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Diffraction of Elastic Waves and Dynamic Stress Concentrations

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Preface

This monograph is an outgrowth of continuous research in diffraction of elastic waves and effects of dynamic loadings on underground openings and structures. The authors first became interested in this problem in early 1960 and soon recognized that although the subject of elastic wave diffraction had been under investigation for over a hundred years and problems of stress concentration had been studied nearly as long, there existed relatively little information and few numerical results for the dynamic stress concentration factor. It was also apparent that in order to have a better understanding of how a ground shock interacts with underground cavities or openings, questions of dynamical effects due to wave diffraction must be answered.

Since the late 1950s, because of the support and impetus provided by various governmental agencies, research in this area has been greatly accelerated, resulting in a flow of publications. By the mid-1960s a substantial number of published theories and numerical results had become available in journals and in reports from government agencies and industry. It is thought that a systematic presentation of elastic wave diffraction and dynamic stress concentration results is warranted because of their wide applicability. These results can be applied not only in the study of hardened underground structures but in machine design, ultrasonics, structural design, the mechanics of composite materials, and the theory of fractures.

Thus, the objectives of the monograph are twofold: (1) to systematically present methods of solution for both steady-state and transient wave as diffracted by an obstacle, and (2) to present numerical results of dynamic stress concentration on obstacles of different geometries. An effort was made to collect information from the open literature as well as from government agencies, industry, and individuals. Because of the time constraints of manuscript preparation and the rapid proliferation of research in this field, many of the latest publications could not be cited in this monograph.

In undertaking this monograph, the authors had no idea it would grow to such a mammoth size. They had planned to write a self-contained monograph and to give so complete a discussion that the reader would not be troubled with checking many references. That plan was obviously too ambitious and unwieldy, and it was later modified somewhat. Undoubtedly many equations and derivations could have been omitted if references to standard texts and original papers had been made.

Omitted from the plan were a section in Chapter IV on the application of the integral equation method to the diffraction of P and SV waves, a section in Chapter V on the application of the

Wiener-Hopf method to the diffraction of P and SV waves by a semi-infinite strip, a section in Chapter VI on the scattering of SV waves by a sphere, a chapter on spheroidal obstacles (elastic waves in spheroidal coordinates), and a final chapter on experimental methods and observations of elastic wave diffraction. However, even if everything originally planned were included, this monograph still would not be a comprehensive treatment on the diffraction of elastic waves. One obvious omission is a detailed discussion of the scattering of sound waves in liquid by an obstacle, with numerical results and graphs. Another omission is the analysis of multiple scattering by many obstacles. The authors wish to be excused if the reader is disappointed by these omissions.

It is said that the publication of a monograph signals the end of active research on the title subject. That will certainly not be true in this case because of the research being done on the diffraction of elastic waves that has not been treated in this monograph. The authors will be most gratified if, in addition to achieving the twofold objectives above, this monograph generates more interest in, and research effort on, the diffraction of elastic waves and dynamic stress concentrations.

During the course of preparation of this monograph, many friends and colleagues gave valuable advice and suggestions. The authors wish to express their special thanks to the following persons who, either as staff members of The Rand Corporation or as consultants, contributed to this work: Richard Schamberg and D. J. Masson for their encouragement and support in beginning the project, /. B. Graham and D. N. Morris for their sustaining support and patience; Tiina Repnau for her able assistance in obtaining many of the new numerical results contained in the text; David Gakenheimer for his critical review of the final draft and his helpful suggestions; and last but not least, Jeanne Dunn, Nancy Hope, Mary McCabe, Maggie Milstead, Alrae Tingley, and B. J. Verdick for their excellent cooperation and perseverance in the preparation of the manuscript.

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In many cases of modern design, the elementary solutions obtained by the application of the theory of strength of materials are insufficient, and recourse has to be made to the general equation of the theory of elasticity in order to obtain satisfactory results. All problems of stress concentration are of this kind.

Stephen Timoshenko, 1925

(from *Transactions of The American Society of Mechanical Engineers, Vol. 47, 1925, p. 237)*

Chapter I

INTRODUCTION

1. A BRIEF HISTORY OF ELASTIC WAVE DIFFRACTION

DIFFRACTION of elastic waves has its origin in the age-old searching for the true nature of light. The name diffraction was given by Fr. Francesco Maria Grimaldi (1618-1663) to the phenomenon that a light beam might be bent slightly while passing the edge of an aperture. It is now applied to a phenomenon of wave propagation when the rays of waves deviate from rectilinear paths, which cannot be interpreted as reflection or refraction. In the first half of the 19th century, light was interpreted as the propagation of a disturbance in an elastic aether, the dynamics of which were described by what is now called the Theory of Elasticity. Thus, the theory of the propagation of elastic waves was developed long before the application of elasticity theory to stress analysis for structures and machinery components.

One of the major problems in stress analysis is the determination of *stress concentration*, which is the sharp increase of stress over a nominal value in a localized region of a structural member due to geometric discontinuities such as holes, corners, and notches. During the first half of the 20th century, the subject of stress concentration had evolved from a mathematical curiosity to an important element in engineering design. However, its understanding was limited to the case of static loading, i.e., when the forces or other sources equivalent to a force are applied gradually and slowly to a structural member in order that the effect of its mass inertia can be neglected, or when the forces have been applied long before the instant of recording such that the observed data show little dependence in time. The investigation of stress concentration under dynamic loading has started only very recently. As in the static case, the analysis of dynamic stress concentration is also based on the theory of elasticity. Hence, it is not surprising to find that dynamic stress concentration is related to the propagation of elastic waves. The effect of a dynamic loading is to generate elastic waves which propagate in a structure or machinery member. When passing through a geometric discontinuity, an elastic wave is diffracted just as the path of a light ray is deviated by the edge of an aperture. Thus, dynamic stress concentration is a result of the *diffraction of elastic waves*.

After the development of the electromagnetic and quantum theories of light, no one would accept the elastic solid theory. In 1888, Lord Rayleigh in the article "Wave Theory of Light" in the *Encyclopaedia Britannica* stated that "The elastic solid theory, valuable as a piece of purely dynamical reasoning, and probably not without mathematical analogy to the truth, can in optics be regarded only as an illustration." It is in this spirit that we begin this section with a brief recount of the "Elastic Solid Theory of Light." By traversing through the historical paths, we will discover how elastic wave theory was developed and trace the links which are common to all waves in nature, including the sound (acoustic) waves, electromagnetic waves, and elastic waves.

The first four subsections are concerned with the historical survey of the diffraction and scattering of light, sound, and elastic waves, together with the related mathematical theories and methods. At various stages, we shall call attention to the prominent achievements of many pioneers and their influences on the modern day theory of diffraction of elastic waves. The subject of static stress concentration is introduced in subsection 5 as a separate entity, and then in the final subsection it is correlated with the diffraction of elastic waves and dynamic stress concentrations, the latter including the static stress concentration as a limiting case when frequency approaches zero.

1.1. *Elastic Solid Theory of Light**

In Grimaldi's book *Physico-Mathesis du Lumine, Coloribus et Iride* which was published posthumously in 1665 at Bologna,† the author described an experiment of letting a beam of light pass through two narrow apertures, one behind the other, and then fall on a black surface. He found that the band of light on the surface was a trifle wider than it was when it entered the first aperture. Therefore, he believed that the beam had been bent outward slightly at the edges of the aperture. This was different from the hitherto observed phenomena of reflection and refraction, and it was named *diffraction*.

The same phenomenon was noticed a few years later by Robert Hooke (1635-1703). Although Hooke, and his contemporary Christian Huygens

*A comprehensive account of the elastic solid theory of light is given by E. T. Whittaker, Ref. 1.1.

†Grimaldi's book is one of the very early scientific books on light. It is perhaps preceded only by G. B. Della Porta's *De Refractione*, Neapoli, 1593, see Ref. 1.2.

(1629-1695), were the early proponents of the "wave theory" of light, diffraction was one of the phenomena they could not explain. For if light was propagated like sound waves,* in the shadow region bounded by an opaque screen light would spread equally and there would be no darkness. Toward the beginning of the 19th century, diffraction, polarization, and double refraction (in crystals) of light were the major difficulties confronting the wave (longitudinal) theory of light.

In 1801, Thomas Young (1773-1829) discovered the law of interference of light waves,[1.3a, 1.3b] which paved the way for Augustin Jean Fresnel (1788-1827) to discover the real cause of diffraction. An interference can be described simply as two waves that when mixed together destroy (or reinforce) each other, either wholly or partially. In the memoir which won him a prize from the French Academy of Sciences in 1818 on the subject of "Diffraction,"[1.4] Fresnel set forth the concept that the diffraction of light is the mutual interference of the secondary waves emitting from an aperture. If the incident waves are conceived to be broken up on arriving at the aperture of a screen, each element of the aperture is then considered as the center of a secondary disturbance according to Huygens' principle. The intensity of the diverging spherical wave does not vary rapidly from one direction to another in the neighborhood of the normal to the incident wave front, and the disturbance at any point of observation is found by taking the aggregate of the disturbances due to all the secondary waves. Since the phase of the motion of each secondary wave is retarded by a quantity corresponding to the distance from its center to the point of

*Sound propagation in the form of a wave was established in Sir Isaac Newton's time (1642-1727).

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observation, the arriving secondary waves interfere with each other, resulting in diffraction.

Shortly afterwards, Francois Arago (1786-1853) and Fresnel jointly discovered experimentally that two beams of light polarized in planes at right angles do not interfere with each other. This discovery led Young to believe that light is a transverse wave in an aether,[(1.5)] and that the motion of the particles in the wave is in a certain constant direction which is at right angles to the direction of propagation of the wave. This phenomenon was called *polarization*.*

Young's explanation of polarization was grasped immediately and expounded further by Fresnel. Based on the concept of transverse waves, Fresnel presented three memoirs[(1.7)] to the French Academy in 1821 and 1822 discussing *double refraction* in crystals. He reasoned that light propagating in any direction through a crystal could be resolved into two plane-polarized components, each with a distinct velocity. Lacking a theory for the transverse wave motion in aether at that time, he found from a purely geometric argument that the two velocities must be the roots of a quadratic equation. He derived this equation by considering the relative displacements resulting from a wave motion in an aether. Thus, in a span of little more than one decade, all major difficulties inherent hitherto in the wave theory of light were resolved. The centuries-old question of the nature of light was answered by stating that light was a transverse motion of waves in elastic aether.

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*The polarization of sunlight upon reflection was first observed by Stephen Louis Malus (1775-1812) in 1809. Biographical accounts of Arago, Malus, Fresnel, and Young and their scientific contributions are contained in Ref. 1.6 with interesting notes and remarks added by the translators.

Although the equation for sound waves (longitudinal waves) in air, also in aether, was already developed by the end of the 18th century,* no general method had been developed for investigating the motion of an elastic aether possessing resistance to both volumetric change and distortion. In 1821, the year Fresnel presented his memoir on crystal optics, Claud Louis-Marie-Henri Navier (1785-1836) presented a molecular theory of an elastic body, giving an equation of motion for the displacement of a particle in elastic solids.(1.8) His theory immediately drew the attention of other members of the Academy who were searching for an equation governing the transverse motion of elastic aether. In the subsequent years, Augustine-Louis Cauchy (1789-1857) started from an entirely different point of view and developed what is known today as the "Mathematical Theory of Elasticity" (See Section 2). He not only introduced the notion of stress, strain, and stress-strain relations, but also correctly established the number of elastic constants, two for an isotropic solid and 21 for a crystal. The equation of motion in Cauchy's theory agrees with Navier's if the bulk modulus equals 5/3 of the shear modulus of the solid.†

Cauchy's theory was contained in a publication in 1828.(1.11) In the same year, Siméon Denis Poisson (1781-1840) succeeded in solving the differential equation of motion for an elastic solid by decomposing the displacement into an irrotational and a circuital (equivoluminal)

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*Jean le Rond D'Alembert (1717-1783) developed the partial differential equation for a vibrating string in 1750. The same equation was derived as a limiting case of a string of beads by Joseph Louis Lagrange (1736-1813) in 1759.

†See Refs. 1.9 and 1.10 for histories of elasticity theory and concise biographical sketches of the scholars' contributions to that theory.

part, each part being a solution of a wave equation.[1.12] Poisson's analysis has been followed to this day in studying wave motions in solids (see Section 2), but his finding of two waves in solids created a new difficulty in the wave theory of light. For if the illuminous aether behaved like an elastic solid, his analysis showed that two waves* instead of one should be visible.

A multitude of modifications in the elastic solid theory were proposed afterwards. In 1837, George Green (1793-1841) used energy and variational principles to derive the equation of motion and correctly established the boundary conditions at the surface of an elastic solid.[1.13] His work stimulated James MacCullagh (1809-1847) to postulate in 1839 a solid of which the potential energy depended only on the rotation of a volume element.[1.14] The equation of motion so derived has the form $\mu\nabla \times \nabla \times \mathbf{u} = \rho\partial^2\mathbf{u}/\partial t^2$, $\nabla\cdot\mathbf{u} = 0$, where $\mathbf{u}$ is the vector displacement, μ the shear rigidity and ρ the density. Since in his theory, the longitudinal wave does not exist and the light wave only propagates with one speed, $(\mu/\rho)^{\frac{1}{2}}$, MacCullagh for the first time really solved the problem of devising a medium whose vibrations, calculated in accordance with the established laws of mechanics, should have the same properties as the vibrations of light.†

The elastic solid theory of light was soon replaced by the electromagnetic theory which had been developing independently for over a century. In a series of papers capped by the memoir "A Dynamic Theory

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*The two waves are designated as P- and S-waves in this volume.

†In terms of electromagnetic theory, $\mathbf{u}$ in MacCullagh's equations corresponds to the magnetic field vector, $\mu\nabla \times \mathbf{u}$ to the electric field vector.

of Electromagnetic Field"* read to the Royal Society in 1864,[1.15] James Clerk Maxwell (1831-1879) presented a unified theory of electromagnetism and concluded that "light itself (including radiant heat, and other radiation if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws." The experimental confirmation of Maxwell's theory in 1888 by Heinrich Rudolf Hertz (1857-1894)[1.16] left no doubt that light was not an elastic wave. However, that did not stop the most illustrious scientists from analyzing the light wave as the propagation of a disturbance in an elastic aether. Toward the end of the 19th century, many important contributions on the diffraction of light--by Gustave Robert Kirchhoff (1824-1887), Lord Rayleigh (John William Strutt, 1842-1919), Horace Lamb (1849-1934), and others--were based upon the theory of elastic waves in solids.

1.2. *Diffraction and Scattering of Light*†

After the elastic solid theory for light was developed, it seemed natural to employ it to investigate the phenomenon of diffraction. The first attempt was made by George Gabriel Stokes (1819-1903) in 1849 when he presented the memoir "On the Dynamical Theory of Diffraction" to the Cambridge Philosophical Society.[1.17] Following Poisson's approach to initial value problems associated with the wave equation, Stokes derived the general solutions of the dynamic equations for the propagation of a disturbance in an elastic medium. He assumed that the disturbance was produced by a given initial disturbance which was

*Part of the corresponding statical theory was developed by M. W. Weber and C. Neumann in 1858.

†The observational diffraction phenomena together with a history of their discovery are described in Ref. 1.19.

confined to a finite portion of the medium. When light is diffracted by an aperture in a screen, each element of the aperture acts like a source which generates secondary waves. Stokes applied his solution to determine the disturbance corresponding to the secondary waves, and he was able to show the polarization and magnitude of the diffracted light at a point far away (when compared with the wavelength) from the screen.

Stokes' paper and his continuous interest in the light wave,(1.18) plus an experimental discovery by Tyndall,* led Rayleigh to investigate the diffraction of light by small particles and to provide the answer to why the sky is blue.

Starting in 1871, Rayleigh discussed in a sequence of papers the scattering of light by small particles.† It should be noted that by that time the electromagnetic wave theory of light was beginning to be accepted and the difference (or resemblance) between a sound wave (an elastic wave) and a light wave was understood. Thus, as far as the mathematical analysis was concerned, light could be treated as either an electromagnetic wave or an elastic wave. Using the elastic solid theory, Rayleigh found the important law of scattering in 1871:(1.22)

> When light is scattered by particles which are very small compared with any of the wave lengths, the ratio of the amplitudes of the vibrations of the scattered and

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*In 1868 Tyndall observed that when a condensed light beam passed through a mixture of air and hydrochloric acid, a cloud was formed which passed in color from the deepest violet through blue. Tyndall remarked in his notebook, "Connect this blue with the colour of the sky." See Ref. 1.20.

†Rayleigh's contributions to the scattering of sound and light waves have been reviewed by Twersky in Ref. 1.21.

> incident light varies inversely as the square of the wave length, and the intensity of the lights themselves as the inverse fourth power.

This law was discovered from a simple dimensional analysis of the wavelength, the amplitude and the size of the particles. It was verified with a mathematical analysis based upon Stokes' work by considering the scattered light as being emitted from a body force in an elastic medium. Since blue has a shorter wavelength in the visible light spectrum, when sunlight is scattered by fine particles (air molecules) in the sky, the blue color with its dominant intensity prevails.

By treating the secondary waves as an emission from a body force in a homogeneous solid, the scattering effect as a consequence of the difference between the refractive power of two media is attributed to a change of density and not to a difference of rigidity. To attack the problem more generally, Rayleigh later assumed a body source in an isotropic but inhomogeneous elastic solid and thus included the difference of rigidity as well as density in the analysis. The result obtained substantiated his law of scattering.(1.23)

Rayleigh followed Stokes' approach to diffraction until 1872 when he treated in detail the scattering of waves (sound) by a spherical obstacle with a finite radius.(1.24) This paper is most important because, aside from its mathematical rigor for a difficult problem, it set the tone for many subsequent analyses of surface scattering. In this paper a velocity-potential of the form $\varphi = \exp\,[ik(x + ct)]$ is assumed and expanded in a series of spherical harmonics. "The whole motion external to the sphere may be divided into two parts; that belonging to the plane waves supposed to be undisturbed, and secondly a motion due

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