their application to the solution of engineering problems, and vector analysis is presented chiefly as a convenient tool.‡

Practical Applications Are Introduced Early. One of the characteristics of the approach used in this book is that mechanics of *particles* is clearly separated from the mechanics of *rigid bodies*. This approach makes it possible to consider simple practical applications at an early stage and to postpone the introduction of the more difficult concepts. For example:

- In *Statics*, the statics of particles is treated first (Chap. 2); after the rules of addition and subtraction of vectors are introduced, the principle of equilibrium of a particle is immediately applied to practical situations involving only concurrent forces. The statics of rigid bodies is considered in Chaps. 3 and 4. In Chap. 3, the vector and scalar products of two vectors are introduced and used to define the moment of a force about a point and about an axis. The presentation of these new concepts is followed by a thorough and rigorous discussion of equivalent systems of forces leading, in Chap. 4, to many practical applications involving the equilibrium of rigid bodies under general force systems.

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†This text is available in separate volumes, *Vector Mechanics for Engineers: Statics,* ninth edition, and *Vector Mechanics for Engineers: Dynamics,* ninth edition.

‡In a parallel text, *Mechanics for Engineers:* fifth edition, the use of vector algebra is limited to the addition and subtraction of vectors, and vector differentiation is omitted.

- In *Dynamics,* the same division is observed. The basic concepts of force, mass, and acceleration, of work and energy, and of impulse and momentum are introduced and first applied to problems involving only particles. Thus, students can familiarize themselves with the three basic methods used in dynamics and learn their respective advantages before facing the difficulties associated with the motion of rigid bodies.

New Concepts Are Introduced in Simple Terms.

Since this text is designed for the first course in statics and dynamics, new concepts are presented in simple terms and every step is explained in detail. On the other hand, by discussing the broader aspects of the problems considered, and by stressing methods of general applicability, a definite maturity of approach is achieved. For example:

- In *Statics,* the concepts of partial constraints and statical indeterminacy are introduced early and are used throughout statics.
- In *Dynamics,* the concept of potential energy is discussed in the general case of a conservative force. Also, the study of the plane motion of rigid bodies is designed to lead naturally to the study of their general motion in space. This is true in kinematics as well as in kinetics, where the principle of equivalence of external and effective forces is applied directly to the analysis of plane motion, thus facilitating the transition to the study of three-dimensional motion.

Fundamental Principles Are Placed in the Context of Simple Applications.

The fact that mechanics is essentially a *deductive* science based on a few fundamental principles is stressed. Derivations have been presented in their logical sequence and with all the rigor warranted at this level. However, the learning process being largely *inductive,* simple applications are considered first. For example:

- The statics of particles precedes the statics of rigid bodies, and problems involving internal forces are postponed until Chap. 6.
- In Chap. 4, equilibrium problems involving only coplanar forces are considered first and solved by ordinary algebra, while problems involving three-dimensional forces and requiring the full use of vector algebra are discussed in the second part of the chapter.
- The kinematics of particles (Chap. 11) precedes the kinematics of rigid bodies (Chap. 15).
- The fundamental principles of the kinetics rigid bodies are first applied to the solution of two-dimensional problems (Chaps. 16 and 17), which can be more easily visualized by the student, while three-dimensional problems are postponed until Chap. 18.

The Presentation of the Principles of Kinetics Is Unified.

The ninth edition of *Vector Mechanics for Engineers* retains the unified presentation of the principles of kinetics which characterized the previous eight editions. The concepts of linear and angular momentum are introduced in Chap. 12, so that Newton's second law of motion can be presented not only in its conventional form $\mathbf{F} = m\mathbf{a}$, but also as a law relating, respectively, the sum of the forces acting on a particle and the

sum of their moments to the rates of change of the linear and angular momentum of the particle. This makes possible an earlier introduction of the principle of conservation of angular momentum and a more meaningful discussion of the motion of a particle under a central force (Sec. 12.9). More importantly, this approach can be readily extended to the study of the motion of a system of particles (Chap. 14) and leads to a more concise and unified treatment of the kinetics of rigid bodies in two and three dimensions (Chaps. 16 through 18).

Free-Body Diagrams Are Used Both to Solve Equilibrium Problems and to Express the Equivalence of Force Systems.

Free-body diagrams are introduced early, and their importance is emphasized throughout the text. They are used not only to solve equilibrium problems but also to express the equivalence of two systems of forces or, more generally, of two systems of vectors. The advantage of this approach becomes apparent in the study of the dynamics of rigid bodies, where it is used to solve three-dimensional as well as two-dimensional problems. By placing the emphasis on "free-body-diagram equations" rather than on the standard algebraic equations of motion, a more intuitive and more complete understanding of the fundamental principles of dynamics can be achieved. This approach, which was first introduced in 1962 in the first edition of *Vector Mechanics for Engineers,* has now gained wide acceptance among mechanics teachers in this country. It is, therefore, used in preference to the method of dynamic equilibrium and to the equations of motion in the solution of all sample problems in this book.

A Four-Color Presentation Uses Color to Distinguish Vectors.

Color has been used, not only to enhance the quality of the illustrations, but also to help students distinguish among the various types of vectors they will encounter. While there is no intention to "color code" this text, the same color is used in any given chapter to represent vectors of the same type. Throughout *Statics,* for example, red is used exclusively to represent forces and couples, while position vectors are shown in blue and dimensions in black. This makes it easier for the students to identify the forces acting on a given particle or rigid body and to follow the discussion of sample problems and other examples given in the text. In *Dynamics,* for the chapters on kinetics, red is used again for forces and couples, as well as for effective forces. Red is also used to represent impulses and momenta in free-body-diagram equations, while green is used for velocities, and blue for accelerations. In the two chapters on kinematics, which do not involve any forces, blue, green, and red are used, respectively, for displacements, velocities, and accelerations.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained.   Because of the current trend in the American government and industry to adopt the international system of units (SI metric units), the SI units most frequently used in mechanics are introduced in Chap. 1 and are used throughout the text. Approximately half of the sample problems and 60 percent of the homework problems are stated in these units, while the remainder

are in U.S. customary units. The authors believe that this approach will best serve the need of students, who, as engineers, will have to be conversant with both systems of units.

It also should be recognized that using both SI and U.S. customary units entails more than the use of conversion factors. Since the SI system of units is an absolute system based on the units of time, length, and mass, whereas the U.S. customary system is a gravitational system based on the units of time, length, and force, different approaches are required for the solution of many problems. For example, when SI units are used, a body is generally specified by its mass expressed in kilograms; in most problems of statics it will be necessary to determine the weight of the body in newtons, and an additional calculation will be required for this purpose. On the other hand, when U.S. customary units are used, a body is specified by its weight in pounds and, in dynamics problems, an additional calculation will be required to determine its mass in slugs (or $lb \cdot s^2/ft$). The authors, therefore, believe that problem assignments should include both systems of units.

The *Instructor's and Solutions Manual* provides six different lists of assignments so that an equal number of problems stated in SI units and in U.S. customary units can be selected. If so desired, two complete lists of assignments can also be selected with up to 75 percent of the problems stated in SI units.

Optional Sections Offer Advanced or Specialty Topics. A large number of optional sections have been included. These sections are indicated by asterisks and thus are easily distinguished from those which form the core of the basic mechanics course. They may be omitted without prejudice to the understanding of the rest of the text.

The topics covered in the optional sections in statics include the reduction of a system of forces to a wrench, applications to hydrostatics, shear and bending-moment diagrams for beams, equilibrium of cables, products of inertia and Mohr's circle, mass products of inertia and principal axes of inertia for three-dimensional bodies, and the method of virtual work. An optional section on the determination of the principal axes and the mass moments of inertia of a body of arbitrary shape is included (Sec. 9.18). The sections on beams are especially useful when the course in statics is immediately followed by a course in mechanics of materials, while the sections on the inertia properties of three-dimensional bodies are primarily intended for the students who will later study in dynamics the three-dimensional motion of rigid bodies.

The topics covered in the optional sections in dynamics include graphical methods for the solution of rectilinear-motion problems, the trajectory of a particle under a central force, the deflection of fluid streams, problems involving jet and rocket propulsion, the kinematics and kinetics of rigid bodies in three dimensions, damped mechanical vibrations, and electrical analogues. These topics will be found of particular interest when dynamics is taught in the junior year.

The material presented in the text and most of the problems require no previous mathematical knowledge beyond algebra, trigonometry, and elementary calculus; all the elements of vector algebra

necessary to the understanding of the text are carefully presented in Chaps. 2 and 3. However, special problems are included, which make use of a more advanced knowledge of calculus, and certain sections, such as Secs. 19.8 and 19.9 on damped vibrations, should be assigned only if students possess the proper mathematical background. In portions of the text using elementary calculus, a greater emphasis is placed on the correct understanding and application of the concepts of differentiation and integration than on the nimble manipulation of mathematical formulas. In this connection, it should be mentioned that the determination of the centroids of composite areas precedes the calculation of centroids by integration, thus making it possible to establish the concept of moment of area firmly before introducing the use of integration.

CHAPTER ORGANIZATION AND PEDAGOGICAL FEATURES

Chapter Introduction.   Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems. Chapter outlines provide students with a preview of chapter topics.

Chapter Lessons.   The body of the text is divided into units, each consisting of one or several theory sections, one or several sample problems, and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally can be covered in one lesson. In a number of cases, however, the instructor will find it desirable to devote more than one lesson to a given topic. *The Instructor's and Solutions Manual* contains suggestions on the coverage of each lesson.

Sample Problems.   The sample problems are set up in much the same form that students will use when solving the assigned problems. They thus serve the double purpose of amplifying the text and demonstrating the type of neat, orderly work that students should cultivate in their own solutions.

Solving Problems on Your Own.   A section entitled *Solving Problems on Your Own* is included for each lesson, between the sample problems and the problems to be assigned. The purpose of these sections is to help students organize in their own minds the preceding theory of the text and the solution methods of the sample problems so that they can more successfully solve the homework problems. Also included in these sections are specific suggestions and strategies which will enable students to more efficiently attack any assigned problems.

Homework Problem Sets.   Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the

text and to help students understand the principles of mechanics. The problems are grouped according to the portions of material they illustrate and are arranged in order of increasing difficulty. Problems requiring special attention are indicated by asterisks. Answers to 70 percent of the problems are given at the end of the book. Problems for which the answers are given are set in straight type in the text, while problems for which no answer is given are set in italic.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Marginal notes are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.

Computer Problems. Each chapter includes a set of problems designed to be solved with computational software. Many of these problems provide an introduction to the design process. In *Statics,* for example, they may involve the analysis of a structure for various configurations and loading of the structure or the determination of the equilibrium positions of a mechanism which may require an iterative method of solution. In *Dynamics,* they may involve the determination of the motion of a particle under initial conditions, the kinematic or kinetic analysis of mechanisms in successive positions, or the numerical integration of various equations of motion. Developing the algorithm required to solve a given mechanics problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply their computer skills to the solution of a meaningful engineering problem.

SUPPLEMENTS

An extensive supplements package for both instructors and students is available with the text.

Instructor's and Solutions Manual. *The Instructor's and Solutions Manual* that accompanies the ninth edition features typeset, one-per-page solutions to all homework problems. This manual also features a number of tables designed to assist instructors in creating a schedule of assignments for their courses. The various topics covered in the text are listed in Table I, and a suggested number of periods to be spent on each topic is indicated. Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. Sample lesson schedules are shown in Tables III, IV, and V.

McGRAW-HILL CONNECT ENGINEERING

McGraw-Hill Connect Engineering is a web-based assignment and assessment platform that gives students the means to better connect with their coursework, their instructors, and the important concepts that they will need to know for success now and in the future. With Connect Engineering, instructors can deliver assignments, quizzes, and tests easily online. Students can practice important skills at their own pace and on their own schedule.

Connect Engineering for *Vector Mechanics for Engineers* is available at **www.mhhe.com/beerjohnston** and includes algorithmic problems from the text, Lecture PowerPoints, an image bank, and animations.

Hands-on Mechanics.   Hands-on Mechanics is a website designed for instructors who are interested in incorporating three-dimensional, hands-on teaching aids into their lectures. Developed through a partnership between the McGraw-Hill Engineering Team and the Department of Civil and Mechanical Engineering at the United States Military Academy at West Point, this website not only provides detailed instructions for how to build 3-D teaching tools using materials found in any lab or local hardware store but also provides a community where educators can share ideas, trade best practices, and submit their own demonstrations for posting on the site. Visit **www.handsonmechanics.com.**

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We are pleased to recognize Dennis Ormond of Fine Line Illustrations for the artful illustrations which contribute so much to the effectiveness of the text.

The authors thank the many companies that provided photographs for this edition. We also wish to recognize the determined efforts and patience of our photo researcher Sabina Dowell.

The authors gratefully acknowledge the many helpful comments and suggestions offered by users of the previous editions of *Vector Mechanics for Engineers.*

E. Russell Johnston, Jr.
David Mazurek
Phillip Cornwell
Elliot R. Eisenberg

List of Symbols

a	Constant; radius; distance
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \ldots$	Reactions at supports and connections
$A, B, C, \ldots$	Points
A	Area
b	Width; distance
c	Constant
C	Centroid
d	Distance
e	Base of natural logarithms
$\mathbf{F}$	Force; friction force
g	Acceleration of gravity
G	Center of gravity; constant of gravitation
h	Height; sag of cable
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	Unit vectors along coordinate axes
$I, I_x, \ldots$	Moments of inertia
$\bar{I}$	Centroidal moment of inertia
$I_{xy}, \ldots$	Products of inertia
J	Polar moment of inertia
k	Spring constant
k_x, k_y, k_O	Radii of gyration
$\bar{k}$	Centroidal radius of gyration
l	Length
L	Length; span
m	Mass
$\mathbf{M}$	Couple; moment
$\mathbf{M}_O$	Moment about point O
$\mathbf{M}_O^R$	Moment resultant about point O
M	Magnitude of couple or moment; mass of earth
M_{OL}	Moment about axis OL
$\mathbf{N}$	Normal component of reaction
O	Origin of coordinates
p	Pressure
$\mathbf{P}$	Force; vector
$\mathbf{Q}$	Force; vector
$\mathbf{r}$	Position vector
r	Radius; distance; polar coordinate
$\mathbf{R}$	Resultant force; resultant vector; reaction
R	Radius of earth
$\mathbf{s}$	Position vector
s	Length of arc; length of cable
$\mathbf{S}$	Force; vector
t	Thickness
$\mathbf{T}$	Force
T	Tension
U	Work

V	Vector product; shearing force
V	Volume; potential energy; shear
w	Load per unit length
W, W	Weight; load
x, y, z	Rectangular coordinates; distances
$\bar{x}, \bar{y}, \bar{z}$	Rectangular coordinates of centroid or center of gravity
α, β, γ	Angles
γ	Specific weight
δ	Elongation
$\delta\mathbf{r}$	Virtual displacement
δU	Virtual work
$\boldsymbol{\lambda}$	Unit vector along a line
η	Efficiency
θ	Angular coordinate; angle; polar coordinate
μ	Coefficient of friction
ρ	Density
ϕ	Angle of friction; angle