

Second Edition

POWER SYSTEM DYNAMICS

Stability and Control

Jan Machowski
Janusz W. Bialek
James R. Bumby

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POWER SYSTEM DYNAMICS

About the Authors

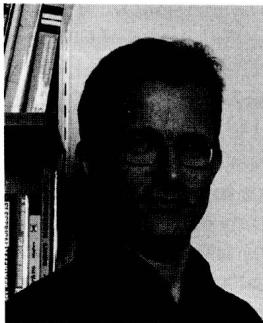


Professor Jan Machowski received his MSc and PhD degrees in Electrical Engineering from Warsaw University of Technology in 1974 and 1979, respectively. After obtaining field experience in the Dispatching Centre and several power plants, he joined the Electrical Faculty of Warsaw University of Technology where presently he is employed as a Professor and Director of the Power Engineering Institute. His areas of interest are electrical power systems, power system protection and control.

In 1989–93 Professor Machowski was a Visiting Professor at Kaiserlautern University in Germany where he carried out two research projects on power swing blocking algorithms for distance protection and optimal control of FACTS devices.

Professor Machowski is the co-author of three books published in Polish: *Power System Stability* (WNT, 1989), *Short Circuits in Power Systems* (WNT, 2002) and *Power System Control and Stability* (WPW, 2007). He is also a co-author of *Power System Dynamics and Stability* published by John Wiley & Sons, Ltd (1997).

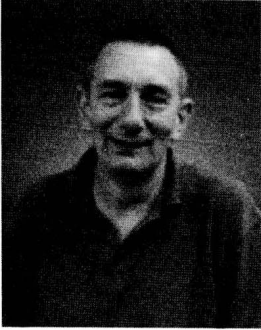
Professor Machowski is the author and co-author of 42 papers published in English in international fora. He has carried out many projects on electrical power systems, power system stability and power system protection commissioned by the Polish Power Grid Company, Electric Power Research Institute in the United States, Electrinstytut Milan Vidmar in Slovenia and Ministry of Science and Higher Education of Poland.



Professor Janusz Bialek received his MEng and PhD degrees in Electrical Engineering from Warsaw University of Technology in 1977 and 1981, respectively. From 1981 to 1989 he was a lecturer with Warsaw University of Technology. In 1989 he moved to the University of Durham, United Kingdom, and since 2003 he has been at the University of Edinburgh where he currently holds the Bert Whittington Chair of Electrical Engineering. His main research interests are in sustainable energy systems, security of supply, liberalization of the electricity supply industry and power system dynamics and control.

Professor Bialek has co-authored two books and over 100 research papers. He has been a consultant to the Department of Trade and Industry (DTI) of the UK government, Scottish Executive, Elexon, Polish Power Grid Company, Scottish Power, Enron and Electrical Power Research Institute (EPRI). He was the Principal Investigator of a number of major research grants funded by the Engineering and Physical Sciences Research Council and the DTI.

Professor Bialek is a member of the Advisory Board of Electricity Policy Research Group, Cambridge University, a member of the Dispute Resolution Panel for the Single Electricity Market Operator, Ireland, and Honorary Professor of Heriot-Watt University, Scotland.



Dr Jim Bumby received his BSc and PhD degrees in Engineering from Durham University, United Kingdom, in 1970 and 1974, respectively. From 1973 to 1978 he worked for the International Research and Development Company, Newcastle-upon-Tyne, on superconducting machines, hybrid vehicles and sea-wave energy. Since 1978 he has worked in the School of Engineering at Durham University where he is currently Reader in Electrical Engineering. He has worked in the area of electrical machines and systems for over 30 years, first in industry and then in academia.

Dr Bumby is the author or co-author of over 100 technical papers and two books in the general area of electrical machines/power systems and control. He has also written numerous technical reports for industrial clients. These papers and books have led to the award of a number of national and international prizes including the Institute of Measurement and Control prize for the best transactions paper in 1988 for work on hybrid electric vehicles and the IEE Power Division Premium in 1997 for work on direct drive permanent magnet generators for wind turbine applications. His current research interests are in novel generator technologies and their associated control for new and renewable energy systems.

Preface

In 1997 the authors of this book, J. Machowski, J.W. Bialek and J.R. Bumby, published a book entitled *Power System Dynamics and Stability*. That book was well received by readers who told us that it was used regularly as a standard reference text both in academia and in industry. Some 10 years after publication of that book we started work on a second edition. However, we quickly realized that the developments in the power systems industry over the intervening years required a large amount of new material. Consequently the book has been expanded by about a third and the word *Control* in the new title, *Power System Dynamics: Stability and Control*, reflects the fact that a large part of the new material concerns power system control: flexible AC transmission systems (FACTS), wide area measurement systems (WAMS), frequency control, voltage control, etc. The new title also reflects a slight shift in focus from solely describing power system dynamics to the means of dealing with them. For example, we believe that the new WAMS technology is likely to revolutionize power system control. One of the main obstacles to a wider embrace of WAMS by power system operators is an acknowledged lack of algorithms which could be utilized to control a system in real time. This book tries to fill this gap by developing a number of algorithms for WAMS-based real-time control.

The second reason for adding so much new material is the unprecedented change that has been sweeping the power systems industry since the 1990s. In particular the rapid growth of renewable generation, driven by global warming concerns, is changing the fundamental characteristics of the system. Currently wind power is the dominant renewable energy source and wind generators usually use induction, rather than synchronous, machines. As a significant penetration of such generation will change the system dynamics, the new material in Chapter 7 is devoted entirely to wind generation.

The third factor to be taken into account is the fallout from a number of highly publicized blackouts that happened in the early years of the new millennium. Of particular concern were the autumn 2003 blackouts in the United States/Canada, Italy, Sweden/Denmark and the United Kingdom, the 2004 blackout in Athens and the European disturbance on 4 November 2006. These blackouts have exposed a number of critical issues, especially those regarding power system behaviour at depressed voltages. Consequently, the book has been extended to cover these phenomena together with an illustration of some of the blackouts.

It is important to emphasize that the new book is based on the same philosophy as the previous one. We try to answer some of the concerns about the education of power system engineers. With the widespread access to powerful computers running evermore sophisticated simulation packages, there is a tendency to treat simulation as a substitute for understanding. This tendency is especially dangerous for students and young researchers who think that simulation is a panacea for everything and always provides a true answer. What they do not realize is that, without a physical understanding of the underlying principles, they cannot be confident in understanding, or validating, the simulation results. It is by no means bad practice to treat the initial results of any computer software with a healthy pinch of scepticism.

Power system dynamics are not easy to understand. There are a number of good textbooks which deal with this topic and some of these are reviewed in Chapter 1. As the synchronous machine plays a decisive role in determining the dynamic response of the system, many of these books start with a detailed mathematical treatment of the synchronous generator in order to introduce Park's equations and produce a mathematical model of the generator. However, it is our experience that to begin a topic with such a detailed mathematical treatment can put many students off further study because they often find it difficult to see any practical relevance for the mathematics. This can be a major obstacle for those readers who are more practically inclined and who want to understand what is happening in the system without having to refer continuously to a complicated mathematical model of the generator.

Our approach is different. We first try to give a qualitative explanation of the underlying physical phenomena of power system dynamics using a simple model of the generator, coupled with the basic physical laws of electrical engineering. Having provided the student with a physical understanding of power system dynamics, we then introduce the full mathematical model of the generator, followed by more advanced topics such as system reduction, dynamic simulation and eigenvalue analysis. In this way we hope that the material is made more accessible to the reader who wishes to understand the system operation without first tackling Park's equations.

All our considerations are richly illustrated by diagrams. We strongly believe in the old adage that an illustration is worth a thousand words. In fact, our book contains over 400 diagrams.

The book is conveniently divided into three major parts. The first part (Chapters 1–3) reviews the background for studying power system dynamics. The second part (Chapters 4–10) attempts to explain the basic phenomena underlying power system dynamics using the classical model of the generator–infinite busbar system. The third part (Chapters 11–14) tackles some of the more advanced topics suitable for the modelling and dynamic simulation of large-scale power systems.

Examining the chapters and the new material added in more detail, Chapter 1 classifies power system dynamics and provides a brief historical overview. The new material expands on the definitions of power system stability and security assessment and introduces some important concepts used in later chapters. Chapter 2 contains a brief description of the major power system components, including modern FACTS devices. The main additions here provide a more comprehensive treatment of FACTS devices and a whole new section on WAMS. Chapter 3 introduces steady-state models and their use in analysing the performance of the power system. The new material covers enhanced treatment of the generator as the reactive power source introducing voltage–reactive power capability characteristics. We believe that this is a novel treatment of those concepts since we have not seen it anywhere else. The importance of understanding how the generator and its controls behave under depressed voltages has been emphasized by the wide area blackouts mentioned above. The chapter also includes a new section on controlling power flows in the network.

Chapter 4 analyses the dynamics following a disturbance and introduces models suitable for analysing the dynamic performance of the synchronous generator. Chapter 5 explains the power system dynamics following a small disturbance (steady-state stability) while Chapter 6 examines the system dynamics following a large disturbance (transient stability). There are new sections on using the Lyapunov direct method to analyse the stability of a multi-machine power system and on out-of-step relaying. Chapter 7 is all new and covers the fundamentals of wind power generation. Chapter 8 has been greatly expanded and provides an explanation of voltage stability together with some of the methods used for stability assessment. The new material includes examples of power system blackouts, methods of preventing voltage collapse and a large new section on self-excitation of the generator. Chapter 9 contains a largely enhanced treatment of frequency stability and control including defence plans against frequency instability and quality assessment of frequency control. There is a large new section which covers a novel treatment of interaction between automatic generation control (AGC) and FACTS devices installed in tie-lines that control the flow of power between systems in an interconnected network. Chapter 10 provides an overview of the main methods of stability enhancement, both conventional and using FACTS devices. The new material

includes the use of braking resistors and a novel generalization of earlier derived stabilization algorithms to a multi-machine power system.

Chapter 11 introduces advanced models of the different power system elements. The new material includes models of the wind turbine and generator and models of FACTS devices. Chapter 12 contains a largely expanded treatment of the steady-state stability of multi-machine power systems using eigenvalue analysis. We have added a comprehensive explanation of the meaning of eigenvalues and eigenvectors including a fuller treatment of the mathematical background. As the subject matter is highly mathematical and may be difficult to understand, we have added a large number of numerical examples. Chapter 13 contains a description of numerical methods used for power system dynamic simulation. Chapter 14 explains how to reduce the size of the simulation problem by using equivalents. The chapter has been significantly expanded by adding novel material on the modal analysis of equivalents and a number of examples.

The Appendix covers the per-unit system and new material on the mathematical fundamentals of solving ordinary differential equations.

It is important to emphasize that, while most of the book is a teaching textbook written with final-year undergraduate and postgraduate students in mind, there are also large parts of material which constitute cutting-edge research, some of it never published before. This includes the use of the Lyapunov direct method to derive algorithms for the stabilization of a multi-machine power system (Chapters 6, 9 and 10) and derivation of modal-analysis-based power system dynamic equivalents (Chapter 14).

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List of Symbols

Notation

Italic type denotes scalar physical quantity (e.g. R , L , C) or numerical variable (e.g. x , y).

Phasor or complex quantity or numerical variable is underlined (e.g. \underline{I} , \underline{V} , \underline{S}).

Italic with arrow on top of a symbol denotes a spatial vector (e.g. \vec{F}).

Italic boldface denotes a matrix or a vector (e.g. \mathbf{A} , \mathbf{B} , \mathbf{x} , \mathbf{y}).

Unit symbols are written using roman type (e.g. Hz, A, kV).

Standard mathematical functions are written using roman type (e.g. e, sin, cos, arctan).

Numbers are written using roman type (e.g. 5, 6).

Mathematical operators are written using roman type (e.g. s , Laplace operator; T , matrix transposition; j , angular shift by 90° ; a , angular shift by 120°).

Differentials and partial differentials are written using roman type (e.g. df/dx , $\partial f/\partial x$).

Symbols describing objects are written using roman type (e.g. TRAF0, LINE).

Subscripts relating to objects are written using roman type (e.g. $\underline{I}_{\text{TRAF0}}$, $\underline{I}_{\text{LINE}}$).

Subscripts relating to physical quantities or numerical variables are written using italic type (e.g. A_{ij} , x_k).

Subscripts A, B, C refer to the three-phase axes of a generator.

Subscripts d, q refer to the direct- and quadrature-axis components.

Lower case symbols normally denote instantaneous values (e.g. v , i).

Upper case symbols normally denote rms or peak values (e.g. V , I).

Symbols

\underline{a} and \underline{a}^2 operators shifting the angle by 120° and 240° , respectively.

B_μ magnetizing susceptance of a transformer.

B_{sh} susceptance of a shunt element.

D damping coefficient.

E_k kinetic energy of the rotor relative to the synchronous speed.

E_p potential energy of the rotor with respect to the equilibrium point.

e_f field voltage referred to the fictitious q-axis armature coil.

e_q steady-state emf induced in the fictitious q-axis armature coil proportional to the field winding self-flux linkages.

e'_d transient emf induced in the fictitious d-axis armature coil proportional to the flux linkages of the q-axis coil representing the solid steel rotor body (round-rotor generators only).

e'_q transient emf induced in the fictitious q-axis armature coil proportional to the field winding flux linkages.

e''_d subtransient emf induced in the fictitious d-axis armature coil proportional to the total q-axis rotor flux linkages (q-axis damper winding and q-axis solid steel rotor body).

e''_q	subtransient emf induced in the fictitious q-axis armature coil proportional to the total d-axis rotor flux linkages (d-axis damper winding and field winding).
\underline{E}	steady-state internal emf.
\underline{E}_f	excitation emf proportional to the excitation voltage V_f .
E_{fm}	peak value of the excitation emf.
E_d	d-axis component of the steady-state internal emf proportional to the rotor self-linkages due to currents induced in the q-axis solid steel rotor body (round-rotor generators only).
E_q	q-axis component of the steady-state internal emf proportional to the field winding self-flux linkages (i.e. proportional to the field current itself).
\underline{E}'	transient internal emf proportional to the flux linkages of the field winding and solid steel rotor body (includes armature reaction).
E'_d	d-axis component of the transient internal emf proportional to flux linkages in the q-axis solid steel rotor body (round-rotor generators only).
E'_q	q-axis component of the transient internal emf proportional to the field winding flux linkages.
\underline{E}''	subtransient internal emf proportional to the total rotor flux linkages (includes armature reaction).
E''_d	d-axis component of the subtransient internal emf proportional to the total flux linkages in the q-axis damper winding and q-axis solid steel rotor body.
E''_q	q-axis component of the subtransient internal emf proportional to the total flux linkages in the d-axis damper winding and the field winding.
\underline{E}_r	resultant air-gap emf.
E_{rm}	amplitude of the resultant air-gap emf.
\underline{E}_G	vector of the generator emfs.
f	mains frequency.
f_n	rated frequency.
\vec{F}	magnetomotive force (mmf) due to the field winding.
\vec{F}_a	armature reaction mmf.
$F_{a AC}$	AC armature reaction mmf (rotating).
$F_{a DC}$	DC armature reaction mmf (stationary).
$\vec{F}_{ad}, \vec{F}_{aq}$	d- and q-axis components of the armature reaction mmf.
\vec{F}_f	resultant mmf.
G_{Fe}	core loss conductance of a transformer.
G_{sh}	conductance of a shunt element.
H_{ii}, H_{ij}	self- and mutual synchronizing power.
i_A, i_B, i_C	instantaneous currents in phases A, B and C.
$i_{A DC}, i_{B DC}, i_{C DC}$	DC component of the current in phases A, B, C.
$i_{A AC}, i_{B AC}, i_{C AC}$	AC component of the current in phases A, B, C.
i_d, i_q	currents flowing in the fictitious d- and q-axis armature coils.
i_D, i_Q	instantaneous d- and q-axis damper winding current.
i_f	instantaneous field current of a generator.
\underline{i}_{ABC}	vector of instantaneous phase currents.
\underline{i}_{DQ}	vector of instantaneous currents in the field winding and the d- and q-axis damper windings.
\underline{i}_{0dq}	vector of armature currents in the rotor reference frame.
\underline{I}	armature current.
I_d, I_q	d- and q-axis component of the armature current.
$\underline{I}_S, \underline{I}_R$	currents at the sending and receiving end of a transmission line.
$\underline{I}_R, \underline{I}_E$	vector of complex current injections to the retained and eliminated nodes.

$\underline{I}_G, \underline{I}_L$	vector of complex generator and load currents.
$\Delta \underline{I}_L$	vector of load corrective complex currents.
J	moment of inertia.
j	operator shifting the angle by 90° .
k_{PV}, k_{QV}	voltage sensitivities of the load (the slopes of the real and reactive power demand characteristics as a function of voltage).
k_{Pf}, k_{Qf}	frequency sensitivities of the load (the slopes of the real and reactive power demand characteristics as a function of frequency).
K_{E_q}	steady-state synchronizing power coefficient (the slope of the steady-state power angle curve $P_{E_q}(\delta)$).
$K_{E'_q}$	transient synchronizing power coefficient (the slope of the transient power angle curve $P_{E'_q}(\delta')$).
$K_{E'}$	transient synchronizing power coefficient (the slope of the transient power angle curve $P_{E'}(\delta')$).
K_i	reciprocal of droop for the i th generating unit.
K_L	frequency sensitivity coefficient of the system real power demand.
K_T	reciprocal of droop for the total system generation characteristic.
l	length of a transmission line.
$L_{AA}, L_{BB}, L_{CC},$ L_{ff}, L_{DD}, L_{QQ}	self-inductances of the windings of the phase windings A, B, C, the field winding, and the d- and the q-axis damper winding.
L_d, L_q	inductances of the fictitious d- and q-axis armature windings.
L'_d, L'_q, L''_d, L''_q	d- and q-axis transient and subtransient inductances.
L_S	minimum value of the self-inductance of a phase winding.
L_{xy}	where $x, y \in \{A, B, C, D, Q, f\}$ and $x \neq y$, are the mutual inductances between the windings denoted by the indices as described above.
ΔL_S	amplitude of the variable part of the self-inductance of a phase winding.
L_R	submatrix of the rotor self- and mutual inductances.
L_S	submatrix of the stator self- and mutual inductances.
L_{SR}, L_{RS}	submatrices of the stator-to-rotor and rotor-to-stator mutual inductances.
M	coefficient of inertia.
M_f, M_D, M_Q	amplitude of the mutual inductance between a phase winding and, respectively, the field winding and the d- and the q-axis damper winding.
N	generally, number of any objects.
p	number of poles.
P_{acc}	accelerating power.
P_D	damping power.
P_e	electromagnetic air-gap power.
$P_{E_q \text{ cr}}$	critical (pull-out) air-gap power developed by a generator.
$P_{E_q}(\delta), P_{E'}(\delta'),$ $P_{E'_q}(\delta')$	air-gap power curves assuming $E_q = \text{constant}$, $E' = \text{constant}$ and $E'_q = \text{constant}$.
P_g	in induction machine, real power supplied from the grid (motoring mode), or supplied to the grid (generating mode).
P_L	real power absorbed by a load or total system load.
P_m	mechanical power supplied by a prime mover to a generator; also mechanical power supplied by a motor to a load (induction machine in motoring mode).
P_n	real power demand at rated voltage.
P_R	real power at the receiving end of a transmission line.
$P_{rI}, P_{rII}, P_{rIII}, P_{rIV}$	contribution of the generating units remaining in operation to covering the real power imbalance during the first, second, third and fourth stages of load frequency control.

$P_{sI}, P_{sII}, P_{sIII}, P_{sIV}$	contribution of the system to covering the real power imbalance during the first, second, third and fourth stages of load frequency control.
P_s	stator power of induction machine or power supplied by the system.
P_S	real power at the sending end of a transmission line or real power supplied by a source to a load or real power supplied to an infinite busbar.
P_{SIL}	surge impedance (natural) load.
$P_{sEq}(\delta)$	curve of real power supplied to an infinite busbar assuming $E_q = \text{constant}$.
P_T	total power generated in a system.
P_{tie}	net tie-line interchange power.
$P_{V_g}(\delta)$	air-gap power curve assuming $V_g = \text{constant}$.
$P_{V_g cr}$	critical value of $P_{V_g}(\delta)$.
Q_L	reactive power absorbed by a load.
Q_G	reactive power generated by a source (the sum of Q_L and the reactive power loss in the network).
Q_n	reactive power demand at rated voltage.
Q_R	reactive power at the receiving end of a transmission line.
Q_S	reactive power at the sending end of a transmission line or reactive power supplied by a source to a load.
R	resistance of the armature winding of a generator.
r	total resistance between (and including) a generator and an infinite busbar.
$R_A, R_B, R_C, R_D,$ R_Q, R_f	resistances of the phase windings A, B, C, the d- and q-axis damper winding, and the field winding.
R_{ABC}	diagonal matrix of phase winding resistances.
R_{fDQ}	diagonal matrix of resistances of the field winding and the d- and q-axis damper windings.
s	Laplace operator.
s	slip of induction motor.
s_{cr}	critical slip of induction motor.
S_n	rated apparent power.
S_{SHC}	short-circuit power.
t	time.
T'_d, T''_d	short-circuit d-axis transient and subtransient time constants.
T'_{do}, T''_{do}	open-circuit d-axis transient and subtransient time constants.
T'_q, T''_q	short-circuit q-axis transient and subtransient time constants.
T'_{qo}, T''_{qo}	open-circuit q-axis transient and subtransient time constants.
T_a	armature winding time constant.
T	transformation matrix between network (a, b) and generator (d, q) coordinates.
v_A, v_B, v_C, v_f	instantaneous voltages across phases A, B, C and the field winding.
v_d, v_q	voltages across the fictitious d- and q-axis armature coils.
v_w	wind speed.
v_{ABC}	vector of instantaneous voltages across phases A, B, C.
v_{fDQ}	vector of instantaneous voltages across the field winding and the d- and q-axis damper windings.
V	Lyapunov function.
V_{cr}	critical value of the voltage.
V_d, V_q	direct- and quadrature-axis component of the generator terminal voltage.
V_f	voltage applied to the field winding.
\underline{V}_g	voltage at the generator terminals.

V_s	infinite busbar voltage.
V_{sd}, V_{sq}	direct- and quadrature-axis component of the infinite busbar voltage.
V_S, V_R	voltage at the sending and receiving end of a transmission line.
V_{sh}	local voltage at the point of installation of a shunt element.
$V_i = V_i \angle \delta_i$	complex voltage at node i .
V_R, V_E	vector of complex voltages at the retained and eliminated nodes.
W	work.
W	Park's modified transformation matrix.
W, U	modal matrices of right and left eigenvectors.
X_a	armature reaction reactance (round-rotor generator).
X_C	reactance of a series compensator.
X_D	reactance corresponding to the flux path around the damper winding.
X_d, X'_d, X''_d	d-axis synchronous, transient and subtransient reactance.
x_d, x'_d, x''_d	total d-axis synchronous, transient and subtransient reactance between (and including) a generator and an infinite busbar.
$x'_{d\text{ PRE}}, x'_{d\text{ F}}, x'_{d\text{ POST}}$	prefault, fault and postfault value of x'_d .
X_f	reactance corresponding to the flux path around the field winding.
X_l	armature leakage reactance of a generator.
X_q, X'_q, X''_q	q-axis synchronous, transient and subtransient reactance.
x_q, x'_q, x''_q	total q-axis synchronous, transient and subtransient reactance between (and including) a generator and an infinite busbar.
X_{SHC}	short-circuit reactance of a system as seen from a node.
Y_T	admittance of a transformer.
Y	admittance matrix.
$Y_{GG}, Y_{LL}, Y_{LG}, Y_{LE}$	admittance submatrices where subscript G corresponds to fictitious generator nodes and subscript L corresponds to all the other nodes (including generator terminal nodes).
$Y_{ij} = G_{ij} + jB_{ij}$	element of the admittance matrix.
$Y_{RR}, Y_{EE}, Y_{RE}, Y_{ER}$	complex admittance submatrices where subscript E refers to eliminated nodes and subscript R to retained nodes.
Z_c	characteristic impedance of a transmission line.
$Z_s = R_s + jX_s$	internal impedance of an infinite busbar.
$Z_T = R_T + jX_T$	series impedance of a transformer.
β	phase constant of a transmission line.
γ	instantaneous position of the generator d-axis relative to phase A; propagation constant of a transmission line.
γ_0	position of the generator d-axis at the instant of fault.
δ	power (or rotor) angle with respect to an infinite busbar.
δ_g	power (or rotor) angle with respect to the voltage at the generator terminals.
$\hat{\delta}_s$	stable equilibrium value of the rotor angle.
δ'	transient power (or rotor) angle between E' and V_s .
δ_{fr}	angle between the resultant and field mmfs.
$\Delta\omega$	rotor speed deviation equal to $(\omega - \omega_s)$.
ε	rotor acceleration.
ζ	damping ratio.
ϑ	transformation ratio.
λ_R	frequency bias factor.
$\lambda_i = \alpha_i + j\Omega_i$	eigenvalue.
ρ	static droop of the turbine-governor characteristic.
ρ_T	droop of the total system generation characteristic.

τ_e	electromagnetic torque.
τ_m	mechanical torque.
τ_ω	fundamental-frequency subtransient electromagnetic torque.
$\tau_{2\omega}$	double-frequency subtransient electromagnetic torque.
τ_d, τ_q	direct- and quadrature-axis component of the electromagnetic torque.
τ_R, τ_r	subtransient electromagnetic torque due to stator and rotor resistances.
φ_g	power factor angle at the generator terminals.
Φ_a	armature reaction flux.
Φ_{ad}, Φ_{aq}	d- and q-axis component of the armature reaction flux.
Φ_{aAC}	AC armature reaction flux (rotating).
Φ_{aDC}	DC armature reaction flux (stationary).
Φ_f	excitation (field) flux.
Ψ_A, Ψ_B, Ψ_C	total flux linkage of phases A, B, C.
$\Psi_{AA}, \Psi_{BB}, \Psi_B$	self-flux linkage of phases A, B, C.
Ψ_{aACr}	rotor flux linkages produced by Φ_{aAC} .
Ψ_{aDCr}	rotor flux linkages produced by Φ_{aDC} .
Ψ_{ar}	rotor flux linkages produced by the total armature reaction flux.
Ψ_D, Ψ_Q	total flux linkage of damper windings in axes d and q.
Ψ_d, Ψ_q	total d- and q-axis flux linkages.
Ψ_f	total flux linkage of the field winding.
Ψ_{fa}	excitation flux linkage with armature winding.
$\Psi_{fA}, \Psi_{fB}, \Psi_{fC}$	excitation flux linkage with phases A, B and C.
Ψ_{ABC}	vector of phase flux linkages.
Ψ_{fDQ}	vector of flux linkages of the field winding and the d- and q-axis damper windings.
Ψ_{0dq}	vector of armature flux linkages in the rotor reference frame.
ω	angular velocity of the generator (in electrical radians).
ω_s	synchronous angular velocity in electrical radians (equal to $2\pi f$).
ω_T	rotor speed of wind turbine (in rad/s)
Ω	frequency of rotor swings (in rad/s)
Ω	rotation matrix.
\mathfrak{R}	reluctance.
$\mathfrak{R}_d, \mathfrak{R}_q$	reluctance along the direct- and quadrature-axis.

Abbreviations

AC	alternating current
ACE	area control error
AGC	Automatic Generation Control
AVR	Automatic Voltage Regulator
BEES	Battery Energy Storage System
d	direct axis of a generator
DC	direct current
DFIG	Doubly Fed Induction Generator
DFIM	Double Fed Induction Machine
DSA	Dynamic Security Assessment
emf	electro-motive force
EMS	Energy Management System
FACTS	Flexible AC Transmission Systems
HV	high voltage
HAWT	Horizontal-Axis Wind Turbine

IGTB	insulated gate bipolar transistor
IGTC	integrated gate-commutated thyristor
LFC	load frequency control
mmf	magneto-motive force
MAWS	mean annual wind speed
PMU	Phasor Measurement Unit
PSS	power system stabiliser
pu	per unit
q	quadrature axis of a generator
rms	root-mean-square
rpm	revolutions per minute
rhs	right-hand-side
SCADA	Supervisory Control and Data Acquisition
SIL	surge impedance load
SMES	superconducting magnetic energy storage
SSSC	Static Synchronous Series Compensator
STATCOM	static compensator
SVC	Static VAR Compensator
TCBR	Thyristor Controlled Braking Resistor
TCPAR	Thyristor Controlled Phase Angle Regulator
TSO	Transmission System Operator
VAWT	Vertical-Axis Wind Turbine
UPFC	unified power flow controller
WAMS	Wide Area Measurement System
WAMPAC	Wide Area Measurement, Protection and Control