SIGNAL PROCESSING DESIGN TECHNIQUES

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TAB Professional and Reference Books
Division of TAB BOOKS Inc.
P.O. Box 40, Blue Ridge Summit, PA 17214

FIRST EDITION

FIRST PRINTING

Printed in the United States of America

Library of Congress Cataloging in Publication Data

Rorabaugh, Britt.
Signal processing design techniques.

Bibliography: p.
Includes index.
1. Signal processing. 2. Electric filters.
I. Title.
TK5102.5.R57 1986 621.38′043 86-5909
ISBN 0-8306-0457-X

SIGNAL PROCESSING DESIGN TECHNIQUES

To Joyce, Geoff, and Amber
for their love and support
and to Glenn whose example
taught me to respect
good engineering

Preface

This book is an overview of the various aspects of signal processing, but it should not be viewed as all-inclusive. The field is huge and growing—particularly in the digital area. Present applications usually involve a mixture of digital and analog techniques because the speed of digital processors limits the bandwidth of signals on which they can be used. However, since the speed of digital processors is increasing, all-digital techniques are beginning to surpass analog techniques in popularity. Furthermore, the flexibility of digital processing allows the use of new techniques that are impossible to realize with analog techniques.

Introduction

The earliest uses of electricity were substitutes for fire—lightbulbs instead of lanterns and motors instead of steam engines. These purely electrical devices quickly led to electronic devices such as phonographs, radios, amplifiers, and televisions which are primarily concerned with processing of signals. Still further developments led to digital devices which process numbers, data, and perhaps even ideas. The modern field of signal processing spans both analog electronics and digital disciplines. Electrical signals can be processed directly using analog circuits such as op-amp active filters or indirectly by first digitizing the signal into a sequence of numeric values and then processing these values in a digital computing device. While amplifiers and radios are indeed ways to process a signal, the term "signal processing" has come to mean a more limited set of techniques which include primarily frequency selective filtering, spectrum analysis, and spectrum shaping techniques. This book will deal with both analog and digital techniques for performing these types of processing.

Chapter 1 begins with a look at signals and spectra and the mathematics conventionally used to represent them. Chapter 2 continues with a similar treatment of linear systems that are presently used in the great majority of all signal processing applications. Chapter 3 takes the general representation and analysis techniques of Chapter 2 and applies them to frequency selective filters which are a particular type of linear system. Two of the most popular filter types—Butterworth and Chebyshev—are presented in great detail in Chapters 4 and 5. The techniques needed to actually implement one of these filters in analog hardware are presented in Chapter 6.

We make the switch to digital signal processing beginning with Chapter 7, which covers such fundamental concepts as sampling and discrete-time signal and system analysis and forms the foundation for all DSP techniques. Chapter 8 then looks at ways to implement the filters of Chapter 4 or 5 in a digital form, and Chapter 9 examines some digital spectrum analysis techniques that really have no counterpart in the analog world.

Contents

Chapter 1

Signals, Spectra, and Noise

THIS BOOK PRESENTS A COLLECTION OF TECHNIQUES FOR THE analysis and design of signal processing systems. Such systems can be as simple as a passive resistor-capacitor lowpass filter, or as sophisticated as a dedicated special-purpose computer for realtime enhancement processing of video signals. Although a trial-and-error approach may occasionally produce something useful, a few mathematical techniques will prove indispensible in the design process. Most of these techniques rely on the use of mathematical functions to represent or model real world electronic signals as shown in Fig. 1-1. Actual electronic signals are very complicated phenomena whose exact behavior may be very difficult or even impossible to describe completely, but simple mathematical models often describe the signals closely enough to produce very useful results in a variety of practical situations. The distinction between a signal and its mathematical representation is not always rigidly observed in signal processing literature—functions which only *model* signals are commonly referred to as signals and properties of these models are often presented as properties of the signals themselves. Purists take warning—this blurring of terminology between a signal and its model crops up everywhere, so you must learn to live with it.

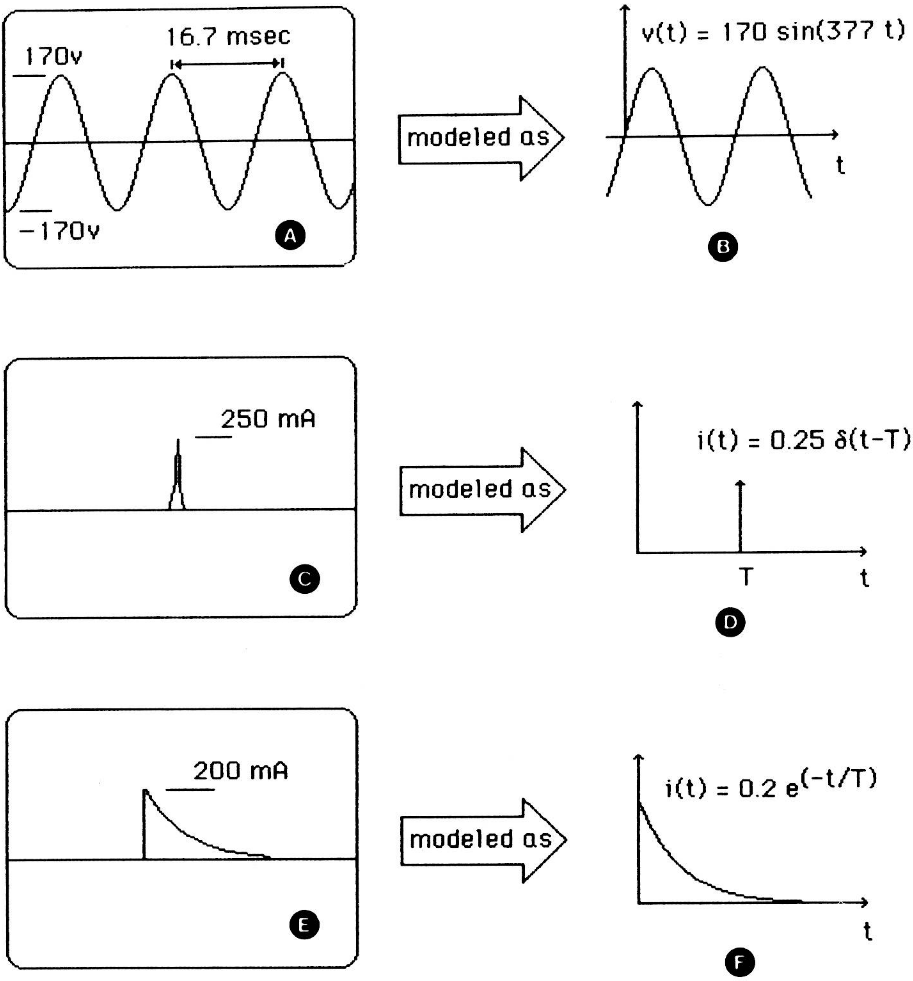


Fig. 1-1. Mathematical models of some practical signals.

This chapter and Chapter 2 are somewhat different from the remainder of the book in that they present theoretical concepts and some fundamental mathematics that are not meant to be used directly for processing actual signals. Instead, this material provides a theoretical basis upon which rest the practical techniques presented in later chapters. Although this material provides valuable insights, a complete understanding of it is not absolutely necessary in order to successfully employ the practical techniques presented in Chapter 3 and beyond.

1.1 SIGNAL MODELS

Mathematical models of signals are generally categorized as either *steady-state* or *transient* models. To understand the difference between these two types, let's examine Fig. 1-2 which shows the typical voltage output from a 1 kHz audio oscillator. This signal exhibits three noticeably different parts—a *turn-on transient* at the beginning, an interval of steady-state operation in the middle, and a *turn-off transient* at the end. We could formulate a single mathematical function to describe all three parts, but for most uses it would be unnecessarily complicated and difficult to work with. In most cases, the primary concern is steady-state behavior, and simplified mathematical representations that ignore the transients are often adequate. The steady-state portion of the oscillator output can be modeled as the sine function shown in Fig. 1-3. Theoretically, this sine function exists for all time, and this might seem to be a contradiction to the obvious fact that the oscillator output only exists for some limited time interval between turn-on and turn-off. However this is really not a problem; over the interval of steady-state operation that we are interested in, the mathematical sine function accurately describes the behavior of the practical oscillator's output voltage. Allowing the mathematical model to assume that the periodic signal exists over all time greatly simplifies matters, since the transients' behavior can be excluded from the model. In situations where the transients are important, they can be modeled

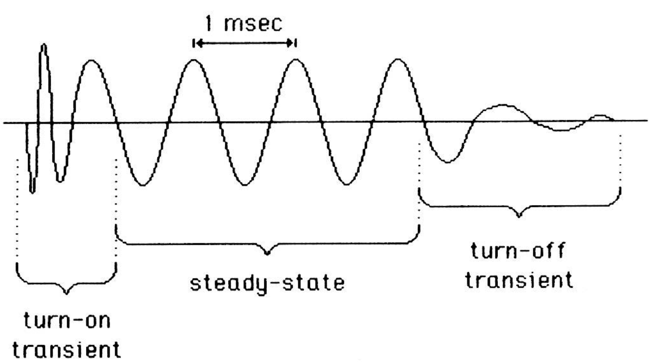


Fig. 1-2. Typical output of an audio oscillator.

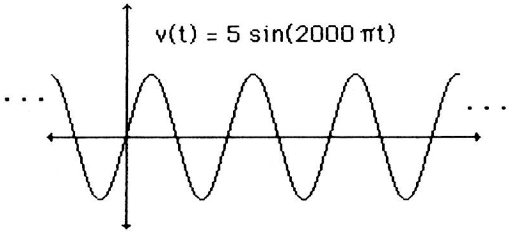


Fig. 1-3. Sine function used to model the steady-state output of the oscillator in Fig. 1-2.

as exponentially saturating and decaying sinusoids as shown in Figs. 1-4 and 1-5. Notice that the amplitude of the saturating exponential envelope continues to increase, but it never quite reaches the steady-state value. Likewise the amplitude of the decaying exponential envelope continues to decrease but it never quite reaches zero. In this context, the steady-state value is sometimes called an *asymptote*, or the envelope can be said to *asymptotically* approach the steady-state value. Of course, such behavior is true only in the pure mathematics of the model—in the real world, signals will eventually get so close to their steady-state values that the difference will be immeasurable.

Steady-state and transient models of signal behavior inherently contradict each other and neither constitutes a "true" description of a particular signal. The selection of an appropriate model requires an understanding of the signal to be modeled and of the implications that a particular choice of model will have for the intended

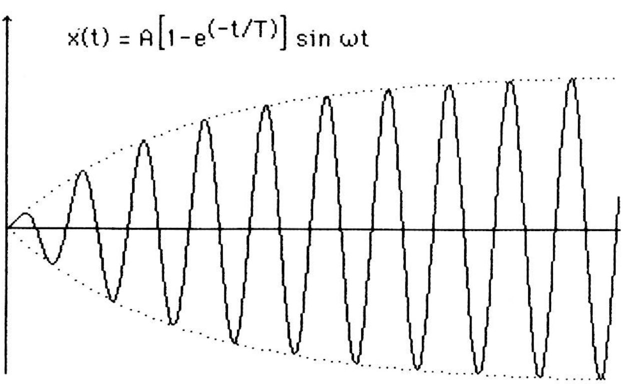


Fig. 1-4. Exponentially saturating sinusoid.

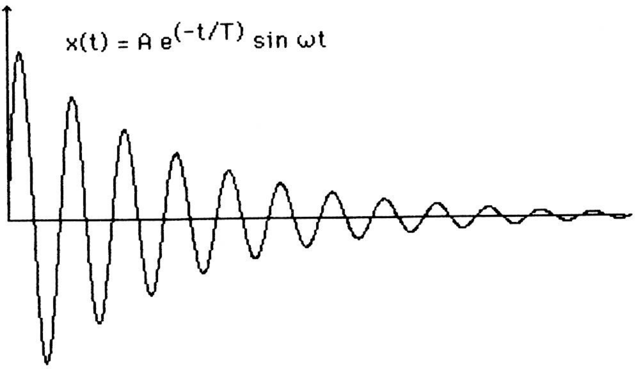


Fig. 1-5. Exponentially decaying sinusoid.

application. The following sections will present details of the more common steady-state and transient signal models.

1.2 STEADY-STATE SIGNALS

Generally, steady-state signals are limited to just sinusoids or sums of sinusoids. This will include virtually any periodic signals of practical interest since such signals can be resolved into sums of weighted and shifted sinusoids using the Fourier analysis techniques presented in Section 1-4. A few basic concepts and definitions will prove useful in work involving steady-state signal models.

1.2.1 Periodicity. Sines, cosines, and squarewaves are all periodic functions The characteristic that makes them periodic is the way in which each of the complete waveforms can be formed by repeating a particular cycle of the waveform over and over at a regular interval as shown in Fig. 1-6. Mathematically, a function $x(t)$ is periodic with a period of T if and only if $x(t+nT) = x(t)$ for all integer values of n.

1.2.2 Symmetry. A function can exhibit a certain symmetry regarding its position relative to the origin. The two major types of symmetry—odd and even—are shown in Fig. 1-7. Symmetry may appear at first to be something that is only "nice-to-know" and not particularly useful in practical applications where the definition of time zero is often somewhat arbitrary. This is far from the case however, because symmetry considerations play an important role in Fourier analysis—especially the discrete Fourier analysis which will be discussed in Chapter 6. Some functions are neither odd or

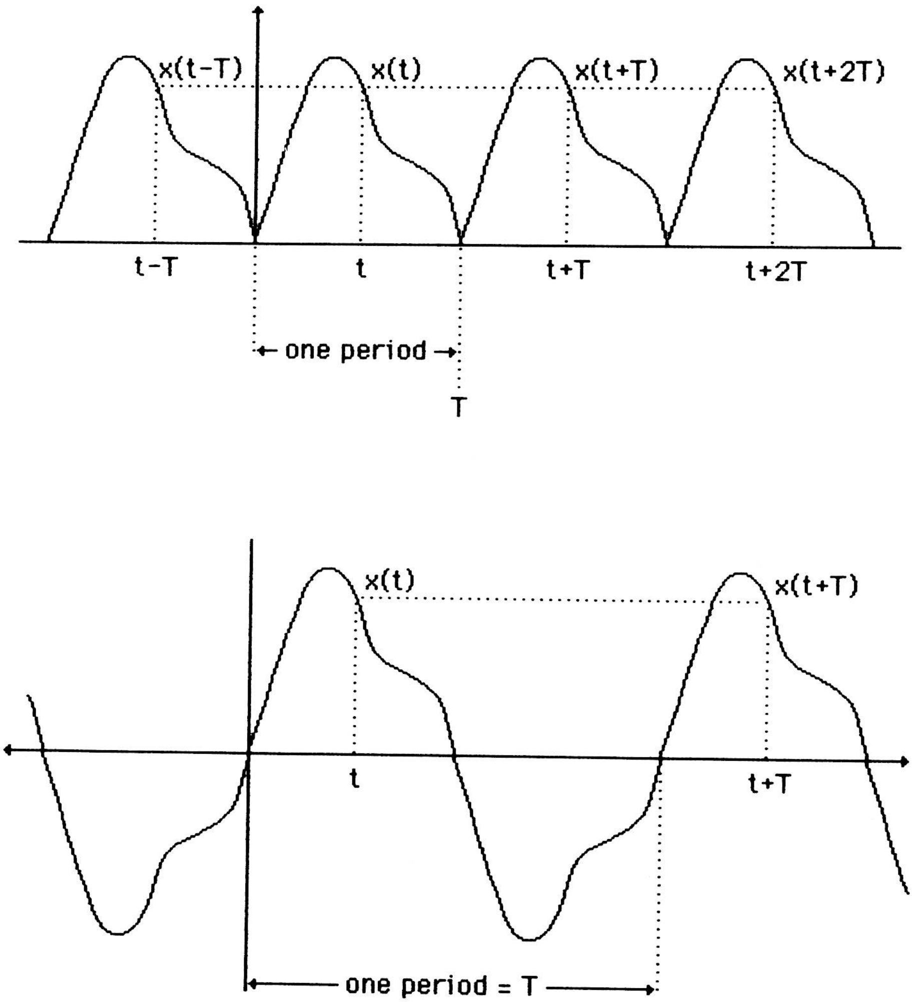
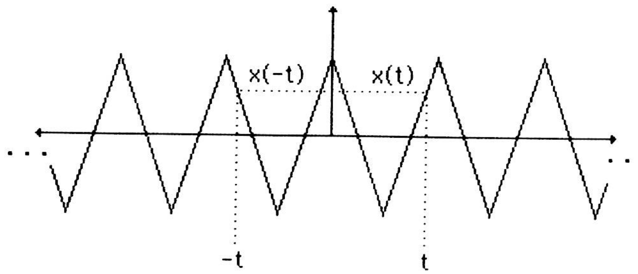


Fig. 1-6. Periodic functions.

even, but Table 1-1 presents formulas that can be used to resolve any periodic function into the sum of an even function and an odd function.

1.3 SINUSOIDS

The sine and cosine functions shown in Fig. 1-8 are together known as *sinusoids*. When a sinusoid is input to a linear system (such



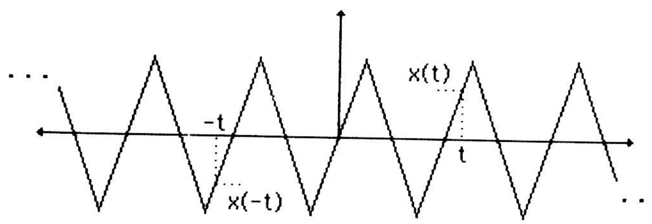


Fig. 1-7. Symmetry of periodic functions.

Table 1-1. Formulas Concerning Symmetry of Periodic Functions.

$$x(t) = x_{even}(t) + x_{odd}(t)\ ) \qquad \text{(Eq. 1.2-1)}$$

$$x_{even}(t) = \frac{1}{2}\left[x(t) + x(-t) \right] \qquad \text{(Eq. 1.2-2)}$$

$$x_{odd}(t) = \frac{1}{2}\left[x(t) - x(-t) \right] \qquad \text{(Eq. 1.2-3)}$$

even function + even function = even function (Eq. 1.2-4)

odd function + odd function = odd function (Eq. 1.2-5)

odd function × odd function = even function (Eq. 1.2-6)

even function × even function = even function (Eq. 1.2-7)

even function × odd function = odd function (Eq. 1.2-8)