

**ELECTRICAL SCIENCE SERIES**

# ANALYSIS OF REFLECTOR ANTENNAS

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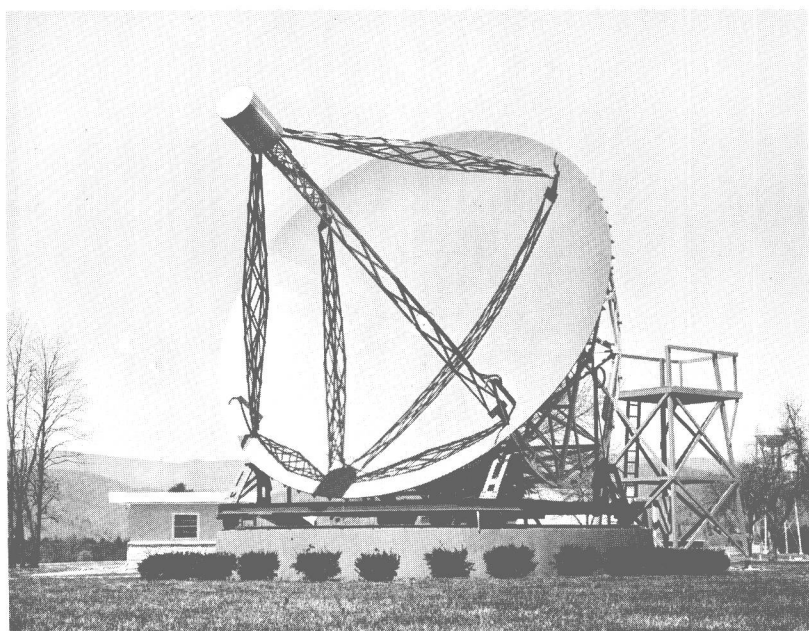
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## ANALYSIS OF REFLECTOR ANTENNAS



Reber's 31.5-foot diameter paraboloidal reflector constructed in 1937 for early radio astronomy experiments. (Courtesy of W. E. Howard, III, National Radio Astronomy Observatory.)

## Preface

Every engineer working in the general area of reflector-antenna analysis or author writing on this subject is indebted to the classic textbook "Microwave Antenna Theory and Design" edited by S. Silver. "Analysis of Reflector Antennas" is not intended to replace that book, but to supplement it with some of the developments that have taken place in the two decades or so since its publication. Some of these developments have been, for example "low-noise" antennas, Cassegrainian and shaped dual-reflector systems, and computer-aided analysis and design.

The work that has resulted in the present book had as its objective the performance optimization of large, paraboloidal reflector-antenna systems used in the Deep Space Instrumentation Facility (DSIF). The program has led to a number of innovations in the state of the art, including high-aperture-efficiency, low-noise, focal-point feed systems, improved Cassegrainian antenna systems, multimode feedhorns, and nutating-subdish systems. In the process of developing such devices, many different types of analysis were performed to assist in understanding, guiding, and even replacing the experimental work.

It would be difficult to include all recent reflector-antenna developments, even in a volume several times the size of this one. The authors have not undertaken to do so but instead, sometimes arbitrarily, have done extensive material selection. No attempt has been made to include specific material on other principal reflector shapes such as spherical since, as will be discussed in Chapter I, many of these reflectors have characteristics similar to the paraboloid, and there is the additional question as to whether the apparently superior scanning performance of other profiles can

be easily achieved in practice. It should be emphasized, however, that the analytical techniques described can be applied to virtually any reflector profile.

This book is intended for use primarily by the research and development engineer who has occasion to be concerned with the analysis of reflector-antenna systems. There are three technical chapters: Chapter II, Equations of the Electromagnetic Field; Chapter III, Performance Analysis; and Chapter IV, Computer-Aided Analysis and Design. In general, the material included is not readily available in the open literature but may have appeared in limited-distribution technical reports or JPL Space Program Summaries (SPS). In some cases continuity is achieved by the inclusion of published or, as in Chapter II, classical material. This book is not intended to be used as a design handbook, but rather to assist in understanding how design data have been and can be derived. Design data for conventional systems are available in several good design handbooks such as "Antenna Engineering Handbook" edited by H. Jasik, Chapters 10 and 12, and Reflecting Systems by L. K. DeSize and J. F. Rumsey in "Microwave Scanning Antennas," Vol. I, edited by R. C. Hansen. Design data for less-conventional, reflector-antenna systems are often available in the references provided. If design data for a particular configuration are unavailable, the data must be generated with analytical techniques such as those described in Chapters II-IV.

The authors have written this volume in order to document recent work in the analysis and design of large reflector-antenna systems, work that has been performed primarily (but not exclusively) at the California Institute of Technology Jet Propulsion Laboratory (JPL) while W.V.T.R., a consultant at JPL, served on the electrical engineering faculty at the University of Southern California.

## Acknowledgments

The authors gratefully acknowledge numerous helpful suggestions from many colleagues in the preparation of the manuscript. These have included Professor J. Meixner, Institute of Theoretical Physics in Aachen, Germany; Dr. A. C. Ludwig, whose name appears frequently throughout the references of this volume; Mr. M. S. Katow, Mr. T. Y. Otoshi, and Mrs. R. A. Rodriguez for invaluable aid with the burden of editing. The authors wish especially to acknowledge the indirect support provided by the National Aeronautics and Space Administration's Office of Tracking and Data Acquisition, and the NASA/JPL Deep Space Network. Without continuing encouragement provided by these organizations this volume could not have been undertaken. Finally, the authors wish to express particular appreciation to their wives, who frequently excused them from their familial responsibilities for the raising of nine young and very active children in order that the manuscript could be completed.



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The history of focusing reflector-antenna systems has been characterized by a highly variable level of general interest. In the period prior to World War II this type of antenna had not achieved wide application, primarily because of a lack of suitable microwave signal sources and sensitive receiving equipment. During World War II the military value of radar devices stimulated a vigorous investigation of microwave reflector antennas, from both the experimental and theoretical points of view. This war-time work in the United States [1] was of such thoroughness as to depress significant further investigations in the decade following the war. During that decade a major microwave-antenna effort was directed toward development of the phased array, a relatively novel approach to the “pencil-beam” requirement. However, by the mid-1950s phased array devices had lost much of their initial attraction. Despite their great potential, many workers found that satisfactory performance was realized only by great effort and expense. In contrast, the paraboloidal reflector antenna was easily designed and almost always performed reasonably well.

In the period since the mid-1950s, several factors have combined to re-create a high level of interest in focusing reflector antennas. The radio astronomy community became interested in large, economical, versatile antennas which could be readily modified to new frequency bands. The reflector-antenna impetus thus provided was strongly accentuated by the growing require-

ment for deep-space communications. These stringent requirements were satisfied by combining the large, paraboloidal ground antenna with sensitive low-noise receivers, a combination for which no satisfactory substitute has yet been found. (See Fig. 1.1.)

### 1.1 The Collection and Collimation of Electromagnetic Energy

Antennas with transverse dimensions that are large compared to the wavelength of operation generally fall into the category of pencil-beam antennas. Because of their large wavelength size, such antennas can be designed using approaches borrowed from optical systems: specifically, the lens and the reflector. Microwave lens systems have advantageous characteristics for special applications. However, they are not generally useful in pencil-beam applications for the same reasons that render them unattractive for large optical systems: i.e., loss, weight, bandwidth limitations, interface matching problems, and design complexity.

On the other hand, the focusing reflector antenna is an obvious selection for pencil-beam applications. A concave reflecting surface is chosen such that an incident bundle of parallel rays is, upon reflection, directed to a localized region of space in which are located suitable receiving devices. Under the special circumstance that the ray bundle is parallel to the axis of a paraboloidal reflector, the reflected rays all pass through a single point (focus). Furthermore, all ray paths between a distant, axial point and the focus are equal in length, resulting in a potentially broad-band system. With the constraint of essentially constant ray path length (excluding stepped or zoned reflector designs), all focusing reflectors must be very nearly paraboloidal to assure (1) energy concentration in the receive mode, and (2) energy collimation in the transmit mode. The paraboloid is thus a “natural” profile for focusing reflector antennas.\*

\* The paraboloidal reflector is a basic prototype for focusing reflectors in the sense that all focusing reflectors have similar scan losses [2].

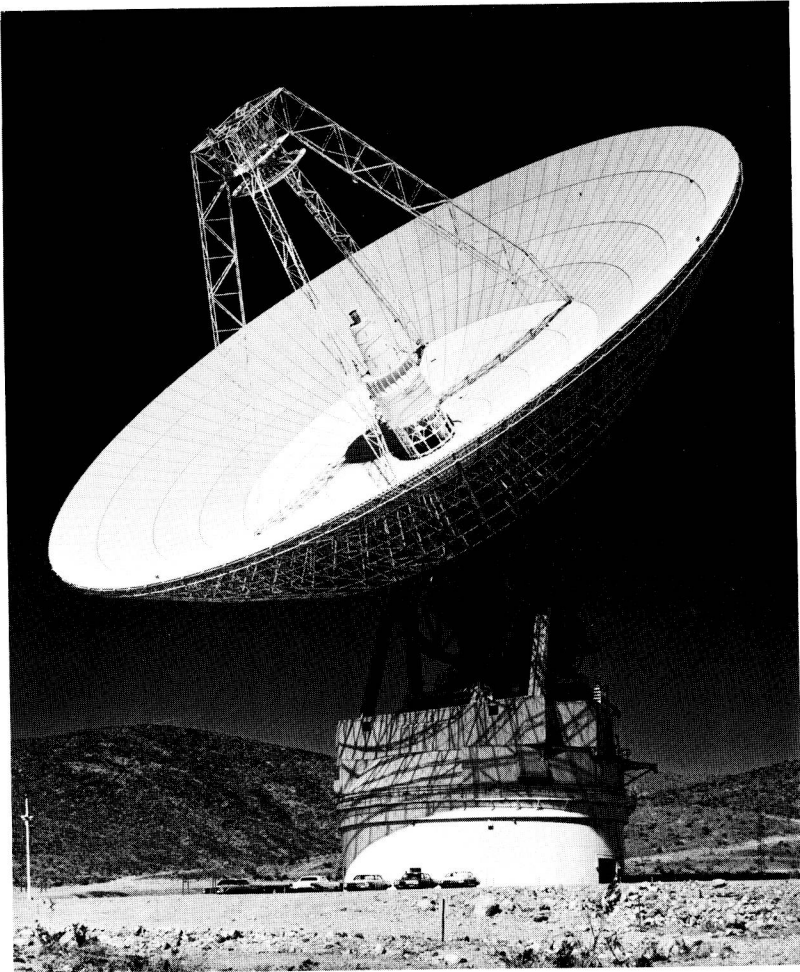


FIG. 1.1. NASA/JPL 210-ft-dia reflector antenna for deep-space communications.

Courtesy of the Jet Propulsion Laboratory, California Institute of Technology, sponsored by National Aeronautics and Space Administration Contract NAS 7-100.

## **1.2 Reflectors versus Electronically Scanned Phased Arrays**

For large ground-antenna systems, the paraboloidal reflector antenna is presently an economically optimized approach [3]. For this particular application the large aperture sizes involved make the costs of construction and material highly significant. Thus it is an economic necessity to construct the aperture of a single reflecting element, rather than a large number of specially constructed array elements.

In certain pencil-beam applications phased arrays are more attractive than paraboloidal reflectors. These special applications typically involve (1) reception and/or transmission of signals in two or more directions separated by at least ten beamwidths, and (2) concurrent reception and/or transmission in those widely separated directions at scan rates exceeding those which are mechanically feasible by moving a single large reflector. Unless both of these conditions exist, the reflector-antenna system or a combination of such systems is superior to the phased array from the standpoints of design simplicity, weight, versatility, and bandwidth potential [3].

A relatively new and powerful approach in reflector-antenna design has been studied by the National Aeronautics and Space Administration (NASA) in connection with the Applications Technology Satellite (ATS) series. In this approach, a highly directive paraboloidal reflector is excited by a multielement feed resembling a small quasiplanar array located in the focal region. Ruze [4] has shown that minimal beam-scan losses result from scans as far as five to ten beamwidths from the axis. The above type of multibeam feed can thus produce several hundred contiguous beams from the same paraboloidal reflector, each with essentially the full gain for that aperture.

## **1.3 Organization of the Material in This Book**

The technical material in this book is organized into three technical chapters. Chapter II contains a brief review of those

electromagnetic definitions and principles which the authors felt to be particularly relevant to the theory of reflector antennas. Maxwell's equations, the continuity equation, boundary conditions, and the wave equation are briefly stated, and solutions of the wave equation in rectangular and spherical coordinates are considered in some detail because of their importance in feed system analysis and the physical-optics approximations.

Because of the quasi-optical nature of many large antenna systems, the geometrical theories are treated in considerable detail. Geometrical optics is first derived as a high-frequency solution of the field equations, then interpreted in terms of ray paths and energy propagation characteristics, and finally applied to reflection from paraboloidal and hyperboloidal surfaces. Geometrical diffraction is discussed qualitatively, but the authors felt that a rigorous mathematical presentation of this subject, covered adequately in the references, would require excessive space, and consequently it was omitted from this monograph. However, two highly significant applications of geometrical diffraction theory to reflector antenna analysis are presented in the final section at the end of the chapter, where the various theories are discussed and compared.

The final third of Chapter II is devoted primarily to the scalar and vector integral theories. Particular emphasis is placed on the physical-optics approximations in the vector theory. Although the physical-optics integral formulas have been known for decades, they have, for the most part, lain dormant until the electronic computers of the 1960s were able to transform them from mathematical formalisms into useful numbers for analysis and design (cf. Chapter IV). Consequently, this integral technique is discussed in detail, in preparation for its frequent application in Chapters III and IV.

Techniques for the analysis and prediction of antenna performance are presented in Chapter III. The initial material of that chapter defines and describes the fundamental antenna properties: gain, directivity, capture area, aperture efficiency,

beam efficiency, polarization, phase center, and effective antenna noise temperature. Polarization properties are treated in considerable detail since this topic is not readily available in the literature in complete form. In succeeding sections of Chapter III the aperture efficiency and radiation pattern are analyzed as functions of the feed-system illumination. The effects of aperture blockage, backlobe interference, and aperture phase error are examined. The final material of the chapter is devoted to the effects of reflector mismatch on antenna performance, since these effects are particularly significant in Cassegrainian-system design.

Chapter IV deals with the uses of digital computers in the analysis and design of large, steerable reflector antennas. High-speed numerical techniques are required to evaluate many of the antenna performance formulas generated in the previous two chapters. Integral-equation techniques are mentioned, but only briefly, since such techniques have not yet achieved wide application to large-aperture antenna analysis. Eventually, however, it is anticipated that the integral equation or related techniques will successfully provide a virtually rigorous solution to the complete antenna problem. Until such a time that appropriate algorithms and/or generations of computers become available, the classical high-frequency techniques of physical optics will continue to serve as the "work horse" of large-aperture reflector-antenna analysis.

Numerical integration procedures are considered in some detail in Chapter IV. The advantages of the Romberg algorithm are stressed. Two methods are considered for the generation of reflector profiles in two-reflector systems. Shaped-reflector systems are currently receiving favorable attention as a solution of the high-aperture-efficiency problem. Considerable material in Chapter IV is devoted to the analysis of scattering from surfaces of revolution using the physical-optics formulas. These somewhat lengthy formulas have not appeared previously in the open literature. However, on many occasions engineers engaged in the



design of large antenna systems have personally sought them from the authors. They are reproduced in Section 4.241, and a FORTRAN listing for their evaluation is given in one of the references. (see Ludwig [12], Chapter IV).

The remaining material in the final chapter describes the application of the physical-optics formulas to such problems as gain and noise-temperature predictions, evaluation of truncation effects, and focal-region studies. A section is devoted to the numerical determination of phase centers from a table of experimental or theoretical field data. Finally, the important subject of mechanical tolerance theory is considered, with special attention given to the uses of computers in the mechanical design of steerable reflector antennas.

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