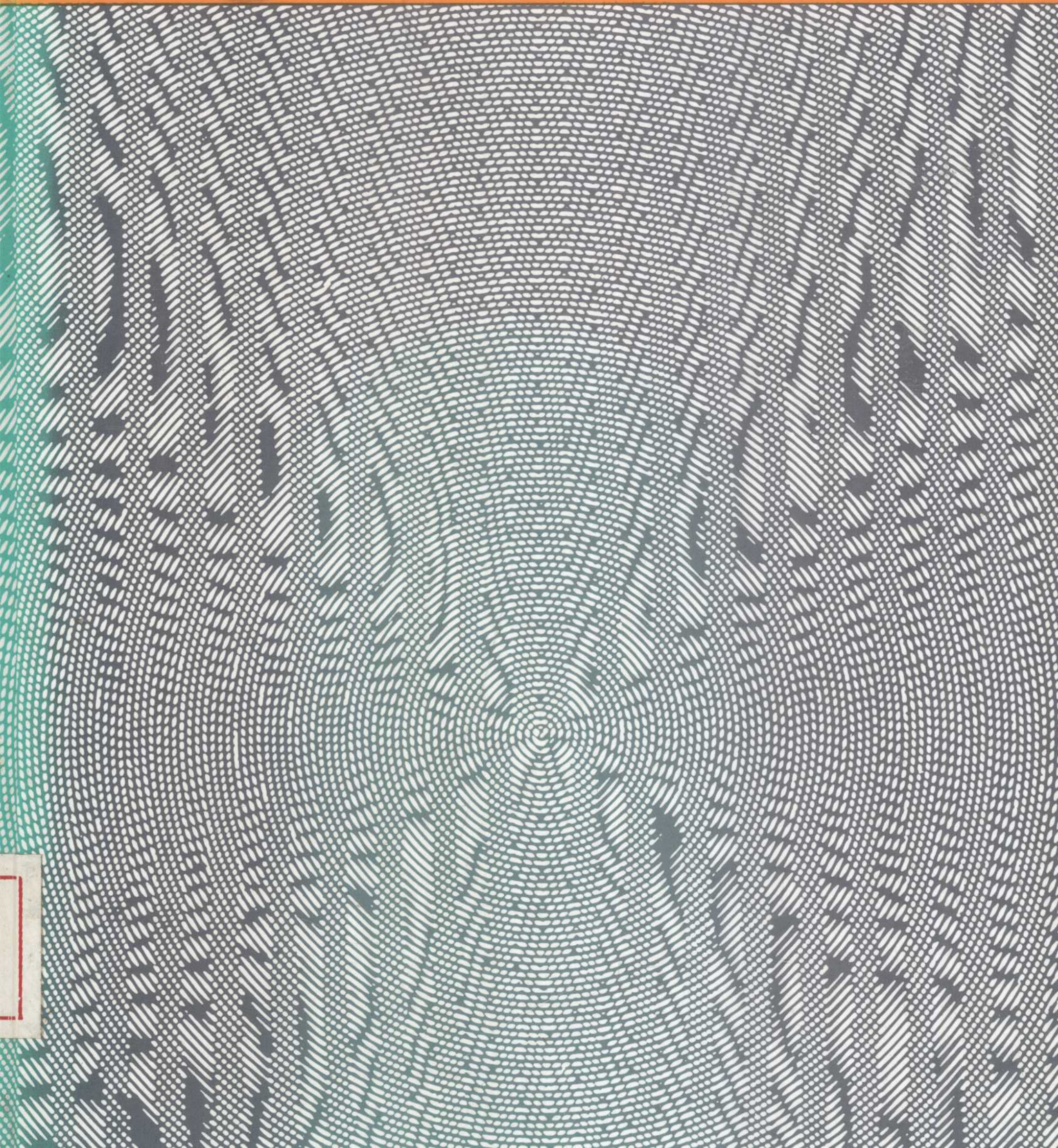
ELLIS HORWOOD SERIES IN ENGINEERING SCIENCE

Vibration Analysis and Control System Dynamics

C. F. BEARDS



VIBRATION ANALYSIS AND CONTROL SYSTEM DYNAMICS

ELLIS HORWOOD SERIES IN ENGINEERING SCIENCE

STRENGTH OF MATERIALS
J. M. ALEXANDER, University College of Swansea.
TECHNOLOGY OF ENGINEERING MANUFACTURE
J. M. ALEXANDER, R. C. BREWER, Imperial College of Science and Technology, University of London, J. R. CROOKALL, Cranfield Institute of Technology.
VIBRATION ANALYSIS AND CONTROL SYSTEM DYNAMICS
CHRISTOPHER BEARDS, Imperial College of Science and Technology, University of London.
COMPUTER AIDED DESIGN AND MANUFACTURE
C. B. BESANT, Imperial College of Science and Technology, University of London.
STRUCTURAL DESIGN AND SAFETY
D. I. BLOCKLEY, University of Bristol.
BASIC LUBRICATION THEORY 3rd Edition
ALASTAIR CAMERON, Imperial College of Science and Technology, University of London.
STRUCTURAL MODELLING AND OPTIMIZATION
D. G. CARMICHAEL, University of Western Australia
ADVANCED MECHANICS OF MATERIALS 2nd Edition
Sir HUGH FORD, F.R.S., Imperial College of Science and Technology, University of London and J. M. ALEXANDER, University College of Swansea.
ELASTICITY AND PLASTICITY IN ENGINEERING
Sir HUGH FORD, F.R.S. and R. T. FENNER, Imperial College of Science and Technology, University of London.
INTRODUCTION TO LOADBEARING BRICKWORK
A. W. HENDRY, B. A. SINHA and S. R. DAVIES, University of Edinburgh
ANALYSIS AND DESIGN OF CONNECTIONS BETWEEN STRUCTURAL JOINTS
M. HOLMES and L. H. MARTIN, University of Aston in Birmingham
TECHNIQUES OF FINITE ELEMENTS
BRUCE M. IRONS, University of Calgary, and S. AHMAD, Bangladesh University of Engineering and Technology, Dacca.
FINITE ELEMENT PRIMER
BRUCE IRONS and N. SHRIVE, University of Calgary
PROBABILITY FOR ENGINEERING DECISIONS: A Bayesian Approach
I. J. JORDAAN, University of Calgary
STRUCTURAL DESIGN OF CABLE-SUSPENDED ROOFS
L. KOLLAR, City Planning Office, Budapest and K. SZABO, Budapest Technical University.
CONTROL OF FLUID POWER, 2nd Edition
D. McCLOY, The Northern Ireland Polytechnic and H. R. MARTIN, University of Waterloo, Ontario, Canada.
TUNNELS: Planning, Design, Construction
T. M. MEGAW and JOHN BARTLETT, Mott, Hay and Anderson, International Consulting Engineers
UNSTEADY FLUID FLOW
R. PARKER, University College, Swansea
DYNAMICS OF MECHANICAL SYSTEMS 2nd Edition
J. M. PRENTIS, University of Cambridge.
ENERGY METHODS IN VIBRATION ANALYSIS
T. H. RICHARDS, University of Aston, Birmingham.
ENERGY METHODS IN STRESS ANALYSIS: With an Introduction to Finite Element Techniques
T. H. RICHARDS, University of Aston, Birmingham.
ROBOTICS AND TELECHIRICS
M. W. THRING, Queen Mary College, University of London
STRESS ANALYSIS OF POLYMERS 2nd Edition
J. G. WILLIAMS, Imperial College of Science and Technology, University of London.

VIBRATION ANALYSIS AND CONTROL SYSTEM DYNAMICS

C. F. BEARDS, B.Sc., Ph.D., C.Eng., M.R.Ae.S.
Department of Mechanical Engineering
Imperial College of Science and Technology
University of London

ELLIS HORWOOD LIMITED
Publishers · Chichester

Halsted Press: a division of
JOHN WILEY & SONS
New York · Brisbane · Chichester · Toronto

First published in 1981 by

ELLIS HORWOOD LIMITED
Market Cross House, Cooper Street, Chichester, West Sussex, PO19 1EB, England

The publisher's colophon is reproduced from James Gillison's drawing of the ancient Market Cross, Chichester.

Distributors:

Australia, New Zealand, South-east Asia:
Jacaranda-Wiley Ltd., Jacaranda Press,
JOHN WILEY & SONS INC.,
G.P.O. Box 859, Brisbane, Queensland 40001, Australia

Canada:
JOHN WILEY & SONS CANADA LIMITED
22 Worcester Road, Rexdale, Ontario, Canada.

Europe, Africa:
JOHN WILEY & SONS LIMITED
Baffins Lane, Chichester, West Sussex, England.

North and South America and the rest of the world:
Halsted Press: a division of
JOHN WILEY & SONS
605 Third Avenue, New York, N.Y. 10016, U.S.A.

British Library Cataloguing in Publication Data

Beards, C. F.
Vibration analysis and control system dynamics –
(Ellis Horwood series in engineering science)
1. Vibration
2. Control theory
I. Title
II. 531'.32 QA865

ISBN 0-85312-242-3 (Ellis Horwood Ltd., Publishers – Library Edn.)
ISBN 0-85312-294-6 (Ellis Horwood Ltd., Publishers – Student Edn.)
ISBN 0-470-27255-4 (Halsted Press)

Typeset in Press Roman by Ellis Horwood Ltd.
Printed in Great Britain by R. J. Acford, Chichester.

Table of Contents

Author's Preface

High costs and questionable availability of materials, land and other resources, together with increasingly sophisticated analysis and manufacturing methods, have led to lightweight structures, high energy sources and finely tuned control systems. These trends, which will surely continue to an increasing extent in the future as ever higher performance levels are demanded, have exacerbated noise, vibration and control system problems.

There is therefore a real need for all practising engineers and scientists as well as students, to have a good understanding of dynamic analysis methods for predicting vibration amplitudes, dynamic stresses and noise levels in a structure, and methods for determining control system performance. It is also essential to be able to understand and contribute to published and quoted data in this field.

There is a great benefit to be gained by studying vibration analysis and control system dynamics together and having this information in a single text, because the analyses of the dynamics of control systems and the vibration of elastic systems are closely linked. This is because in many cases the same equations of motion occur in the control system as in the vibrating system, and thus the techniques developed in the analysis of one system can be used in the other and vice versa. Furthermore, the results of the analysis of one system can be applied to another.

This book is intended to give practising engineers and scientists, and students of engineering and science to first degree level, a thorough understanding of the principles involved in the analysis of vibration and control system dynamics, and to provide a sound theoretical basis for further study. A number of worked examples have been included.

C. F. Beards, September 1980

CHAPTER 1

Introduction

The vibration which occurs in most machines and structures is undesirable, not only because of the unpleasant motions, the noise and the dynamic stresses, which may lead to fatigue and failure of the structure, but also because of the energy losses and the reduction in performance which accompany the vibrations. The literature is heavy with accounts of system failures brought about by excessive vibration of one component or another. Because of the devastating effects which unwanted vibrations can have on machines and structures, vibration analysis should be carried out as an inherent part of the design, when necessary modifications can most easily be made in an effort to eliminate vibration, or to reduce it as much as possible. Modifications made after this stage, to prototypes or production samples, are usually costly, difficult to implement and often unsatisfactory. Current trends towards lightweight structures and high speed machines are making this analysis increasingly important: as these trends continue so will the need for vibration analysis grow.

The demands made on automatic control systems are also increasing. Whatever the duty of the system, from satellite tracking to controlling sheet thickness in a steel rolling mill, there is a continual effort to improve performance whilst making the system cheaper and more compact. Accurate and relevant analysis of control system dynamics is therefore very necessary, in order to determine the effects of proposed modifications on the system response, as well as predicting the response of new system designs.

There are two reasons why we should study vibration analysis and control system dynamics together. Firstly, because we need to consider control in relation to mechanical engineering using mechanical analogies, rather than as a specialised and isolated aspect of electrical engineering, and, secondly, because the basic equations governing the behaviour of vibration and control systems are the same: different emphasis is placed on the different forms of the solution available, but they are all dynamic systems. Each subject benefits from techniques developed in the other.

Dynamic analysis can be carried out most conveniently in the following three stages:

Stage I. Form a mathematical or physical model of the system to be analysed.
Stage II. From the model write the equations of motion.
Stage III. Evaluate the system response to relevant specific excitation.

These stages are now considered in greater detail.

Stage I. The mathematical model

To model any real system a number of simplifying assumptions have to be made. For example, a distributed mass may be considered as a lumped-mass, or the effect of damping in the system may be neglected particularly if only resonant frequencies are sought or a non-linear spring may be considered linear, or certain elements may be neglected altogether if their effect is likely to be small. Furthermore the directions of motion of the mass elements are usually restrained to those of immediate interest to the analyst.

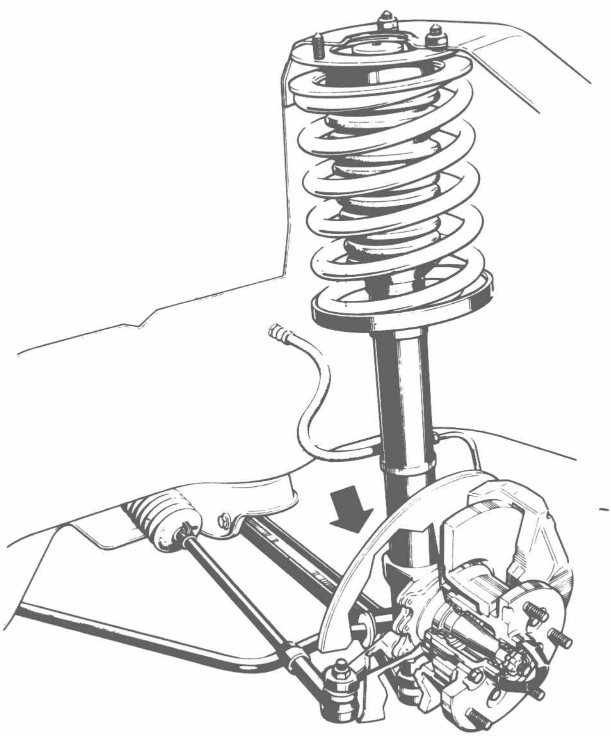


Fig. 1.1 – Rover 3500 – front suspension (By courtesy of British Leyland).

Thus the model is usually a compromise between a simple representation which is easy to analyse but may not be very accurate, and a complicated but realistic model which is difficult to analyse but does give useful results. Consider, for example, the analysis of the vibration of the front wheel of a motor car. Fig. 1.1 shows a typical suspension. Fig. 1.2(a) is a very simple model of this same system, which considers translational motion in a vertical direction only: this model is not going to give much useful information, although it is easy to analyse. The model shown in Fig. 1.2(b) is capable of producing some meaningful

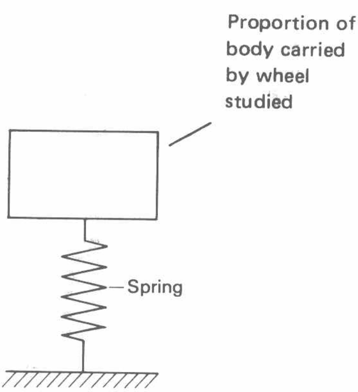


Fig. 1.2(a) – Simplest model – Motion in a vertical direction only can be analysed.

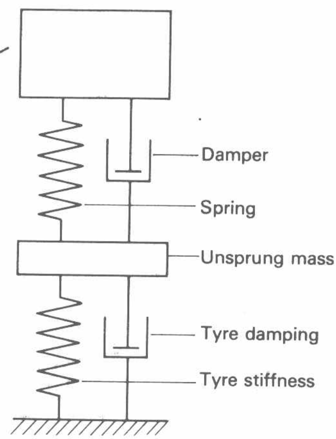


Fig. 1.2(b) – Motion in a vertical direction only can be analysed.

results at the cost of increased labour in the analysis, but the analysis is still confined to motion in a vertical direction only. A more refined model, shown in Fig. 1.2(c), shows the whole car considered, translational and rotational motion of the car body being allowed.

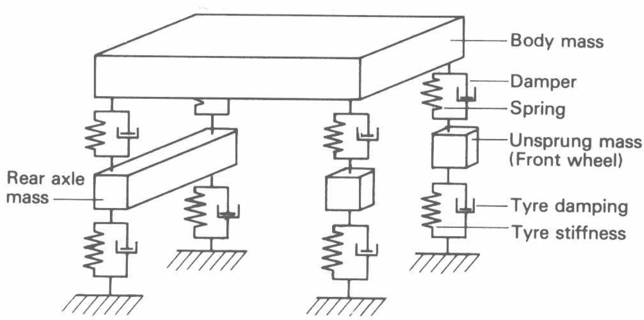


Fig. 1.2(c) – Motion in a vertical direction, roll, and pitch can be analysed.

If the modelling of the car body as a rigid mass is not acceptable, a finite element approach to the analysis may be necessary.

A block diagram model is usually used in the analysis of control systems. For example, a system used for controlling the rotation and position of a turntable about a vertical axis is shown in Fig. 1.3. The turntable can be used for

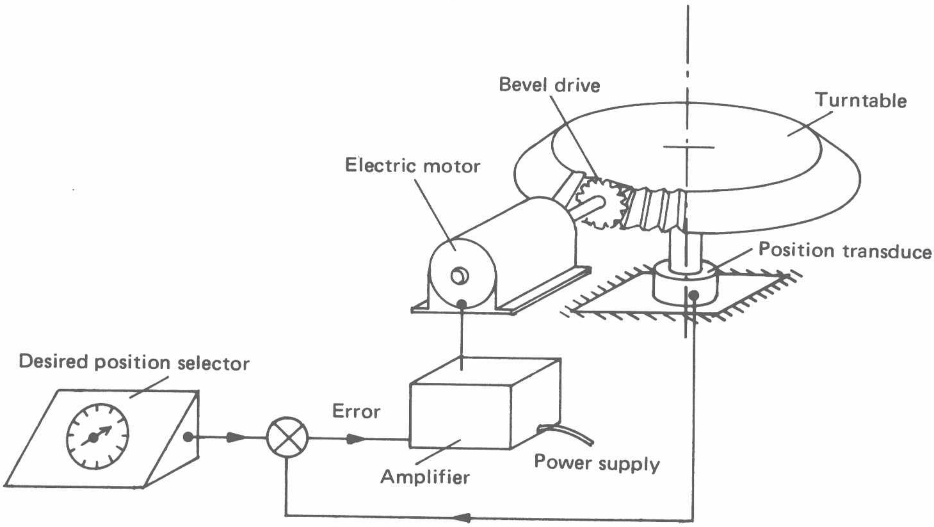


Fig. 1.3 – Turntable position control system.

mounting a telescope or gun, or if it forms part of a machine tool it can be used for mounting a workpiece for machining. Fig. 1.4 shows the block diagram model used in the analysis.

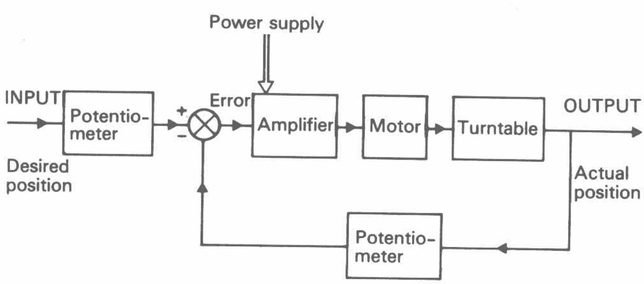


Fig. 1.4 – Turntable position control system: block diagram model.

Model Parameters

Because of the approximate nature of most models, whereby small effects are neglected and the environment is made independent of the system motions, it is usually reasonable to assume constant parameters and linear relationships. This means that the coefficients in the equations of motion are constant and the equations themselves are linear: these are real aids to simplifying the analysis. Distributed masses can often be replaced by lumped mass elements to give ordinary rather than partial differential equations of motion. Usually the numerical value of the parameters can, substantially, be obtained directly, from the system being analysed. However, model system parameters are sometimes difficult to assess, and then an intuitive estimate is required, engineering judgement being of the essence.

It is not easy to create a relevant mathematical model of the system to be analysed, but such a model does have to be produced before Stage II of the analysis can be started. Most of the material in subsequent chapters is presented to make the reader competent to carry out the analyses described in Stages II and III. A full understanding of these methods will be found to be of great help in formulating the mathematical model referred to above in Stage I.

Stage II. The equations of motion

Several methods are available for obtaining the equations of motion from the mathematical model, the choice of method often depending on the particular model and personal preference. For example, analysis of the free-body diagrams drawn for each body of the model usually produces the equations of motion quickly: but it can be advantageous in some cases to use an energy method such as the Lagrange equation.

From the equations of motion the characteristic or frequency equation is obtained, yielding data on the natural frequencies, modes of vibration, general response, and stability.

Stage III. Response to specific excitation

Although Stage II of the analysis gives much useful information on natural frequencies, response and stability, it does not give the actual system response to specific excitations. It is necessary to know the actual response in order to determine such quantities as dynamic stress or noise for a range of system inputs, either force or motion, including harmonic, step and ramp. This is achieved by solving the equations of motion with the excitation function present.

Remember:

Stage I	Model
Stage II	Equations
Stage III	Excitation.

So far we have considered a few examples of how real systems can be modelled and the principles of their analysis. Obviously, if we are to be competent to analyse system models, it is essential to be able to analyse single degree of freedom systems, which are considered in Chapter 2, systems with more than one degree of freedom in Chapter 3, and continuous systems in Chapter 4. Some aspects of automatic control system analysis which require special consideration, particularly their stability and the system frequency response, are discussed in Chapter 5. Chapter 6 contains a number of exercises for the reader to try.

CHAPTER 2

The vibrations of systems having one degree of freedom

A system with one degree of freedom is the simplest case to analyse because only one coordinate is necessary to completely describe the motion of the system. Some real systems can be modelled in this way, either because the excitation of the system is such that the vibration can be described by one coordinate although the system could vibrate in other directions if so excited, or the system really is simple, as for example in the case of a clock pendulum. It should also be noted that a one degree of freedom model of a complicated system can often be constructed where the analysis of a particular mode of vibration is to be carried out. To be able to analyse one degree of freedom systems is therefore an essential ability in vibration analysis.

2.1 FREE UNDAMPED VIBRATION

2.1.1 Translational vibration

In the system shown in Fig. 2.1 a body of mass m is free to move along a fixed horizontal surface. A spring of constant stiffness k which is fixed at one end is attached at the other end to the body. Displacing the body to the right (say) from the equilibrium position causes a spring force to the left (a restoring force). This force gives the body an acceleration to the left. When the body reaches its equilibrium position the spring force is zero, but the body has a velocity which

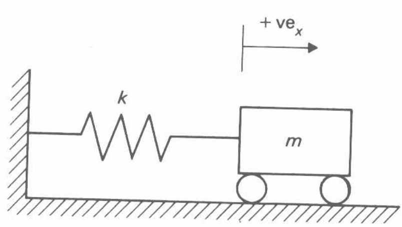


Fig. 2.1

carries it further to the left although it is retarded by the spring force which now acts to the right. When the body is arrested by the spring, the spring force is to the right so that the body moves to the right, past its equilibrium position, and hence reaches its initial displaced position. In practice this position will not quite be reached because damping in the system will have dissipated some of the vibrational energy. However, if the damping is small its effect can be neglected.

If the body is displaced a distance x_0 to the right and released, the free-body diagrams (FBDs) for a general displacement x are as shown in Figs. 2.2(a) and (b).

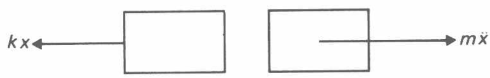


Fig. 2.2(a) – Applied force. (b) – Effective force.

The effective force is always in the direction of positive x. If the body is being retarded $\ddot{x}$ will calculate to be negative. The mass of the body is assumed contant: this is usually so, but not for example in the case of a rocket burning fuel. The spring stiffness k is assumed contant: this is usually so within limits, see section 2.1.3. It is assumed that the mass of the spring is negligible compared to the mass of the body.

From the free-body diagrams the equation of motion for the system is

$$m\ddot{x} = -kx \quad \text{or} \quad \ddot{x} + (k/m)x = 0 \tag{2.1}$$

This will be recognised as the equation for simple harmonic motion.

The solution is $\quad x = A \cos \omega t + B \sin \omega t \qquad (2.2)$

where A and B are constants which can be found by considering the initial conditions, and ω is the circular frequency of the motion. Substituting (2.2) into (2.1) we get

$$-\omega^2(A \cos \omega t + B \sin \omega t) + (k/m)(A \cos \omega t + B \sin \omega t) = 0$$

since $A \cos \omega t + B \sin \omega t \neq 0$ (otherwise no motion), $\omega = \sqrt{(k/m)}$ rad/s,

and $\quad x = A \cos \sqrt{(k/m)}\, t + B \sin \sqrt{(k/m)}\, t.$

Now $\quad x = x_0$ at $t = 0$ so $x_0 = A$, and $\dot{x} = 0$ at $t = 0$ so $0 = B\sqrt{(k/m)}$

that is, $\quad x = x_0 \cos \sqrt{(k/m)}\, t \qquad (2.3)$