RALPH E. STEUER

MULTIPLE CRITERIA OPTIMIZATION: THEORY, COMPUTATION, AND APPLICATION

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Multiple Criteria Optimization:

Theory, Computation, and Application

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Multiple Criteria Optimization:

Theory, Computation, and Application

To my wife Judy, my son Evan, and
my daughters Andrea and Catherine

Preface

If there ever was an area of management science/operations research that is exciting and challenging but fraught with deceptions and pitfalls, it is multiple criteria optimization. The field is fascinating because it is clearly an art as well as a science. When studying multiple criteria optimization, we learn how not to be naive. We learn about where the "bodies are buried" in mathematical programming and how to be innovative in overcoming the variety of solution set and solution procedure difficulties that may arise.

To me, multiple criteria optimization is a breath of fresh air: We can now openly admit that a problem has multiple objectives when it possesses multiple conflicting criteria. There is no need to ignore or gloss over the fact. We can deal with multiple objectives head on, because we now have the tools to solve large-scale multiple criteria optimization problems.

Multiple criteria optimization is also interesting because of its international scope. The subject is not strictly a product of North American and Western European culture. It is also of interest to the socialist economics of Eastern Europe and Third World nations because of its application possibilities in centralized planning. The international nature of the field is apparent in the references at the conclusion of this text's chapters, as contributions to the field have come from all over the world.

Purpose of Book

This book serves both as a teaching text and as a comprehensive reference volume concerning the principles and practices of multiple criteria optimization. The pedagogical approach relies on examples and graphical illustrations; hence many graphs are presented. In addition, numerous computational examples are included to stress the computer implementability of the methods discussed.

Mathematical Background

Only a modest mathematical background is needed to reach the state of the art in multiple criteria optimization. The only prerequisites are (a) familiarity with set theory and linear algebra, (b) knowledge of the simplex method equivalent to what one would obtain in a first course in operations research, (c) a bit of calculus, and (d) an acquaintance with computers to the extent that the user can edit files and not be intimidated by mathematical programming software. Apart from calculus and computers, the book is self-contained because of the review material in Chapter 2 (Mathematical Background) and Chapter 3 (Single Objective Linear Programming).

Organization

Chapters 4 through 9 develop the theory of multiple objective linear programming (MOLP). In particular, Chapter 4 discusses the computation of all *optimal* extreme points in single objective linear programming (LP). Chapter 5, which concerns objective row parametric programming, is essentially multiple objective programming with two objectives. This chapter forms the bridge between conventional LP and MOLP.

Chapters 6, 7, and 8 discuss the subleties involved with the solution set notion of efficiency (Pareto optimality), and Chapter 9 presents the theory of linear vector-maximization for computing all efficient points. In Chapter 9, we see how generalized versions of the methods discussed in Chapter 4 can be used to compute all *efficient* extreme points.

Chapter 10 addresses goal programming. One might say that goal programming is not so much covered in Chapter 10 as it is *dissected*. Goal programming is analyzed in terms of contours, and the usefulness of the deviational variable to multiple objective programming in general is stressed. Chapter 11 discusses *filtering*, or how representative subsets of larger sets can be obtained.

Chapter 12 deals with multiple objective linear fractional programming (MOLFP), in which the objectives are fractional in the sense that they have linear numerators and linear denominators. This is a research topic, as much work remains to be done in MOLFP.

The discussion of interactive procedures in Chapters 13, 14, and 15 is the climax of the book. In Chapter 13, we discuss **STEM**, the Geoffrion-Dyer-Feinberg procedure, and the Zionts-Wallenius procedure among others. In Chapters 14 and 15, we discuss the Tchebycheff procedure. Three interactive applications are given in Chapter 16. Comments about the future, particularly in regard to the use of graphics at the computer/user interface, are made in Chapter 17. Also, Chapter 17 contains bibliographies concerning some specialized multiple criteria topics.

How to Use the Text

With my graduate course in multiple criteria optimization at the University of Georgia, we cover Chapters 6, 7, 8, and part of 9, along with Chapters 10, 11, 13, 14, and 15. We also discuss the use of computer graphics as outlined in Chapter 17. Students are asked to review material in Chapters 2 and 3 on their own in order to fill any gaps in their background. Chapters 6, 7, 8, 9, 10, and 11 provide the tools for multiple criteria analysis. Chapters 13, 14, and 15 describe their interactive application. Because the emphasis is on computer implementation, computers are used intensively in the course from day one. We use the **MINOS** linear/nonlinear programming code and the **ADBASE** vector-maximum code

(Section 9.11). Also, we use an automated package such as the one discussed in Chapter 15 for implementing the interactive weighted-sums and Tchebycheff procedures of Chapters 13 and 14. In addition, the `LAMBDA` and `FILTER` codes (Sections 11.7 and 11.9) are used for a variety of small supporting chores.

Problem Exercises

Problem exercises are found at the ends of the chapters. Those exercises whose numbers carry a "C" suffix are computer problems. Those without a "C" suffix are paper and pencil problems. A solutions manual for all problem exercises is available.

Numbering

Within a given chapter, the numbering of equations, programs, formulations, definitions, lemmas, and theorems come from the same sequence. For instance, in Chapter 9, Theorem 9.17 appears after Definition 9.13, and in Chapter 14, Lemma 14.17 falls before formulation (14.19). It is hoped that after the reader becomes accustomed to the numbering convention, it will be of some convenience, particularly in the longer chapters. Note that tables and figures are numbered separately as usual.

Notation

Set theoretic notation is used throughout the text. A listing of the most frequently used notation is given in Section 1.6. It should also be pointed out that the book utilizes two specialized items of notation. One is the *convex combination operator* γ, and the other is the *unbounded line segment operator* μ. Most readers will not be familiar with this notation; however, it is very convenient. The use of γ and μ makes the expression of convex sets in set theoretic notation particularly straightforward. The convex combination and unbounded line segment operators are described in Section 2.3.2.

Acknowledgments

I would first like to express my appreciation to all of the researchers in various disciplines from around the world who, by virtue of their contributions over the past fifteen years, have created the field of multiple criteria optimization. Without the foundations that they laid, the procedures that they pioneered, and the maturity that they have rendered to the field, a book such as this would not have been possible.

I want to thank those with whom I have conducted joint multiple criteria research. The list includes K. R. Balachandran (New York University), Eng-Ung Choo (Simon Fraser University), John P. Evans (University of North Carolina), Heinz Isermann (University of Bielefeld), Jonathan S. H. Kornbluth (Hebrew University), Kenneth D. Lawrence (AT & T Long Lines), Albert T. Schuler (U.S. Forest Service), Joe Silverman (U.S. Navy Personnel Research and Development

Center), Marc J. Wallace, Jr. (University of Kentucky), Alan W. Whisman (U.S. Navy Personnel Research and Development Center), and Eric F. Wood (Princeton University).

I want to take this opportunity to give special thanks to John P. Evans, who was my Ph.D. advisor at the University of North Carolina. He suggested multiple objective linear programming as my dissertation topic, and it was the work that we did together, much of which is covered in the first half of Chapter 9, that introduced me to the field.

I also want to give special thanks to Stanley Zionts (State University of New York at Buffalo). His colleagueship and advice over the years have been greatly appreciated. He has been a leader in the field and many of his contributions to the literature are reflected in this book, particularly in Chapters 9 and 13. In addition, special thanks go to Eng-Ung Choo for contributing the MOLFP example with a nonlinear efficient boundary in Section 12.7. Also, I want to acknowledge Erick C. Duesing (University of Scranton) who has been my very good multiple criteria friend.

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Although not explicitly covered in this book, I would like to acknowledge the work that has been conducted by Andrzej P. Wierzbicki, Manfred Gauer, and others at IIASA (International Institute for Applied Systems Analysis, Laxenburg, Austria) on the *reference point method* of multicriteria optimization. This is important work and I plan to include it in the text if this book is ever revised.

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Ralph E. Steuer

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