

HANDBOOK OF SINGLE-PHASE CONVECTIVE HEAT TRANSFER

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PREFACE

The field of heat transfer has grown enormously in the last 20 years with the explosion of scientific and engineering research. It has increased tremendously the depth of our understanding. It is no longer possible for a single individual to be intimately familiar with and/or be an expert in even some major subfields of heat transfer. One such subfield of great industrial importance is single-phase convective heat transfer. This is the subject that we have tried here in considerable depth with the dedicated effort of 25 specialists.

This handbook is intended to furnish the latest design and research information in the area of single-phase convective heat transfer to practicing engineers, researchers, academicians, and students. It consists of 22 chapters, a brief description of which is provided next.

Chapter 1. This chapter provides the reader with basic concepts and fundamentals of heat transfer. Four general laws are stated in terms of a system, and then control-volume forms are given. Particular laws of heat transfer are stated. Governing equations of convective heat transfer are formulated and are presented in tabular form for rectangular, cylindrical, and spherical coordinates. Boundary-layer approximations for laminar and turbulent flow are presented.

Chapter 2. This chapter deals with external flow forced convection. Fundamental equations in general form and definitions are presented first. Then, Reynolds equations for turbulent flow are described; reduced forms for inviscid flow and viscous flow are given. Summaries of the common turbulent models are presented. The flow over a flat plate and in other geometries is discussed; important correlations for heat transfer and friction coefficients are provided. General formulas and data correlations for use in preliminary design and as benchmark checks for computer codes are also discussed in this chapter. In addition, the capabilities of computational procedures for forced convection over external surfaces are also discussed.

Chapter 3. This chapter presents an up-to-date compilation of analytical solutions for laminar fluid flow and forced convection heat transfer in circular and noncircular ducts. The solutions are presented for four types of laminar flows of Newtonian fluids, viz., fully developed, hydrodynamically developing, thermally developing, and simultaneously developing flows. The most solutions are given in terms of mathematical expressions and in graphical form to elicit the general trends. In all, results are presented for 70 duct geometries covering a variety of thermal boundary conditions.

- Chapter 4.* In this chapter, the heat transfer and fluid flow characteristics of turbulent flows in ducts are considered. The turbulent pressure drop and heat transfer for various entry and wall surface conditions are presented. Other effects such as the influences of thermal boundary conditions, entrance shape, high velocity, porous walls, Prandtl number, body force, and internal heat generation on turbulent forced convection are discussed. Turbulent flows in planar ducts, rectangular ducts, and other-shaped ducts are presented. Various turbulence models are discussed. Correlations covering a number of geometries and conditions for turbulent flow in ducts are provided, along with constraints, if any, on their practical application.
- Chapter 5.* Based on an extensive literature search, dimensionless heat transfer and flow friction design data are presented for curved ducts with circular, square, and rectangular cross sections. These design data are based on the theoretical analyses and experimental measurements on laminar and turbulent flows of Newtonian and non-Newtonian fluids through curved ducts. The results are presented in terms of design correlations, graphs, or tables.
- Chapter 6.* This chapter deals with convective heat transfer in cross flow. Correlations for the local and average heat transfer coefficients of single tubes and bodies for cross flow are presented, and the factors influencing heat transfer are discussed. Heat transfer from smooth and rough tube bundles, and the drag of smooth and yawed tube bundles are treated, as well as heat transfer from finned tube bundles. Heat transfer correlations are presented in tabular forms. Extensive design information for convective heat transfer in cross flow is also provided in graphical form.
- Chapter 7.* Longitudinal flow over tube or rod bundles is common in most fuel elements of nuclear power reactors. Other applications of this geometry are encountered in shell-and-tube heat exchangers, boilers, condensers, etc. This chapter provides information on heat transfer and friction coefficients for laminar, transitional, and turbulent flow over rod bundles for various conditions. Various correlations useful to the designers are presented, and effects of spacers are discussed. Valuable information is provided in graphical forms.
- Chapter 8.* This chapter provides the reader with a short introduction to the fundamentals of liquid metal heat transfer, followed by detailed discussions of laminar and turbulent liquid metal heat transfer correlations. Thermal entry lengths, variable fluid properties, and natural convection are also treated in this chapter. Flows in various geometries, including round pipes, annuli, parallel plates, and various tube-bank geometries, are also covered. Both laminar and turbulent entry-length correlations are presented. Correlations covering a number of plate geometries in natural convection heat transfer are presented, as well as the correlation for heat transfer from horizontal cylinders.
- Chapter 9.* This chapter discusses the convective heat transfer with electric and magnetic fields. The important basic concepts of electrohydrodynamics (EHD) and of magnetohydrodynamics (MHD) are presented. EHD in external boundary layers and in confined flows is treated. The experimental and mathematical limitations of the existing literature have been emphasized. Governing equations and dimensionless groups are presented. The basic physics of magnetic field effects in electrically

conducting liquids is discussed. MHD in confined flows, in external flows, and in natural convection are presented.

Chapter 10. Bends and fittings are most commonly used in pipelines, for which considerable pressure drop information has been summarized in the literature. A first attempt is made here to compile the available heat transfer information for bends with 90° , 180° , and other angles. Experimental and theoretical results for laminar and turbulent flow friction factors and Nusselt numbers are presented for bends having circular, square, and rectangular cross sections.

Chapter 11. This chapter is mainly concerned with the transient response of duct flows. The parallel-plate channel and circular duct, which are the two commonly encountered geometries in practice, are considered with both laminar and turbulent flows. The results of transient laminar forced convection in ducts for a step change in wall temperature and in wall heat flux are presented. Transient laminar forced convection in circular ducts with arbitrary time variations in wall temperature and with unsteady flow is also considered. The chapter also discusses transient turbulent forced convection in circular ducts with a step change in wall temperature, and in parallel-plate channels for a step change in wall temperature and wall heat flux.

Chapter 12. This chapter presents the basic considerations for the study of natural-convection heat transfer. The governing equations, along with their important simplifications, are presented to indicate the dimensionless parameters that arise and the basic nature of the transport process. Laminar natural convection over flat surfaces, cylinders, and spheres is discussed in detail, and the resulting heat transfer expressions presented. Transient effects and turbulent flow are outlined, since many practical problems involve these effects. Recommended empirical correlations for a variety of external natural convection heat transfer processes are given, along with the constraints, if any, on their application to physical problems.

Chapter 13. This chapter provides important basic information on the physics of the natural convection phenomena in enclosures; the formalism of the mathematical formulation of the natural convection problem; the available solution techniques; some significant results in the field, including both theoretical and experimental data and their correlation; and a brief description of recent studies of basic natural convection phenomena interacting with other heat transfer processes in an enclosure. Some emphasis is placed on the modern development in this field, including numerical and experimental techniques, laminar and turbulent flows, and two-dimensional and three-dimensional phenomena. Areas of future research are also delineated.

Chapter 14. This chapter deals with mixed convection in external flows. Results on the local Nusselt number are presented for various flow configurations, and instability studies conducted for these flows are described. This chapter summarizes and presents comprehensive correlations for the local and average Nusselt numbers that cover the entire mixed-convection regime, from pure forced convection to pure free convection, for various flow configurations of engineering interest, and for laminar as well as turbulent flows under the heating conditions of uniform wall temperature (UWT) and uniform surface heat flux (UHF). The instability characteristics of

laminar mixed convection along flat plates, with regard to both wave and vortex instability, are also summarized.

Chapter 15. This chapter discusses combined free and forced convection in internal flow. The best available information is summarized, and correlations are presented for ducts in the vertical and horizontal orientations. Results are categorized according to laminar, transitional, and turbulent flow, and according to duct geometry, heating conditions, buoyancy-assisted or buoyancy-opposed flow, and whether or not the flow is hydrodynamically or thermally fully developed. The effects of secondary flow in horizontal ducts are indicated, and the contrasting influences of buoyancy forces on mixed convection in laminar flow and in turbulent flow are discussed.

Chapter 16. This is an up-to-date review of the literature on convective heat transfer in porous media. It emphasizes *scale analysis* as a means of identifying and sorting out the proper heat transfer scales of forced and natural convection through porous media. The engineering heat transfer correlations assembled in this chapter are all scaling-correct correlations, i.e., their analytical forms are the ones recommended by the appropriate scale analysis. In many instances, classical experimental and numerical results are here condensed into scaling-correct correlations.

Chapter 17. This chapter discusses special heat transfer surface geometries that yield higher heat transfer coefficients than “plain” surfaces do. The major emphasis of the chapter is on forced convection of gases and liquids. The enhancement geometries covered include finned surfaces for gases and tube-side enhancement for laminar and turbulent flow of gases. Design equations are provided to calculate the heat transfer coefficient and friction factor for all of the enhancement geometries discussed. Performance Evaluation Criteria are described, which are used to calculate the performance improvement provided by an enhanced surface, relative to that of a plain surface for specific design objectives and operating constraints.

Chapter 18. This chapter deals with the effect of temperature-dependent fluid properties on convective heat transfer. Correlations for heat transfer and friction coefficients which take into account temperature-dependent properties are described for viscous liquids and gases both for laminar and turbulent flow. Many solutions are surveyed which have been proposed to describe these effects as they occur in practical applications. Tabular forms of correlations for heat transfer and friction coefficients for turbulent flow in ducts with variable physical properties are provided. The particular characteristics of fluids at supercritical pressure are described, the role of property variations in influencing flow and heat transfer is discussed, and correlations of supercritical forced convection are presented. The chapter also surveys and summarizes the available solutions and experimental studies on temperature-dependent effects as they occur in natural convection.

Chapter 19. When heat transfer by radiation is of the same order of magnitude as convection, radiation and convection need to be treated simultaneously. In such situations, in the analysis of the problem distinction should be made between cases involving a completely transparent fluid and a fluid that absorbs, emits, and perhaps scatters radiation. In this chapter, radiation transfer in nonparticipating and participating media is discussed for such cases, and typical heat transfer results are

presented on the effects of radiation parameters such as the conduction-to-radiation parameter, optical thickness, surface reflectivity, and single-scattering albedo on the Nusselt number and temperature distribution for forced convection over a flat plate and inside a parallel-plate duct.

Chapter 20. The basic definitions of non-Newtonian fluids and their rheological properties are presented. Methods of measuring these rheological properties are described. Analyses are made of the flow and heat transfer characteristics of non-Newtonian flows in both ducts and over external surfaces in laminar and turbulent flows. Both free and forced convection are considered. Emphasis has been placed on the presentation of results in a form suitable for engineering design calculations.

Chapter 21. This chapter is concerned with the fouling of heat transfer surfaces; a reasonable balance between gas-side and liquid-side fouling is maintained throughout. The treatment is design-oriented and includes tabulated values of liquid-side and gas-side fouling factors, along with properties of representative fouling deposits. Detailed procedures are presented for taking into account the effects of fouling on both pressure drop and heat transfer. A considerable amount of material on techniques available to combat fouling is also presented and discussed.

Chapter 22. Tables are given of the thermophysical properties: specific volume, specific enthalpy, specific entropy, specific heat at constant pressure, viscosity, thermal conductivity, and Prandtl number as a function of temperature. For saturated liquid and vapor, these are given for air, carbon dioxide, cesium, lithium, mercury, potassium, Refrigerant 12, Refrigerant 22, rubidium, sodium, and steam. Similar saturated tables for ice-water-steam are given as a function of pressure. Wherever possible, the different tables are presented at equal temperatures and temperature increments to facilitate comparisons in design and also computer programming of the data.

Most of the results of engineering utility are presented in terms of equations, tables, and/or figures. Where appropriate, most results are presented in nondimensional form; and the dimensional results are presented in two unit systems—the International System (SI) and the U.S. Customary System (USCS)—to allow for the worldwide use of this handbook. Although the nomenclature is listed at the end of each chapter, the editors have made a diligent effort to make most of the symbols common throughout the handbook.

The success of this handbook rests with the quality of the information provided by the contributors. We are grateful to them for providing excellent material in a timely manner and within the length limitations. The editorial staff at John Wiley has provided superb cooperation and continued support throughout the compilation of this handbook, from the inception of the idea to the final production. In particular, we sincerely appreciate the cordial and prompt support of Mr. Frank Cerra on all matters of concern. We also gratefully acknowledge the outstanding editorial work of Mr. Joseph Fineman, the efficient work and outstanding cooperation of Lisa VanHorn during the production and the excellent figures prepared by Mr. Ali Akgüneş of the Middle East Technical University, Ankara, Turkey. We also wish to thank the professional staff of John Wiley & Sons, Inc. who were involved with the publication of this Handbook. The first editor would like to express his appreciation to Norman Einspruch, Dean of the College of Engineering at the University of Miami for suggesting the idea of preparing a handbook.

Every effort has been made by the editors to minimize typographical errors. Each chapter has been independently reviewed by other experts in the field to enhance the quality and the correctness. If any errors come to the attention of readers, we would greatly appreciate being informed of them so that they can be eliminated in the subsequent printing. Of course, we would also appreciate any more general comments related to any chapter.

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CONTENTS

1. BASICS OF HEAT TRANSFER
S. Kakaç and Y. Yener
2. EXTERNAL FLOW FORCED CONVECTION
R. H. Pletcher
3. LAMINAR CONVECTIVE HEAT TRANSFER IN DUCTS
R. K. Shah and M. S. Bhatti
4. TURBULENT AND TRANSITION FLOW CONVECTIVE HEAT TRANSFER IN DUCTS
M. S. Bhatti and R. K. Shah
5. CONVECTIVE HEAT TRANSFER IN CURVED DUCTS
R. K. Shah and S. D. Joshi
6. CONVECTIVE HEAT TRANSFER IN CROSS FLOW
A. A. Žukauskas
7. CONVECTIVE HEAT TRANSFER OVER ROD BUNDLES
K. Rehme
8. CONVECTIVE HEAT TRANSFER IN LIQUID METALS
C. B. Reed
9. CONVECTIVE HEAT TRANSFER WITH ELECTRIC AND MAGNETIC FIELDS
F. A. Kulacki, J. H. Davidson, and P. F. Dunn
10. CONVECTIVE HEAT TRANSFER IN BENDS AND FITTINGS
S. D. Joshi and R. K. Shah
11. TRANSIENT FORCED CONVECTION IN DUCTS
Y. Yener and S. Kakaç
12. BASICS OF NATURAL CONVECTION
Y. Jaluria
13. NATURAL CONVECTION IN ENCLOSURES
K. T. Yang
14. MIXED CONVECTION IN EXTERNAL FLOW
T. S. Chen and B. F. Armaly
15. MIXED CONVECTION IN INTERNAL FLOW
W. Aung

16. CONVECTIVE HEAT TRANSFER IN POROUS MEDIA
A. Bejan
 17. ENHANCEMENT OF SINGLE-PHASE HEAT TRANSFER
R. L. Webb
 18. THE EFFECT OF TEMPERATURE-DEPENDENT FLUID
PROPERTIES ON CONVECTIVE HEAT TRANSFER
S. Kakaç
 19. INTERACTION OF RADIATION WITH CONVECTION
M. N. Özışık
 20. NON-NEWTONIAN FLUID FLOW AND HEAT TRANSFER
T. F. Irvine, Jr. and J. Karni
 21. FOULING WITH CONVECTIVE HEAT TRANSFER
W. J. Marner and J. Sutor
 22. THERMOPHYSICAL PROPERTIES
P. Liley
- INDEX

1

BASICS OF HEAT TRANSFER

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- 1.1 Introduction**
- 1.2 Modes of Heat Transfer**
- 1.3 Statements of General Laws**
 - 1.3.1 Law of Conservation of Mass
 - 1.3.2 Newton's Second Law of Motion
 - 1.3.3 First Law of Thermodynamics
 - 1.3.4 Second Law of Thermodynamics
- 1.4 Statements of Particular Laws**
 - 1.4.1 Fourier's Law of Heat Conduction
 - 1.4.2 Newton's Law of Cooling
 - 1.4.3 Stefan-Boltzmann Law of Radiation
- 1.5 Governing Equations of Convective Heat Transfer**
 - 1.5.1 Continuity Equation
 - 1.5.2 Equations of Motion
 - 1.5.3 Energy Equation
- 1.6 Boundary-Layer Approximations — Laminar Flow**
- 1.7 Turbulent Flow**
- 1.8 Final Remarks**
- Nomenclature**
- References**

1.1 INTRODUCTION

Convective heat transfer is the study of heat transport processes between the layers of a fluid when the fluid is in motion and/or between a fluid in motion and a boundary surface in contact with it when they are at different temperatures.

Heat is that form of energy which crosses the boundary of a thermodynamic system by virtue of a temperature difference existing between the system and its surroundings. *Heat flow* is vectorial in the sense that it is in the direction of negative temperature gradients, i.e., from higher toward lower temperatures.

The science of heat transfer is based upon foundations comprising both theory and experiment. As in other engineering disciplines, the theoretical part is constructed from one or more *physical* (or *natural*) laws. The physical laws are statements, in terms of various concepts, which have been found to be true through many years of experimental observations. A physical law is called a *general law* if its application is independent of the medium under consideration. Otherwise, it is called a *particular law*. There are, in fact, the following four general laws among others upon which all the analyses concerning heat transfer, either directly or indirectly, depend:

1. The law of conservation of mass
2. The first law of thermodynamics
3. The second law of thermodynamics
4. Newton's second law of motion

In addition to the general laws, it is usually necessary to bring certain particular laws into an analysis. Examples are Fourier's law of heat conduction, Newton's law of cooling, the Stefan-Boltzmann law of radiation, Newton's law of viscosity, the ideal-gas law, etc.

1.2 MODES OF HEAT TRANSFER

The mechanism by which heat is transferred in a heat exchange or an energy conversion system is, in fact, quite complex. There appear, however, to be three rather basic and distinct *modes* of heat transfer. These are *conduction*, *convection*, and *radiation*.

Conduction is the process of heat transfer by molecular motion, supplemented in some cases by the flow of free electrons, through a body (solid, liquid, or gaseous) from a region of high temperature to a region of low temperature. Heat transfer by conduction also takes place across the interface between two bodies in contact when they are at different temperatures.

The mechanism of heat conduction in liquids and gases has been postulated as the transfer of kinetic energy of the molecular movement. Transfer of thermal energy to a fluid increases its internal energy by increasing the kinetic energy of its vibrating molecules, and is measured by the increase of its temperature. Heat conduction is thus the transfer of kinetic energy of the more energetic molecules in the high-temperature region by successive collisions to the molecules in the low-temperature region.

Heat conduction in solids with crystalline structure, such as quartz, depends on energy transfer by molecular and lattice vibrations and free-electron drift. In general, energy transfer by molecular and lattice vibrations is not as large as the transfer by free electrons. This is the reason why good electrical conductors are always good heat

conductors, and electrical insulators are usually good heat insulators. In the case of amorphous solids, such as glass, heat conduction depends only on the molecular transport of energy.

Thermal radiation, or simply *radiation*, is heat transfer in the form of electromagnetic waves. All substances, solid bodies as well as liquids and gases, emit radiation as a result of their temperature, and they are also capable of absorbing such energy. Furthermore, radiation can pass through certain types of substances (called *transparent* and *semitransparent* materials) as well as through vacuum, whereas for heat conduction to take place a material medium is absolutely necessary.

Conduction is the only mechanism by which heat can flow in *opaque* solids. Through certain transparent or semitransparent solids, such as glass and quartz, energy flow can be by radiation as well as by conduction. With gases and liquids, if there is no observable fluid motion, the heat transfer mechanism will be conduction (and radiation). However, if there is macroscopic fluid motion, energy can also be transported in the form of internal energy by the movement of the fluid itself. The process of energy transport by the combined effects of heat conduction (and radiation) and the movement of fluid is referred to as *convection* or *convective heat transfer*.

1.3 STATEMENTS OF GENERAL LAWS

In the following sections, the four general laws referred to in Sec. 1.1 are stated first in terms of a system, and then the control-volume forms are given.

1.3.1 Law of Conservation of Mass

A *system* is any arbitrary collection of matter of fixed identity bounded by a closed surface, which can be a real or an imaginary one. All other systems that interact with the system under consideration are known as its surroundings. The law of conservation of mass simply states that, in the absence of any mass-energy conversion, the mass of a system remains constant. Thus, for a system

$$\frac{dm}{dt} = 0, \quad \text{or} \quad m = \text{constant} \quad (1.1)$$

where m is the mass of the system.

A *control volume* is any defined region in space, across the boundaries of which matter, energy, and momentum may flow, within which matter, energy, and momentum storage may take place, and on which external forces may act. Its position and/or size may change with time. Consider now a control volume fixed in space and of fixed size and shape, as illustrated in Fig. 1.1. Matter (e.g., a fluid) flows across its boundaries. The law of conservation of mass for this control volume can then be expressed as

$$\frac{\partial m_{c.v.}}{\partial t} = \dot{m}_{in} - \dot{m}_{out} \quad (1.2a)$$

where $m_{c.v.}$ is the instantaneous mass inside the control volume, and \dot{m}_{in} and \dot{m}_{out} are the instantaneous mass flow rates into and out of the control volume, respectively. Equation (1.2a) can also be written as [1, 2]

$$\frac{\partial}{\partial t} \int_{c.v.} \rho \, dV = - \int_{c.s.} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \quad (1.2b)$$

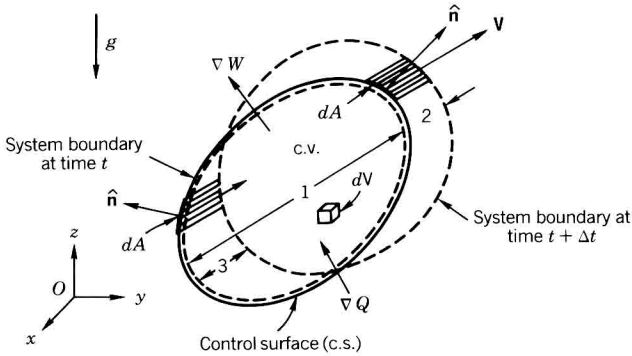


Figure 1.1. Flow through a control volume.

where dV is an element of the control volume, ρ is the local density of that element, and c.v. designates the control volume fixed in space and bounded by the control surface (c.s.). Finally, \mathbf{V} is the velocity vector, and $\hat{\mathbf{n}}$ is the outward-pointing unit vector normal to the control surface.

The control-volume form of the law of conservation of mass states that the net rate of mass flow into a control volume is equal to the time rate of change of mass within the control volume.

1.3.2 Newton's Second Law of Motion

Newton's second law of motion states that the net force \mathbf{F} acting on a system in an inertial coordinate system is equal to the time rate of change of the total linear momentum \mathbf{M} of the system; that is,

$$\mathbf{F} = \frac{d\mathbf{M}}{dt} \tag{1.3}$$

which, for the control volume shown in Fig. 1.1, reduces to [1, 2]

$$\mathbf{F} = \frac{\partial}{\partial t} \int_{c.v.} \mathbf{V}\rho \, dV + \int_{c.s.} \mathbf{V}\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{1.4}$$

This result is usually called the *momentum theorem* or the *law of conservation of linear momentum* and states that the net force acting instantaneously on a control volume is equal to the time rate of change of linear momentum within the control volume plus the net flow rate of linear momentum out of the control volume.

Equation (1.4) is a vector relation. Referred to the rectangular coordinates x , y , and z , the component in the x direction, for example, can be written as

$$F_x = \frac{\partial}{\partial t} \int_{c.v.} u\rho \, dV + \int_{c.s.} u\rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \tag{1.5}$$

where u and F_x are the x components of the velocity vector \mathbf{V} and the force vector \mathbf{F} , respectively.